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Teaching optics

Comparison study of imaging quality of refracting, diffractive and hybrid single lenses

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This paper is of didactic value. A possibility of aberration correction as exemplified by simple cases of single refractive, diffractive and hybrid lenses is shown. The imaging quality assessment was carried out based on the point spread function, wave aberration and incohenent MTF. These functions were calculated numerically by evaluating the diffraction integral. In the authors opinion, these simple examples render it possible to show clearly the influence of the aberrations on the quality of the image produced by the optical elements operating on either the refraction or diffraction basis or both.

When teaching optics one faces difficult didactic problems *i.e.*, how to show the influence of the imaging system aberrations on the quality of image. The simplest optical element, which can be used to demonstrate the above problem, is a single lens. In the contemporary optics, the single lens can be realised on refraction, diffraction or hybrid bases, therefore a comparison study may be very instructive. This paper deals with all the three cases. For the sake of precision we start with the relevant definitions. The refractive lens is understood as a glass body of two spherical surfaces characterized by the curvatures R_1 and R_2 and a refractive index n (Fig. 1a). The diffractive lens is understood as a thin diffractive element with a microstructure corresponding to the pattern of the interference fringes occurring due to interference of two spherical waves of respective curvature radii z_α and z_β incident on a plane or spherical surface of the curvature radius R (Fig. 1b). Such a lens may be produced using holographic or synthetic techniques, and the magnitudes z_α and z_β are parameters describing the geometry of the diffractive structure. The hybrid lens is a combination of refractive and diffractive lenses (Fig. 1c), while the diffractive structure can be deposited either on the first or on the second refracting surface of the lens.

When dealing with the diffractive lenses their small sizes and little mass are usually mentioned as their merits. It is emphasized that the imaging quality of such a lens is high, especially for the object located on the axis when the imaging can be stigmatic. While taking into account the possibilities of the hybrid lens, it should be stressed that it has the merits of both refractive and diffractive lenses.

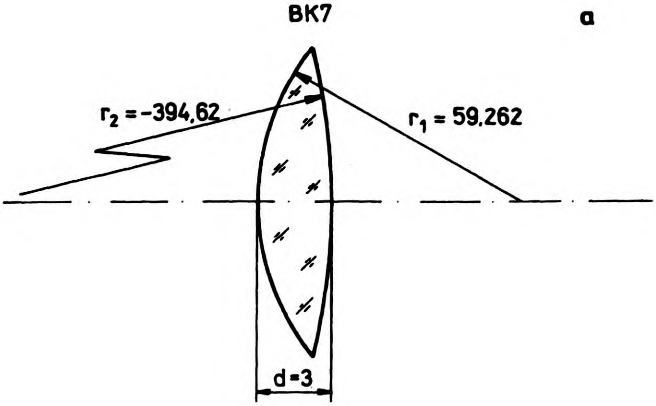

Fig. 1a

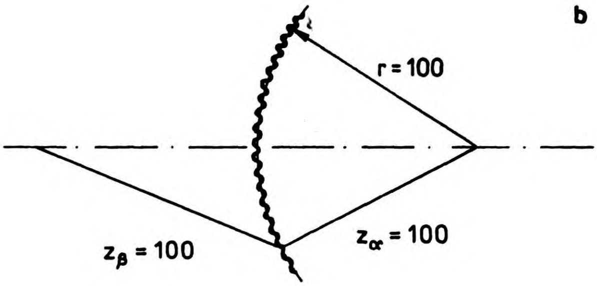

Fig. 1b

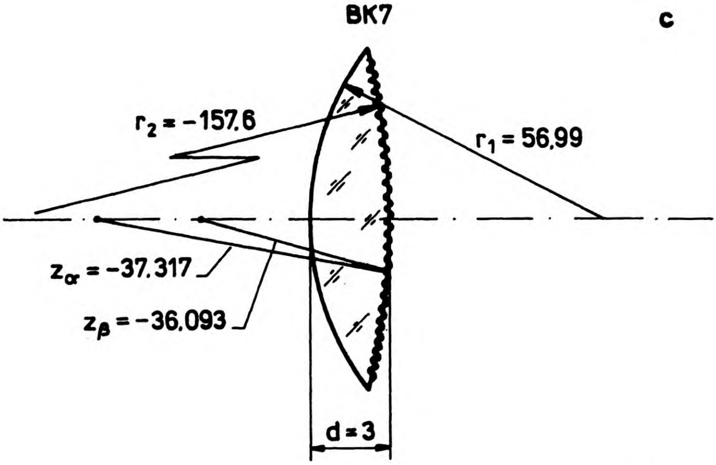

Fig. 1c

Fig. 1. Construction data for the lenses examined: a — classic (glass) lens, , b — holographic lens, c — hybrid lens

The purpose of this paper is to compare the imaging qualities of the aforementioned three types of optical elements. From the aberration theory of the third order it is well known that a single refractive lens can be entirely free of aberrations [1], [2]. Due to a suitably chosen ratio of R_1 to R_2 only one aberration, preferably the spherical aberration, can be minimized. It can be shown that if spherical aberration is minimized coma is also not great.

The diffractive lens allows the spherical aberrations to be fully corrected (by the proper choice of the parameters z_α and z_β), which is also true for coma. However, the correction of coma requires the diffraction structures to be deposited on a spherical substrate [3]–[5].

Similar correction can be achieved also for the hybrid lens. The diffractive structure can be deposited either on the first or the second refractive surface [6], [7].

Thus, it can be stated that both the diffractive lens and the hybrid lens enable achievement of aplanatic correction, while in the case of the refractive lens no aberration can fully be corrected.

In the present paper, we consider only monochromatic aberrations. In fact, the hybrid lens can be achromatic but then it is characterized by an enormously wide secondary spectrum. The other lenses can act only in the monochromatic light. We assume additionally that the illumination is incoherent and the lens is thin, while the entrance pupil coincides with the lens holder.

The quality of imaging will be compared taking account of three examples. Let us assume that both the focal length $f' = 100$ mm and the relative aperture 1:10 are the same in three analysed lenses, while the field of view does not exceed 0.05 rad, and the object is located at infinity.

The condition minimizing the spherical aberration for the refractive lens has the form [1]

$$\frac{R_2}{R_1} = \frac{n(n+1)}{2n^2-n-4}. \tag{1}$$

The construction parameters of the refractive lens, which result from this formula are given in Fig. 1a.

In the case of diffraction lens, the spherical aberration is corrected if one of the parameters z_α or z_β is equal to the object distance. In order to correct coma, the curvature radius of the surface, on which the diffraction structure is recorded, must fulfil the condition [3]

$$R = f'. \tag{2}$$

Hence, the construction parameters should be like those given in Fig. 1b.

Unfortunately, the conditions for aplanatic correction for the hybrid lens of spherical surfaces cannot be formulated in such a simple way. In Figure 1c, the construction parameters of such a lens are given taking advantage of the results reported in paper [7] without any justification of this choice.

In order to estimate the imaging quality, the methods offered by the geometrical optics can be used, *i.e.*, we plot the curves characterizing the particular Seidel aberrations or determine the corresponding spot diagram. Also, the diffraction model of imaging can be assumed and the wave aberration determined. Taking the latter as a basis, the intensity distribution in the aberration spot (in other words, the point spread function) can be defined. In the time of speedy personal computers of large memories, the solution of the due diffraction integral is not a problem. If we assume the isoplanatism the modulation transfer function characterizing the imaging of an extended object can be determined, provided that we know the point spread function.

The geometric criteria are of some significance, at least when designing an optimal refractive lens or an achromatic doublet; however, the diffraction critera seem to be more precise if we compare the quality of the imaging given by the well corrected optical systems. Therefore in this paper we use only the diffraction criteria.

In the first stage, we calculate the light intensity distribution in the meridional cross-section (*i.e.*, in the *YOZ* plane) of the aberration spot for the angles of the field of view increasing from 0 to $y/z = 0.05$. In Figure 2 some results referring to the field angle $y/z = 0.04$ are shown. Figure 2a shows the relevant results for the refractive lens, while in Fig. 2b,c the relevant results for the diffractive lens and the hybrid lens are presented. In each of the figures, the cross means the position of the Gaussian image, while a circle denotes a point in which the light intensity in the aberration spot is the greatest and thus this point is a conventional centre of the image of the object point or, in other words, that of the aberration spot. A shift of this point with respect to the Gaussian image is observed which results from aberrations.

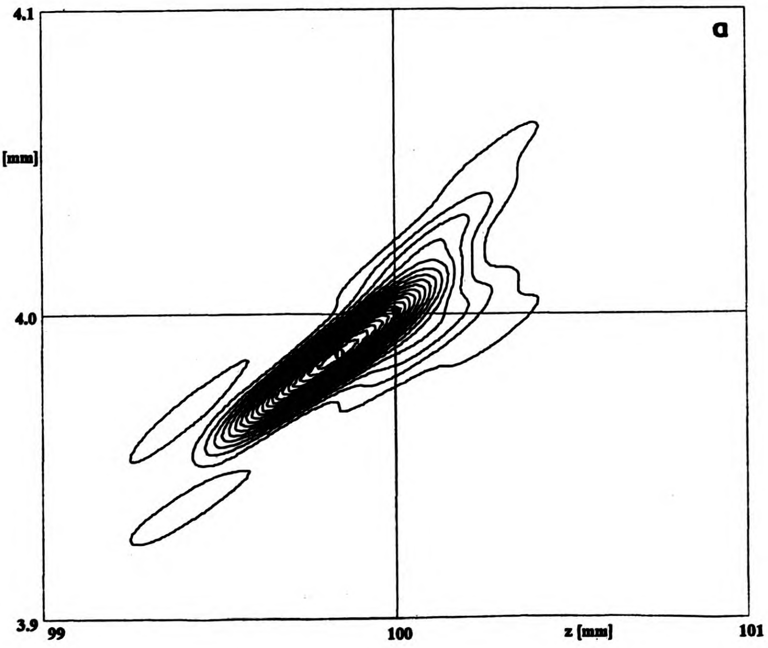

Fig. 2a

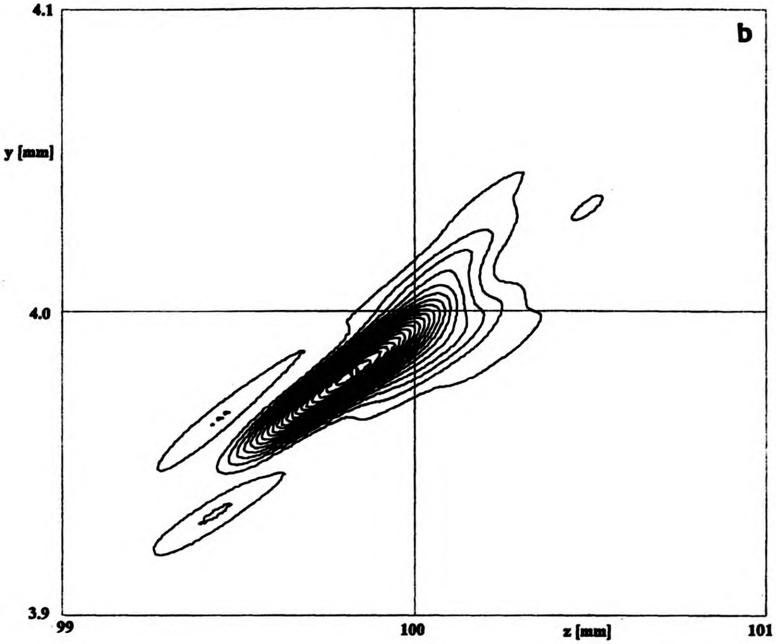

Fig. 2b

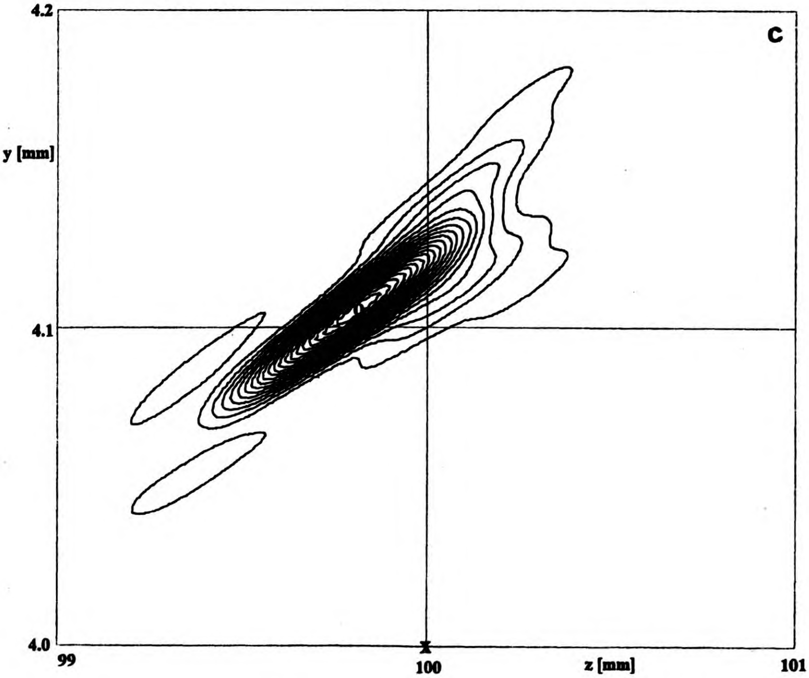

Fig. 2c

Fig. 2. Light intensity distribution in the meridional cross-section of the image given by: a – refractive lens, b – diffractive lens, c – hybrid lens, all for the field angle $y/z = 0.04$

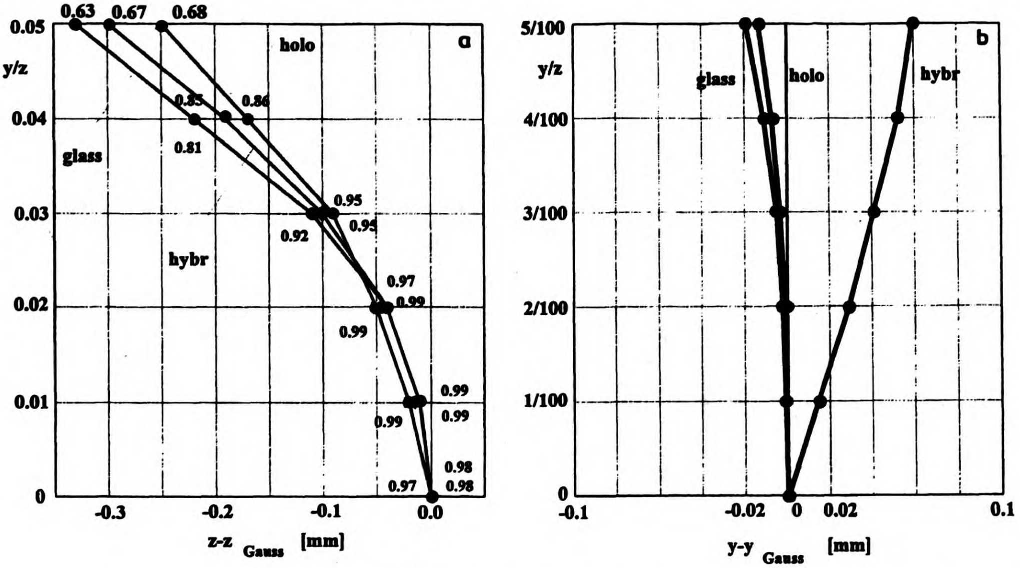

Fig. 3. Meridional cross-section of the best imaging for the three lenses examined (a). Lateral shift of the centre of the aberration spot with respect to the Gaussian image for the three lenses examined (b)

The plots in Figures 3**a** and **b** present the shift of the centre of the aberration spot understood in the above sense with respect to the Gaussian image. In Fig. 3**a**, illustrating the shift of the maximum light intensity points in front of the Gaussian plane, which means that the surface of the best imaging becomes curved, the values of the light intensity at those points are additionally given. In Figure 3**b**, showing the lateral shift of the aberration spot centre with respect to the Gaussian image, the influence of distortion can be noticed in the case of hybrid lens.

This preliminary analysis of the light intensity in the meridional cross-sections of the aberration spots at different angles of the field of view already shows that the best imaging is offered by the diffractive lens.

The choice of the plane of the best imaging, in which the aberration spots are being determined, is always a problem. Due to the field curvature this plane is different from the Gaussian plane. In the present paper, it has been assumed that the plane of the best imaging passes through the point in which the light intensity at the centre of the spot produced by the object point lying on the axis drops to the value equal to about 80% of the maximum intensity. This means that the criterion of Marechal is satisfied.

In the surface of optimal imaging defined in such a way, the aberration spots have been calculated for subsequent field angles. The light intensity distributions in the spots corresponding to the respective field angles equal to $y/z = 0$, $y/z = 0.04$ and $y/z = 0.05$ for the refractive, diffractive and hybrid lenses are presented in Figs. 4**a**, **b** and **c**, (see insert). For the same lenses and the same field angles, the wave aberrations determined with respect to the reference sphere of the centre coinciding each time with the centre of the corresponding diffraction spot are presented in Figs. 5a, b and c (see insert).

The characteristic numerical parameters describing both diffraction spots and wave aberrations are collected in Tab. 1.

Table 1. Some parameters characterizing the diffraction spot and the wave aberration

y/z	I			$M_2 \times 10^{-5}$ [mm]		
	refractive	diffractive	hybrid	refractive	diffractive	hybrid
0	0.80	0.80	0.79	1.7	1.9	1.9
0.04	0.71	0.84	0.81	2.4	1.7	2.0
0.05	0.34	0.56	0.47	4.9	3.1	4.9

y/z	$\sigma_W [\lambda]$		
	refractive	diffractive	hybrid
0	0.0030	0.0017	0.0017
0.04	0.0034	0.0015	0.0016
0.05	0.0063	0.0027	0.0031

This table includes: values of the maximum light intensities $I_{\max}$ in the aberration spots (in other words, the Strehl number), the value of the second moment of the light intensity distribution in the meridional cross-section of the aberration spot M_2 and the standard deviation of the wave aberration σ_W. The magnitude M_2 determined from formula

$$M_2 = \frac{\iint y^2 I(x,y)\mathrm{d}x\mathrm{d}y}{\iint I(x,y)\mathrm{d}x\mathrm{d}y} \tag{3}$$

describes "the moment of inertia" of the spot and is a measure of its spreading.

When comparing the corresponding values we state that the diffractive and hybrid lenses are characterized by similarity of imaging (though the diffractive lens is slightly better) and their aberration characteristics are definitely better than those of the refractive lens. In particular, it can be observed for the field angle $\omega = 0.05$ rad.

Knowing the point spread function we can determine the shape of the modulation transfer function which is a commonly accepted measure of the imaging quality for the extended object imaged in the incoherent light. In Figures 6a, b and c, there are shown the curves representing this function for the field angles equal to 0, 0.04 and 0.05 rad, which characterize the corresponding imaging performed by the refractive, diffractive and hybrid lenses. The spatial frequencies are normalized to the value of the diffraction-limited spatial frequency ν_0 resulting from the lens aperture. In order to compare better these functions, the values of frequencies normalized in this way are collected in Tab. 2; for these frequencies MTF drops to 1/2 and 1/4 for the same cases.

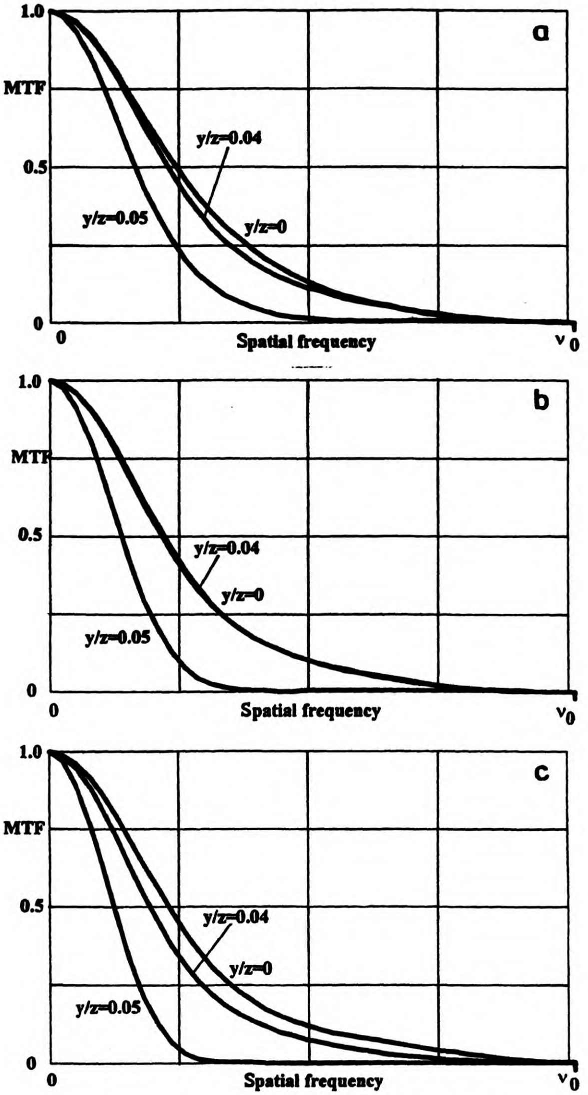

Fig. 6. Incoherent modulation (optical) tranfer function for three chosen field angles $y/z = 0$, 0.04 and 0.05 in the images given by: **a** — refractive lens, **b** — diffractive lens, and **c** — hybrid lens

Comparing MTF we state again that the best imaging is offered by the diffractive lens. According to this criterion the imaging quality offered by the hybrid lens is comparable to that of the refractive lens.

Summing up, we state that the imaging performed by the diffractive lens is the best. It seems that the hybrid lens presents also an interesting case since when analysed according to all criteria considered it is characterized by better imaging than that offered by the refractive lens, while its diffraction structure is technologic-

Table 2. Selected parameters characterizing the modulation transfer function

y/z	MTF	Refractive	Diffractive	Hybrid
0	1/2	32	35	32
	1/4	52	52	49
0.04	1/2	27	35	30
	1/4	44	57	49
0.05	1/2	18	25	20
	1/4	25	35	29

ally much simpler than that of a pure diffractive lens. In the typical case, the focusing power of the diffractive element of the hybrid lens does not exceed 5–10% of the total focusing power of this lens which means that its diffraction structure is characterized by much less cut-off spatial frequency than in the case of purely diffractive lens. This allows us to treat the hybrid lens as being composed of a glass lens with a diffractive corrector of the aberrations.

Finally, it should be stated that in practice such a simple optical element as that discussed is usually a component of a more complex optical system offering, for instance, significant diminishing of the system dimensions. The analysis of imaging performed by such a system is complex [9], and from our viewpoint unnecessary when teaching optics at the optical engineering level.

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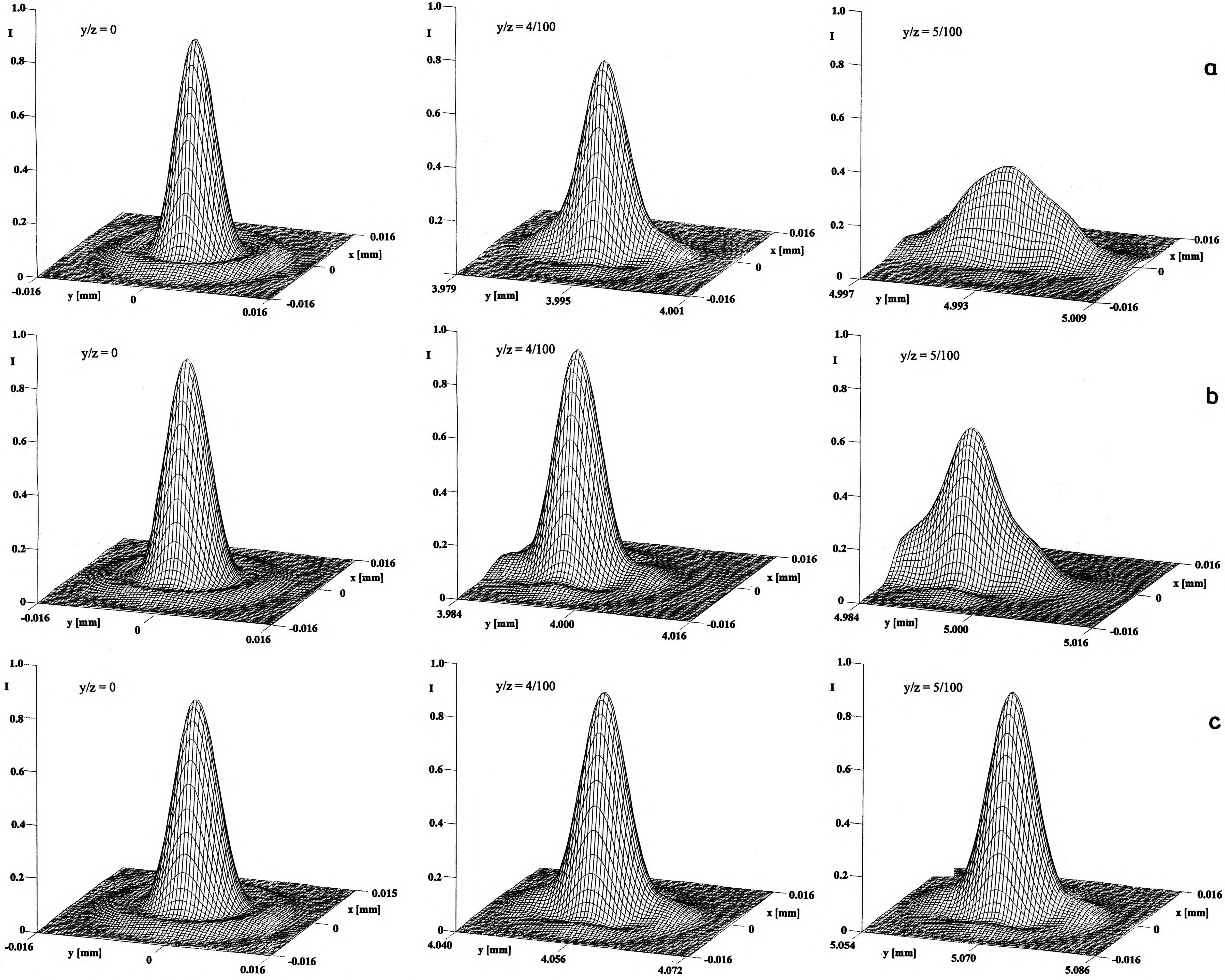

Fig. 4. Light intensity distribution in the diffractive spot for three chosen field angles as given by $y/z = 0$, 0.04, 0.05 in the images given by: **a** — refractive lens, **b** — diffractive lens, **c** — hybrid lens

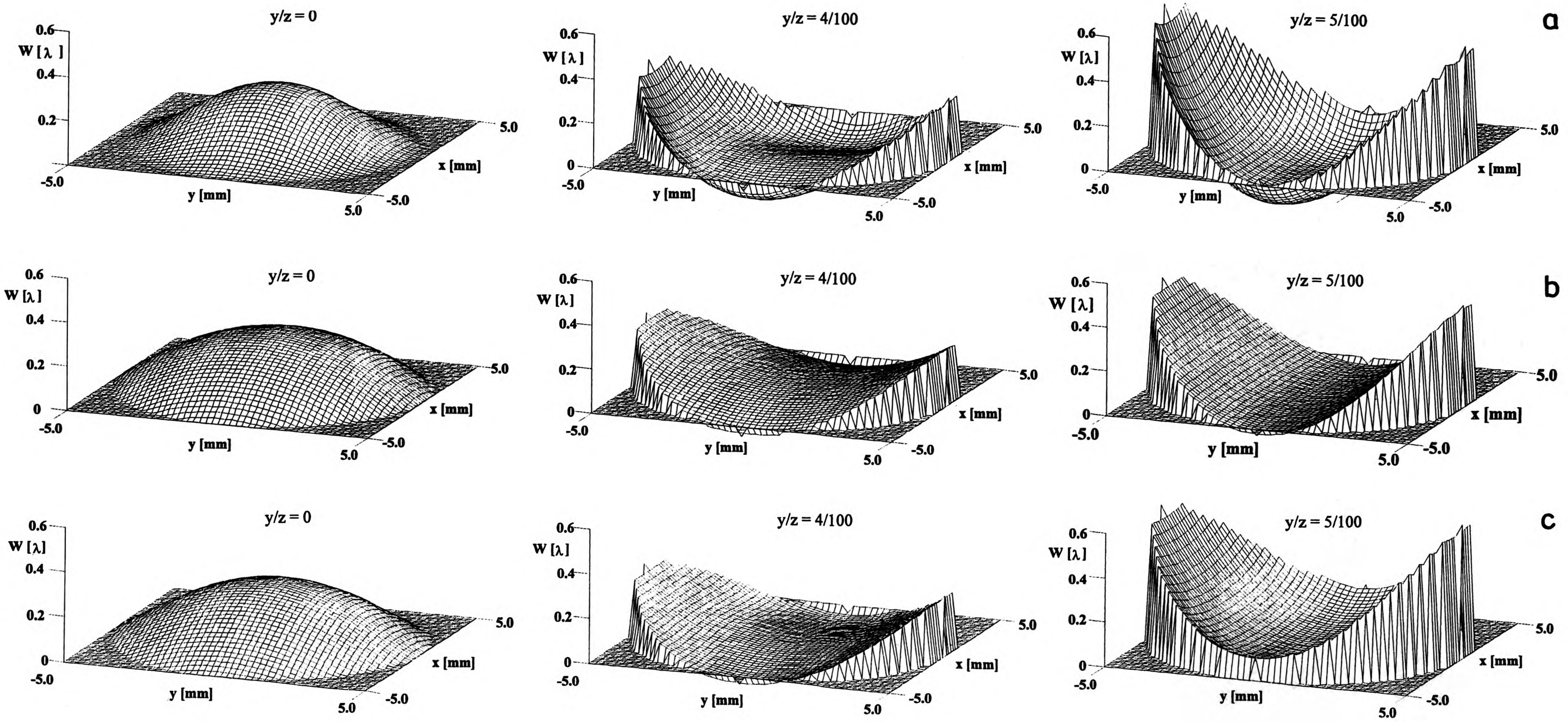

Fig. 5. Wave aberration for three chosen field angles $y/z = 0$, 0.04, 0.05 in the images given by: a — refractive lens, b — diffractive lens, c — hybrid lens