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Nonlinear cone optical fiber – propagation characteristic

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Numerical analysis of the nonlinear cone optical fiber using the coupled mode theory and the split-step Fourier method is described. The nonlinear cone optical fiber is divided into a large number of appropriate cylindrical segments. In the analysis of light propagation between two segments of various diameters, the coupled mode theory is used. Propagation of a pulse, in the cylindrical nonlinear segment, is analyzed using the split-step Fourier method. Some results of computation for the selected cone conlinear optical fibers are also presented.

1. Introduction

A cone waveguide is a guiding structure for electromagnetic waves that gradually becomes narrower towards one of the ends [1]. The analysis of such guides can be carried out in two steps. In the first one, the fiber is approximated by a large number of segments of appropriate diameter (the staircase model of the optical fiber), Fig. 1.

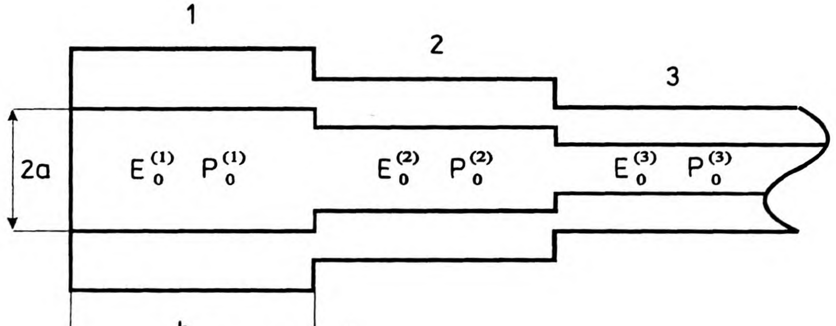

Fig. 1. Staircase model of optical fiber

The field distribution on the boundary of two segments of different diameter is described using the coupled mode theory [2] – [6]. In the case of the staircase of the nonlinear optical fiber (Fig. 1), the exchange of power between two segments is described by the equation [3]

$$E_0^{(2)}(x,y) = \sqrt{P_0^{(1)}}\,|\Phi_{12}|\,\hat{E}_0^{(2)}(x,y) \tag{1}$$

where E_0 is the electric field, Φ_{12} is the amplitude coupling coefficient

$$\Phi_{12} = \frac{\iint\limits_{-\infty}^{\infty} E_0^{(1)} E_0^{(2)*} \mathrm{d}x\mathrm{d}y}{\sqrt{\iint\limits_{-\infty}^{\infty} |E_0^{(1)}|^2 \mathrm{d}x\mathrm{d}y \iint\limits_{-\infty}^{\infty} |E_0^{(2)}|^2 \mathrm{d}x\mathrm{d}y}}, \tag{2}$$

$\hat{E}_0$ is the normalized electric field

$$\hat{E}_0 = \frac{E_0}{\sqrt{|P_0|}} \tag{3}$$

where P_0 represents the power of the mode in the segment.

In the second step, the nonlinear Schrödinger equation (NLSE) of the form [7], [8]

$$j\frac{\partial A}{\partial z} - \frac{1}{2}\beta_2 \frac{\partial^2 A}{\partial T^2} + \gamma|A|^2 A = 0 \tag{4}$$

is solved for each segment. In Eq. (4), A represents the envelope function, $\gamma = \frac{n_{\mathrm{NL}}k}{A_{\mathrm{ef}}}$ denotes the nonlinearity parameter, $\beta_2 = \frac{d^2\beta}{d\omega^2}$ is the dispersion parameter of the fiber, n_{NL} is the nonlinear refractive coefficient, which for SiO_2 is equal to $1.2\cdot 10^{-22}$ m^2/V^2, $k = \frac{2\pi}{\lambda}$, A_{ef} is the mode effective area [7], [8], β is the phase constant, λ is the wavelength. Equation (4) is valid for pulses wider than 100 fs. If losses are taken into consideration, the component of $j\frac{\alpha}{2}A$ must be added to Eq. (4), where α represents attenuation.

The idea of the split-step Fourier method is based on the mutual physical interaction of nonlinearity and dispersion. The solution of the NLSE by the split-step Fourier method is carried out in two steps. In the first one, the influence of nonlinearity on the pulse shape is analysed, in the second step, it is just the dispersion effect that is analysed. After some transformations, Eq. (4) takes the form [9]

$$\frac{\partial A}{\partial z} = (D + N)A \tag{5}$$

where

$$D = -\frac{j}{2}\beta_2 \frac{\partial^2}{\partial T^2} - \frac{\alpha}{2} \tag{6}$$

is the differential operator which describes the dispersion and the losses in a nonlinear guide,

$$N = j\gamma |A|^2 \tag{7}$$

is a nonlinear differential operator describing the nonlinear effects occurring during the propagation of the optical pulses. After some mathematics, we obtain

$$A(z+h,T) = \exp(h\boldsymbol{D})\exp(h\boldsymbol{N})A(z,T). \tag{8}$$

To implement the method, the guide is divided into a large number of segments of width h. The pulse is carried twice through each segment [9], [10], once as a nonlinear segment, where $\boldsymbol{D} = 0$, and then as a dispersive segment, where $\boldsymbol{N} = 0$ (Fig. 2).

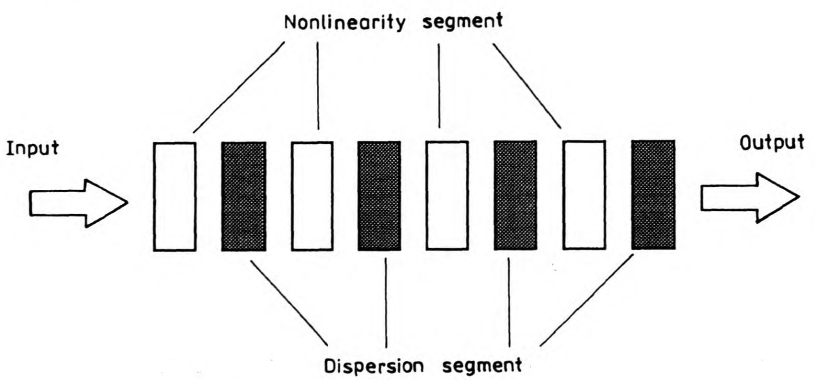

Fig. 2. Scheme of calculations

In the case of the nonlinear segment, Eq. (8) has the form

$$A_1(z+h,T) = \exp(h\boldsymbol{N})A(z,T). \tag{9}$$

The exponential operator $\exp(h\boldsymbol{D})$, which represents the dispersive segment, is convenient for calculations in the frequency domain

$$A(z+h,T) = F^{-1}\{\exp[h\boldsymbol{D}(j\omega)]F[A_1(z+h,T)]\} \tag{10}$$

where F denotes the Fourier transform operation, $\boldsymbol{D}(j\omega)$ is given by Eq. (6) when the differential operator $\partial/\partial T$ is replaced by $(j\omega)$, namely

$$\boldsymbol{D} = j\frac{1}{2}\beta_2\omega^2 - \frac{\alpha}{2}. \tag{11}$$

To accelerate the computation, the fast Fourier transform can be used.

2. Numerical results

To prove how the nonlinearity γ and dispersion β_2 vary, calculations of the cone nonlinear optical fiber for shifted dispersion characteristics have been carried out. Results of the computation for $\Delta = 0.9\%$, where $\Delta = \frac{n_1 - n_2}{n_2}$ is the relative core-cladding difference, have been presented (Figs. 3–5).

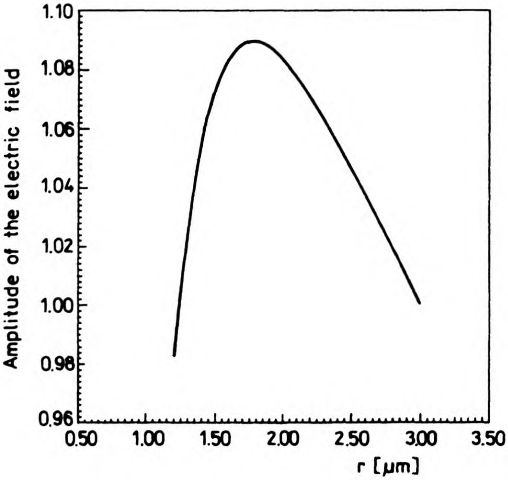

Fig. 3. Amplitude of the electric field vs core radius ($\Delta = 0.9\%$, $\lambda = 1.55$ μm, r [μm])

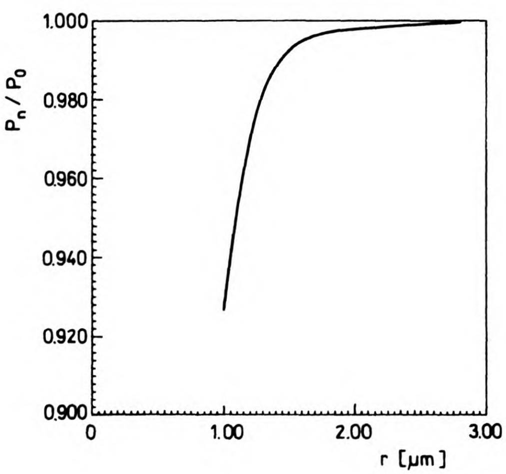

Fig. 4. Average power vs. core radius ($\Delta = 0.9\%$, $\lambda = 1.55$ μm, r [μm])

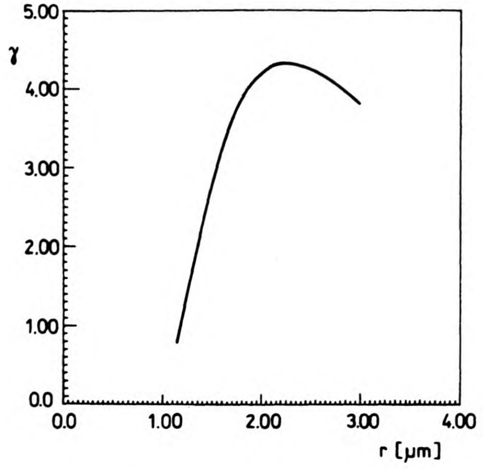

Fig. 5. Nonlinearity γ vs. core radius ($\Delta = 0.9\%$, $\lambda = 1.55$ μm, r [μm], γ [W^{-1} m^{-1}])

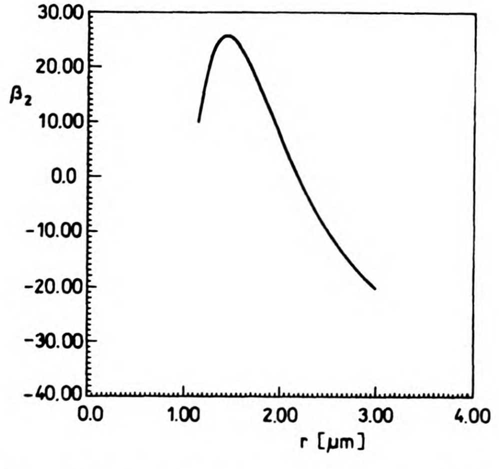

Fig. 6. Dispersion β_2 vs. core radius ($\Delta = 0.9\%$, $\lambda = 1.55$ μm, r [μm], β_2 [ps^2/km])

Using the nonlinearity γ and dispersion β_2, calculation of the electric field distribution, for shifted dispersion characteristic, has been carried out (Fig. 6). In Figure 7, fundamental solitons in the cylindrical and the cone nonlinear optical fiber are presented, where $\Delta = 0.95\%$.

3. Conclusions

Owing to the cone structure of the optical guide the values of nonlinear and dispersion parameters are influenced in a such way that dispersion due to losses is partially compensated. As the diameter of the guide is getting smaller the dispersion diminishes and the intensity of light becomes greater in respect to its initial

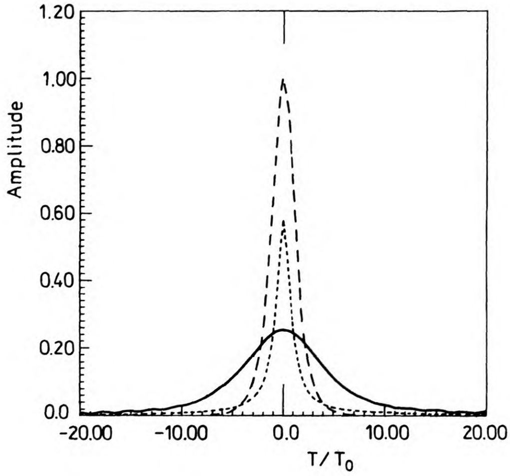

Fig. 7. Fundamental soliton in the cone nonlinear optical fiber ($\Delta = 0.95\%$, $\lambda = 1.55$ μm, $\alpha = 0.3$ dB/km, $z = 5$ km). Dashed line – input shape. Dotted line – the tapered nonlinear optical fiber where initial value of radius **a** is 2.27 μm, final value of **a** is 2.17 μm. Solid line – the nonlinear optical fiber where $a = 2.27$ μm

magnitude, so mutual compensation of the nonlinear and dispersion effects is preserved over longer distance than in cylindrical structure. Both numerical methods, coupled mode theory and split-step Fourier method, have proved to be successful; high accuracy (better than 1%) is achieved when the segments are 2 m long. The staircase model of the cone guide enables a simulation of imperferctions, which may occur due to technological process.

References

[1] SPORLEDER F., UNGER H. G., IEE Electromagnet. Wave Series **6** (1979).

[2] SNYDER A. W., IEEE Trans. Microwave Theory Tech. **18** (1970), 383.

[3] EBELING K. J., *Integrierte Optoelektronik, Wellenleiteroptik, Photonik, Halbleiter*, Springer-Verlag, Berlin 1989.

[4] YAJIMA H., IEEE J. Quantum Electron. **14** (1978), 749.

[5] PETYKIEWICZ J., *Integrated Optics: Physical Fundamentals*, (in Polish), [Ed.] WNT, Warszawa 1989.

[6] MARCUSE D., *Theory of Dielectric Waveguides*, Academic Press, New York 1974.

[7] MAJEWSKI A. *Solitons in Optical Guides*, (in Polish), [Ed.] Wydawnictwa Politechniki Warszawskiej, Warszawa 1991.

[8] MAJEWSKI A., *Nonlinear Fiber Optics*, (in Polish), [Ed.] Wydawnictwa Politechniki Warszawskiej, Warszawa 1993.

[9] AGRAWAL G. P., *Nonlinear Fiber Optics*, Academic Press 1989.

[10] HAGESAWA A., TAPPERT F., Appl. Phys. Lett. **23** (1973), 142.

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