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Limitations of diffraction measurements by means of axicons

I. G. Palchikova, S. V. Smirnov

Institute of Automation and Electrometry, Sib. Br. RAS, Prosp. Koptyug, 1, Novosibirsk, 630090 Russia.
Novosibirsk State University, Pirogova Str., 2, Novosibirsk, 630090, Russia.

Asymptotic and numerical evaluation of the Fresnel diffraction integral for intensity distribution in the axial focal segment of conical axicon was accomplished to find the H coordinate of maximum in cases of monochromatic or polychromatic illumination. The experimental and theoretical values of H coordinate were found to be in good agreement. It has been shown that the axicon focal segment can be used as a tape-line for the length measuring.

1. Introduction

Refractive optical elements with axial symmetry (for example, with conical surfaces) that are able to form multiple images of a point light source along the optical axis or a continuous light line are called axicons [1]. Refractive optical axicons are not used commercially because of difficulties in fabricating and testing conical surfaces. This problem is solved with the diffractive analogies of the axicon [2]. The technology for diffractive optical elements production removes the difficulties of producing glass conic surfaces and makes it possible to multiply elements by photolithography and stamping methods.

One of the significant aims of research and development of axicons is the study of their possible applications in controlling the straightness of guides in machine tools and aligning machinery [3]. We propose using an axicon as the main component of the length measuring instrument.

In the geometry optics approximation the axicon with a linear phase function [4] should exhibit a linear increase in intensity distribution along the focal segment axis and direct dependence of the segment length on the aperture. However, axial intensity oscillations appear due to the truncation of an illuminating beam on the sharp edges of the aperture [5]. This effect can change the position of the intensity maximum [6]–[8] and limit the applicability of axicons. This paper presents the results of calculations and laboratory study of the intensity distribution in the focal segment produced by the conical diffractive axicon.

2. Theory

The refractive axicon transforms the plane light wave into the conic one, and behind axicon the axial symmetry system of rays is formed. In a homogeneous medium, where the rays are straight, any system of rays has two caustics surfaces since in each point the wave front is characterised by two main radii of curvature. One caustic of the conic wave front [9] is the focal segment along the optical axis. The second caustic of the conic wave front is a ring and, similar to the caustic of the plane wave, it tends to infinity since the rays generating it are parallel to each other.

Below, the Fresnel diffraction integral for intensity distribution in the axial focal segment is asymptotically and numerically evaluated in order to find the coordinate of maximum in cases of monochromatic or polychromatic illumination.

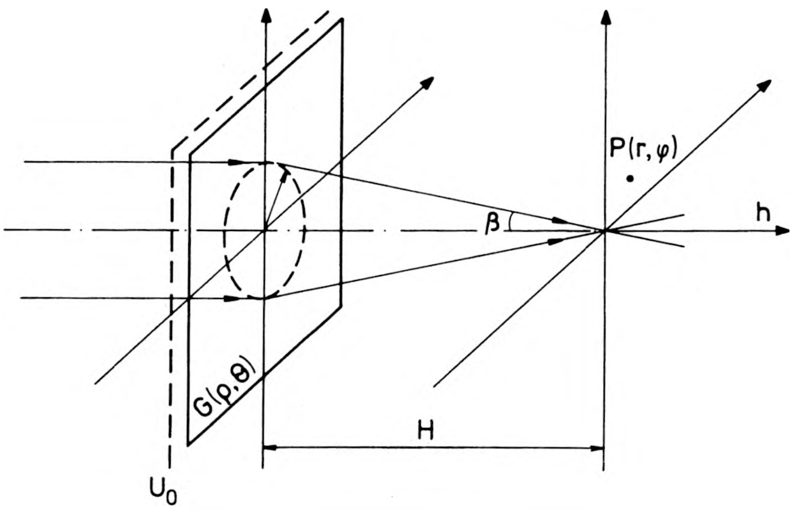

Fig. 1. Beam travelling through the thin axicon.

Figure 1 shows the beam travelling through the thin axicon, placed in the plane G, ρ, θ, 0 are the coordinates on the exit pupil plane. The phase transmission function of axicon can be written as $f(\rho) = -\gamma\rho = \sin\beta\,\rho$, where:

$\gamma = \alpha(n-1)$ – for the refractive conical axicon (α is the angle of the vertex of the cone, n – the refractive index);

$\gamma = 2\alpha$ – for the reflective conical axicon;

$\gamma = \lambda/T$ – for the first diffraction order of the diffractive conical axicon (T is the diffraction structure period);

$\gamma = f'(\rho)$ – for an arbitrary diffractive axicon (the prime is used to indicate the derivative function).

When calculating the field amplitude $U(r, h)$ at arbitrary point $P(r, \varphi)$ we proceed from the Fresnel diffraction integral for the optical system with axial symmetry [6]

$$U(r,h)=\frac{k}{ih}\int_0^a U_0(\rho)\exp\left\{ik\left[h+\frac{r^2+\rho^2}{2h}+f(\rho)\right]\right\}J_0\left(\frac{k\rho r}{h}\right)\rho\cdot\mathrm{d}\rho \tag{1}$$

where: a is the radius of the axicon aperture, $U_0(\rho)$ – an amplitude of the incident wave, λ – the wavelength, $k = 2\pi/\lambda$, J_0 – Bessel function of the first kind and zero order.

For the purpose of asymptotical evaluation of integral (1) we use the stationary phase method (SPM). The field amplitude $U(r,h)$ is determined by the neighbourhood of appropriate critical points disposed at the circumference with radius $\tilde{\rho}$ on the plane of exit pupil G

$$U(r,h)=kU_0(\tilde{\rho})\exp\left\{ik\left[h+\frac{r^2+\tilde{\rho}^2}{2h}+f(\tilde{\rho})\right]-\frac{j\pi}{4}\right\}f'(\tilde{\rho})\,J_0(krf')\sqrt{\frac{\lambda h}{1+hf''(\tilde{\rho})}} \tag{2}$$

where $\tilde{\rho} = -f'h$.

The optical intensity $I(r,\varphi,h)$ behind the axicon can be expressed as $I(r,\varphi,h) \sim |U(r,\varphi,h)|^2$.

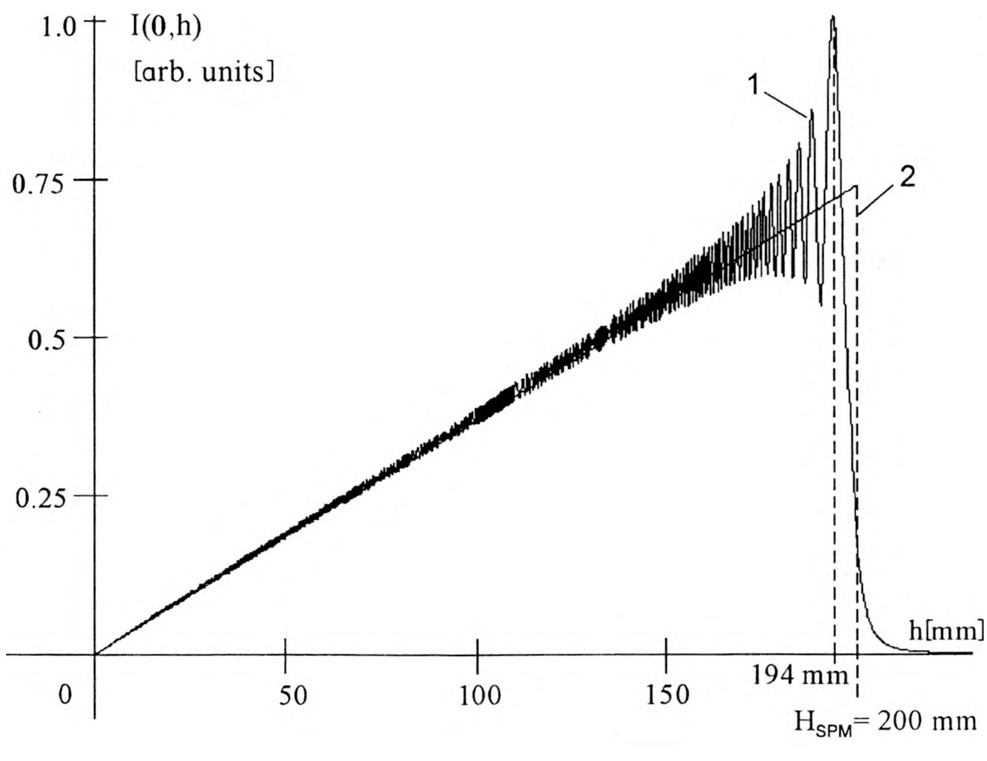

Fig. 2. Typical on-axis intensity distribution $I(0,h)$ vs. distance h behind conical axicon, obtained as a result of computations according to Eq. (1).

The axial focusing region has the form of the body of revolution. Based on the SPM we can conclude the following about its dimensions. The transverse intensity distribution in the focal segment is proportional to the Bessel function of the first kind and zero order. The focal spot diameter is $d = 5/kf'(\tilde{\rho}) = 5/k\gamma$, and it may vary along the optical axis. These variations depend on the function $\gamma(\rho)$. The length H_{SPM} of the focal segment is equal to $H_{SPM} = a\sqrt{1-[f'(a)]^2}/f'(a) \approx a/\gamma(a)$. For the diffractive conical axicon $d = 5T/2\pi$, and $H_{SPM} = aT/\lambda$. If the amplitude $U_0(\rho, \theta) =$ const., the linear increase of the on-axis intensity $I(0, h)$ behind conical axicon follows from (2), and the intensity reaches its peak at $h = H_{SPM}$, as shown in Fig. 2 (curve 2).

In the numerical analysis we assumed a monochromatic illumination with the wavelength $\lambda_1 = 0.63 \cdot 10^{-3}$ mm. The parameters of the conical axicon were: $a =$ 10 mm, $H_{SPM} = 200$ mm. In Fig. 2, curve 1 represents the function $I(0, h)$ for conical axicon calculated from formula (1). It is normalised to its maximum. The numerical solution shows the presence of oscillations (Fig. 2, curve 1), and the results of SPM (Fig. 2, curve 2) are the average approximation. Obviously, oscillations appear to the truncation of the illuminating beam on the sharp edges of the aperture. The peak position H_{SPM}, predicted with the use of SPM, does not coincide with that obtained in the exact solution $h = 194$ mm. If the wavelength is changed, the peak position is displaced. Figure 3**a** demonstrates this displacement: $\lambda_1 = 0.63 \cdot 10^{-3}$ mm for curve 1, $\lambda_2 = 0.58 \cdot 10^{-3}$ mm for curve 2, $\lambda_3 = 0.54 \cdot 10^{-3}$ mm for curve 3.

If a conical axicon is illuminated by a polychromatic source, different kinds of oscillations are observed. Figure 3**b** shows the on-axis intensity distribution corresponding to the case of a polychromatic incident plane wave, which is a sum of three monochromatic plane waves such as those presented in Fig. 3a. This plot confirms that the main peak of on-axis intensity is preserved for any illuminating beam.

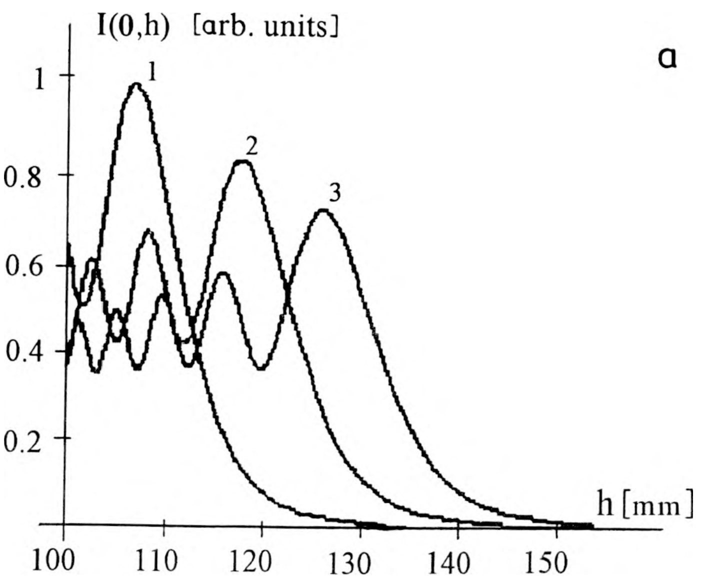

Fig. 3a

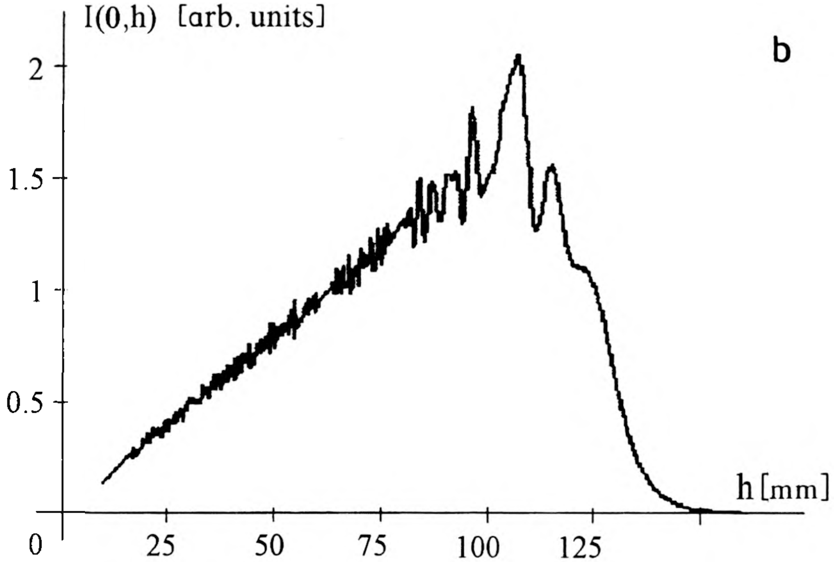

Fig. 3b

Fig. 3. On-axis intensity distribution at the focal segment for the different wavelengths: $\lambda_1 = 0.63 \cdot 10^{-3}$ mm for curve 1, $\lambda_1 = 0.58 \cdot 10^{-3}$ mm for curve 2, $\lambda_1 = 0.54 \cdot 10^{-3}$ mm for curve 3 (a). On axis intensity distribution $I(0, h)$ vs. distance h at the focal segment for the polychromatic incident plane wave (b).

3. Experimental results

The diffractive conical axicon was made on the glass substrate of high optical quality. The two-step phase profile in glass was produced by ion etching. The amplitude original of the element was fabricated on a thin film of chromium with a circular photoplotter [2]. The axicon was calculated for radiation at the wavelength $\lambda_1 = 0.63 \cdot 10^{-3}$ mm with the period $T = 30 \cdot 10^{-3}$ mm, and radius of 18 mm.

The optical scheme used for the study of intensity distributions is represented in Fig. 4, in which: 1 – the light source (halogen source or He-Ne laser), 2 – a small

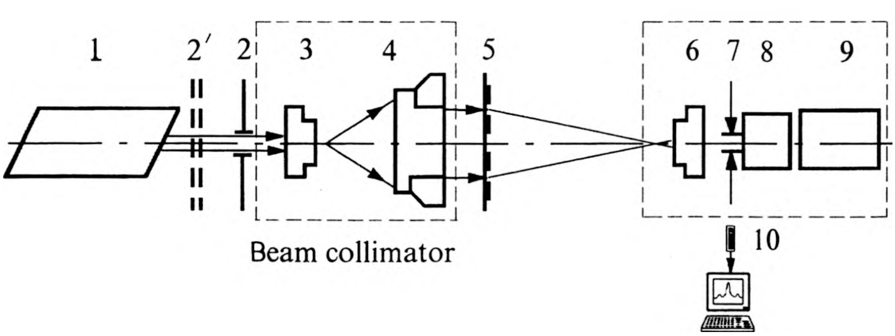

Fig. 4. Experimental set-up (for explanation, see the text).

aperture, 2′ – coloured glass (red glass KC11, orange – OC14, blue CC2) in the scheme with halogen source, 3 – objective microlens, 4 – the lens with an iris diaphragm, 5 – axicon, 6 – microobjective, 7 – pinhole, 8 – photomultiplier tube, 9 – ammeter. The objective 6 enlarges the focal spot. The objective 6, the pinhole 7 and the photomultiplier tube 8 can be moved together by microscrews. The computer controlled line of the silicon diodes 10 consists of 1024 cells. It is used instead of the set with the photomultiplier tube. The spurious diffraction orders of

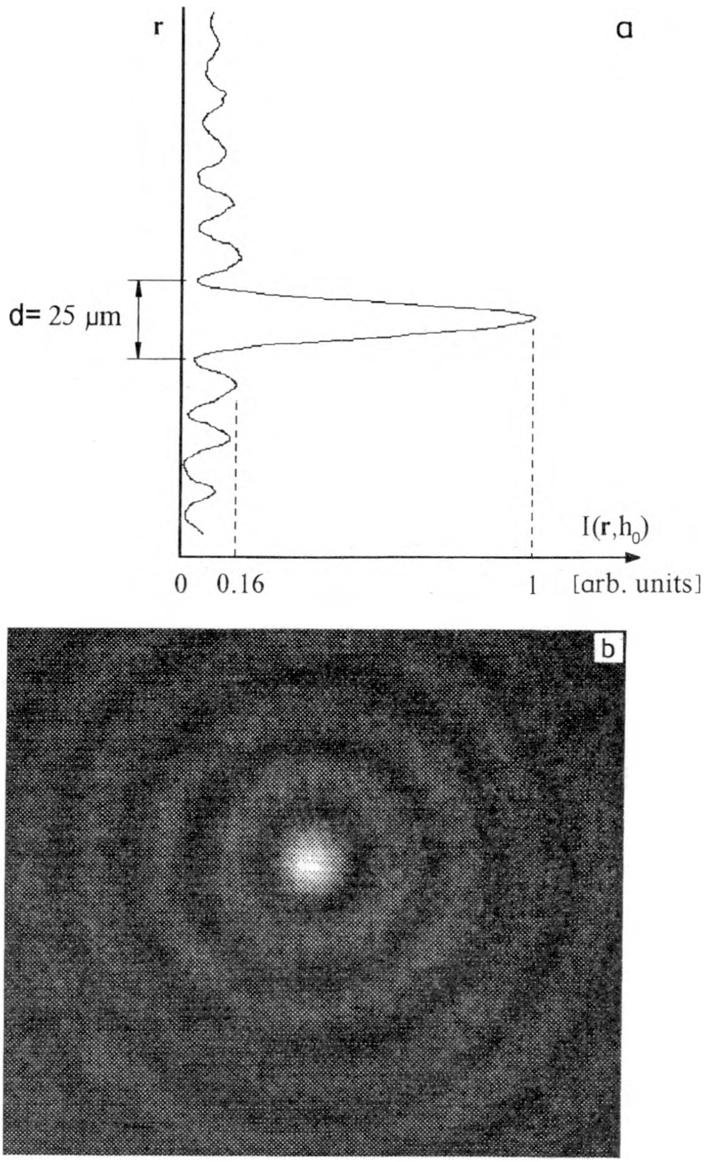

Fig. 5. Typical transverse intensity distribution at the focal segment ($\lambda_1 = 0.63\cdot 10^{-3}$ mm).

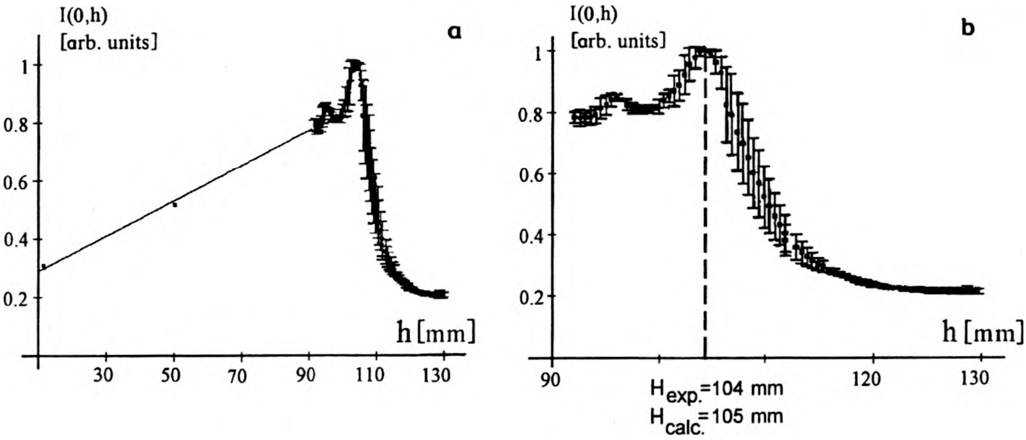

Fig. 6. Experimental data. $I(0, h)$ vs. distance h with laser source ($\lambda_1 = 0.63 \cdot 10^{-3}$ mm).

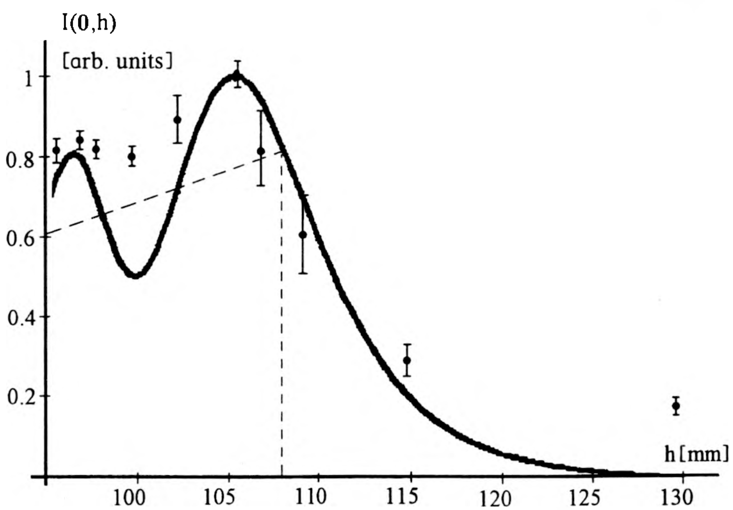

Fig. 7. $I(0, h)$ vs. distance h with laser source ($\lambda_1 = 0.63 \cdot 10^{-3}$ mm). The computational result (solid line) in comparison with experimental data (dots) and SPM results (dashed line).

the diffractive axicon have no influence upon the intensity distribution at the end of the focal segment.

Figures 5–7 present the results of experiments with the laser source $\lambda_1 = 0.63 \cdot 10^{-3}$ mm. A typical transverse intensity distribution at the focal segment is shown in Fig. 5. The spot diameter d is the same as one predicted by SPM. The on-axis energy density distribution at the end of the focal segment was measured with a photomultiplier with a diaphragm of 46 μm after the focal spot was increased 200 times in diameter. Experimental data for a series of measurements (21 test runs) are shown in Fig. 6, the point marks the mean quantity and the vertical segment marks the dispersion. Figure 6**b** demonstrates a fragment of the curve from

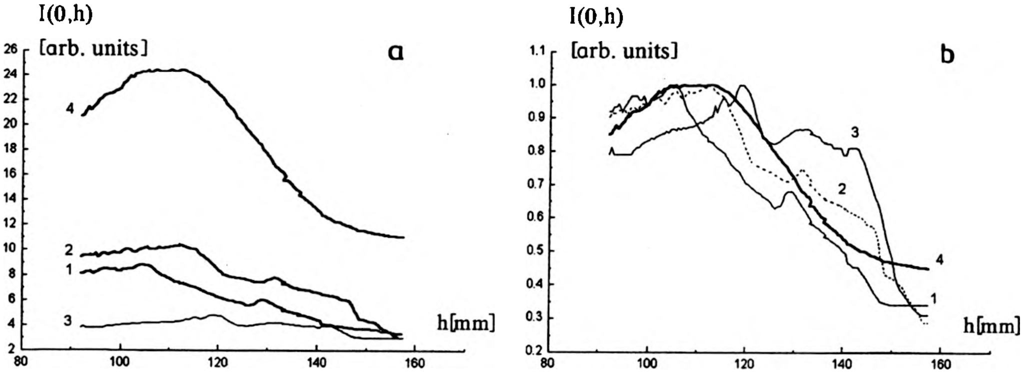

Fig. 8. $I(0, h)$ vs. distance h obtained during measurements in the white light — curve 4, the red light (red glass KC11) — curve 1, the orange light (orange glass OC14) — curve 2, the blue light (blue glass CC2) — curve 3. The curves in figure **a** are not normalised, the same curves in figure **b** are normalised at their maximum.

Fig. 6a. We find the main peak of intensity at $H_{exp} = 104$ mm, the theoretically predicted peak being at $H_{calc} = 105$ mm. In Fig. 7, computation results (solid line) for the on-axis intensity distribution at the end of focal segment are compared with experimental data (dots) and SPM results (dashed line). Here the error of H_{SPM} is about 9%.

The experimental results for the case where polychromatic light was used are given in Fig. 8**a**, **b**. Curve 4 represents the experimental on-axis energy distributions obtained when measurements were performed in the white light from halogen source. Polychromatic light was obtained with the help of coloured glasses. Curve 1 represents average experimental results when the red glass KC11 was used, with maximum transmission at $\lambda_1 = 0.60 \cdot 10^{-3}$ mm, curve 2 — the orange glass OC14 ($\lambda_{max} = 0.58 \cdot 10^{-3}$ mm), curve 3 — the blue glass CC2 ($\lambda_{max} = 0.54 \cdot 10^{-3}$ mm). The curves in Fig. 8**a** are not normalised, the same curves in Fig. 8**b** are normalised at their maximum. It results from experimental curves that on-axis intensity has one main peak at the end of the focal segment irrespective of the type of illumination.

4. Conclusions

The experimental and theoretical values for h-coordinate of the peak of on-axis intensity behind the conical axicon were found to be in good agreement. So we propose to use the axicon focal segment as a tape line for the length measuring. The scale of the aperture can serve as a scale of the axicon tape-line, but it has to be calibrated with respect to the influence of diffraction at the end of the segment.

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