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Diffraction by a perfectly conducting open-ended waveguide in a homogeneous biisotropic medium

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In this paper, the diffraction of an electromagnetic wave within perfectly conducting parallel-plates embedded in a homogeneous biisotropic medium is examined. The vector diffraction problem is reduced to the scattering of a single scalar field, the latter being the normal component of either a left-handed or a right-handed Beltrami field. The scattering of the left-handed field component is explicitly analyzed, with that of the other scalar field being analogously tractable.

1. Introduction

The concept of Beltrami fields and flows has a long and distinguished history in fluid mechanics. A mathematical formulation of Beltrami flows was first published in the year 1889 [1]. The reintroduction of Beltrami flows must be credited to CHANDRASEKHAR [2], who was then deeply involved in a study of force-free magnetic fields. In electromagnetism, following early work by SILBERSTEIN [3], the Beltrami field concept has been repeatedly rediscovered throughout this century (see, *e.g.*, [4]–[6]), though its antecedents have generally remained muddled. It is fair to state that these approaches mainly viewed the Beltrami field concept as a convenient tool to rearrange the time-harmonic electromagnetic field equations. Only in recent years, with considerable interest in the study of complex media, has there been a shift in emphasis: Beltrami fields are essential for the description of time harmonic electromagnetic fields in chiral and biisotropic media [7], [8], for which reason they have occurred quite often in very recent literature on electrical engineering.

During the last few years, considerable interest has been demonstrated by the electromagnetics community in the study of the Beltrami fields in a biisotropic medium. A number of developments have been given in [9]–[15]. To these may be added papers on diffraction by a half-plane [16], [17]. Whereas guided wave propagation in biisotropic and isotropic chiral media has been much attended to of late, canonical diffraction problems have not been tackled. A notable exception is a paper by FISANOV [18] on chiral wedges. In continuation of this line of seeking a better understanding of the interaction of electromagnetic fields in a biisotropic medium, the objective of this paper is to obtain the diffraction of an electromagnetic wave by a perfectly conducting open-ended waveguide. Since time-harmonic electromagnetic fields in a biisotropic medium are not more difficult to analyze than in an isotropic chiral medium, we let the open-ended waveguide lie in a homogeneous biisotropic medium. The diffraction of a left-handed Beltrami plane wave incidence is discussed using the Wiener–Hopf technique [19].

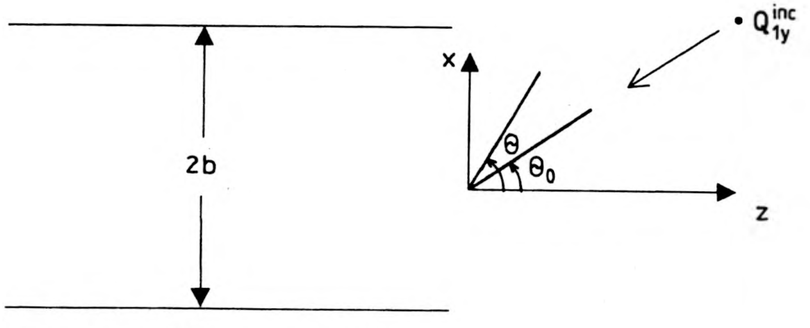
2. Formulation of the problem

Let all space be occupied by a homogeneous biisotropic medium with the exception of the perfectly conducting waveguide $z < 0$, $x = \pm b$. The geometry of the problem is shown in the Figure. In the Fedorov representation [20], biisotropic medium is characterized by the frequency-domain constitutive relations:

$$D = \varepsilon E + \varepsilon\alpha\nabla \times E, \tag{1}$$

$$B = \mu H + \mu\beta\nabla \times H \tag{1a}$$

where ε and μ are the permittivity and permeability scalars, respectively, while α and β are the biisotropy pseudoscalars. The biisotropic medium with $\alpha = \beta$ is reciprocal and is then called a chiral medium. The time dependence $\exp(i\omega t)$ is assumed throughout.



Open ended parallel plates waveguide.

Using the constitutive relations (1a) and (1b), the Maxwell curl postulates, $\nabla \times E = -i\omega B$, and $\nabla \times H = i\omega D$, may be written as:

$$\nabla \times Q_1 = \gamma_1 Q_1, \tag{2a}$$

$$\nabla \times Q_2 = -\gamma_2 Q_2, \tag{2b}$$

with the Beltrami fields [21]

$$Q_1 = \frac{\eta_1}{\eta_1 + \eta_2}[E + i\eta_2 H], \tag{3a}$$

$$Q_2 = \frac{\eta_1}{\eta_1 + \eta_2}[H + iE/\eta_1], \tag{3b}$$

in terms of the electric field E and the magnetic field H. The two wave numbers γ_1 and γ_2 in this medium are given by:

$$\gamma_1 = \frac{k}{(1-k^2\alpha\beta)}\{\sqrt{1+k^2(\alpha-\beta)^2/4}+k(\alpha+\beta)/2\}, \tag{4a}$$

$$\gamma_2 = \frac{k}{(1-k^2\alpha\beta)}\{\sqrt{1+k^2(\alpha-\beta)^2/4}-k(\alpha+\beta)/2\}, \tag{4b}$$

and the two impedances η_1 and η_2 by:

$$\eta_1 = \eta\{\sqrt{1+k^2(\alpha-\beta)^2/4}+k(\alpha-\beta)/2\}^{-1}, \tag{4c}$$

$$\eta_2 = \eta\{\sqrt{1+k^2(\alpha-\beta)^2/4}+k(\alpha-\beta)/2\} \tag{4d}$$

where $k = -\omega\sqrt{\varepsilon\mu}$ and $\eta = \sqrt{\mu/\varepsilon}$ are merely shorthand notations. Furthermore, while Q_1 is E-like (left-handed Beltrami field), Q_2 is H-like (right-handed). Because of our interest in scattering with a prescribed y-variation, it is appropriate to decompose the Beltrami fields as [15]

$$Q_1 = Q_{1t}+\hat{y}Q_{1y}, \tag{5a}$$

$$Q_2 = Q_{2t}+\hat{y}Q_{2y} \tag{5b}$$

where the fields Q_{1t} and Q_{2t} lie in the $x-z$ plane and $\hat{y}$ is a unit vector along the y-axis, so that $\hat{y}\cdot Q_{1t} = 0$ and $\hat{y}\cdot Q_{2t} = 0$. Next, on assuming that all field vectors have an implicit $\exp(-ik_y y)$ dependence on the variable y, we get from Eqs. (2a), (2b)

$$Q_{1x} = \frac{-1}{k_{1xz}^2}\left[ik_y\frac{\partial Q_{1y}}{\partial x}+\gamma_1\frac{\partial Q_{1y}}{\partial z}\right], \tag{6a}$$

$$Q_{1z} = \frac{1}{k_{1xz}^2}\left[-ik_y\frac{\partial Q_{1y}}{\partial z}+\gamma_1\frac{\partial Q_{1y}}{\partial x}\right], \tag{6b}$$

$$Q_{2x} = \frac{1}{k_{2xz}^2}\left[-ik_y\frac{\partial Q_{2y}}{\partial x}+\gamma_2\frac{\partial Q_{2y}}{\partial z}\right], \tag{6c}$$

$$Q_{2z} = \frac{-1}{k_{2xz}^2}\left[ik_y\frac{\partial Q_{2y}}{\partial z}+\gamma_2\frac{\partial Q_{2y}}{\partial x}\right] \tag{6d}$$

where:

$$\left.\begin{aligned} k_{1xz}^2 &= (\gamma_1^2-k_y^2) \\ k_{2xz}^2 &= (\gamma_2^2-k_y^2) \end{aligned}\right\}. \tag{6e}$$

Now, note that if we explore the scattering of the scalar fields Q_{1y} and Q_{2y}, then we can completely determine the other components Q_1 and Q_2 by using Eqs. (6a)– (6d). With the help of Eqs. (2a), (2b), it can be easily shown that the scalar fields Q_{1y} and Q_{2y} satisfy the reduced scalar Helmholtz equations:

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right]Q_{1y}+k_{1xz}^2 Q_{1y} = 0, \tag{7a}$$

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right]Q_{2y}+k_{2xz}^2 Q_{2y}=0. \tag{7b}$$

On the perfectly conducting plates the tangential component of electric field must vanish. This implies that $E_x = 0 = E_y$ [22], [23] for $z < 0$, $x = \pm b$. Using this fact in Eqs. (3a), (3b) we obtain:

$$Q_{1y}-i\eta_2 Q_{2y}=0, \quad z<0,\ x=\pm b, \tag{8a}$$

$$Q_{1x}-i\eta_2 Q_{2x}=0, \quad z<0,\ x=\pm b. \tag{8b}$$

With the help of Eqs. (6a), (6c) and (8a), (8b), we have:

$$\frac{\partial Q_{1y}}{\partial x}\mp\delta\frac{\partial Q_{1y}}{\partial z}=0, \quad z<0,\ x=\pm b, \tag{9}$$

where

$$\delta=(\gamma_1 k_{2xz}^2+\gamma_2 k_{1xz}^2)[ik_y(k_{1xz}^2-k_{2xz}^2)]^{-1}$$

and $\mp$ signs in Eq. (9) correspond to the upper and lower sides of each plate, respectively. It is worthwhile noting that the boundary conditions (9) are of the same form as impedance boundary conditions [19].

Because the fields radiated by any finite-sized source can be represented in terms of an angular spectrum of plane waves, without loss of generality we can set the incident field to be a plane wave. Thus,

$$Q_1(x,z)=Q_1^{\mathrm{inc}}(x,z)+Q_1^{\mathrm{sca}}(x,z) \tag{10}$$

with

$$Q_{1y}^{\mathrm{inc}}(x,z)=e^{-i(k_y y+k_{1x}x+k_{1z}z)}, \tag{11}$$

where $k_{1xz}^2=k_{1x}^2+k_{1z}^2=(\gamma_1^2-k_y^2)$. Our aim is to find $Q_{1y}^{\mathrm{sca}}(x,z)$ such that Eq. (9) is satisfied by

$$Q_{1y}(x,z)=Q_{1y}^{\mathrm{inc}}(x,z)+Q_{1y}^{\mathrm{sca}}(x,z), \tag{12}$$

and we recall the implicit $\exp(-ik_y y)$ dependence of all fields on the variable y.

For a unique solution of the problem we also require

$$Q_{1y}^{\mathrm{sca}}(x,z)=O[\exp(-\mathrm{Im}\,k_{1xz})z] \text{ as } z\to\infty,$$

$$Q_{1y}^{\mathrm{sca}}=O[\exp(\mathrm{Im}\,k_{1z})z] \text{ as } z\to-\infty, \tag{13a}$$

and the edge condition [24]:

$$Q_{1y}(x,z)=O(1),$$

$$\frac{\partial}{\partial x}Q_{1y}(x,z)=O(r^{-1/2}) \text{ as } r\to 0. \tag{13b}$$

In Equation (13b), r is the distance from (b,z) to $(b,0)$ or $(-b,z)$ to $(-b,0)$, respectively, with $z>0$.

We note that the field Q_{2y} also satisfies the same set of conditions. *i.e.*, Eqs. (9) and (13a), (13b). We must also observe at this juncture that, in effect, we need to consider the diffraction of only one scalar field, *i.e.*, either Q_{1y} or Q_{2y}, at a time, but the presence of the other scalar field is reflected in the complicated nature of the boundary conditions (9).

From Equations (7a) and (12)

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}+k_{1xz}^2\right]Q_{1y}^{sca}=0 \tag{14}$$

where: $k_{1xz}=\mathrm{Re}(k_{1xz})+i\mathrm{Im}(k_{1xz})$, and $\mathrm{Im}(k_{1xz})\to 0^+$ is the loss factor of the medium.

3. Wiener–Hopf method

We now suppress the y-dependence in our analysis. If we introduce the Fourier transform

$$\psi(x,v)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}Q_{1y}^{sca}(x,z)\,e^{ivz}\mathrm{d}z \tag{15}$$

where $v=\sigma+i\tau$, then it follows from the asymptotic behaviour of $Q_{1y}^{sca}(x,z)$ that $\psi(x,v)$ is regular in the strip defined by

$$\mathrm{Im}(k_{12})>\mathrm{Im}(v)>-\mathrm{Im}(k_{1xz}). \tag{16}$$

The transformed wave equation (14) reads

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}x^2}-\varkappa^2\right]\psi(x,v)=0 \tag{17}$$

where $\varkappa=(v^2-k_{1xz}^2)^{1/2}$ and the proper branch for the double mixed function $\varkappa$ is chosen such that $\mathrm{Re}\,\varkappa>0$. Introducing the Fourier integrals as

$$\psi_+(x,v)=\frac{1}{\sqrt{2\pi}}\int_{0}^{\infty}Q_{1y}^{sca}(x,z)\,e^{ivz}\mathrm{d}z,$$

$$\psi_-(x,v)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0}Q_{1y}^{sca}(x,z)\,e^{ivz}\mathrm{d}z,$$

we express $\psi(x,v)$ as

$$\psi(x,v)=\psi_+(x,v)+\psi_-(x,v) \tag{18a}$$

where the subscripts "$-$" and "$+$" indicate that $\psi_-(x,v)$ is regular in the lower-half v plane defined by $\mathrm{Im}(v)<\mathrm{Im}(k_{1z})$, and $\psi_+(x,v)$ is regular in the upper-half v plane defined by $\mathrm{Im}(v)>-\mathrm{Im}(k_{1xz})$, respectively.

The inverse Fourier transform is

$$Q_{1y}^{sca}(x, z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(x, \nu) e^{-i\nu z} d\nu. \tag{18b}$$

Using Eqs. (11) and (12) in Eq. (9) and then taking Fourier transform with respect to z of the resulting expression we obtain

$$\psi_-(b^+, \nu) - (i/\nu\delta)\psi'_-(b^+, \nu) + \frac{i}{(2\pi)^{1/2}\nu\delta}\left[\frac{k_{1x} - \delta k_{1z}}{\nu - k_{1z}}\right] e^{-ik_{1x}b} = 0, \tag{19a}$$

$$\psi_-(b^-, \nu) - (i/\nu\delta)\psi'_-(b^-, \nu) - \frac{i}{(2\pi)^{1/2}\nu\delta}\left[\frac{k_{1x} + \delta k_{1z}}{\nu - k_{1z}}\right] e^{-ik_{1x}b} = 0, \tag{19b}$$

$$\psi_-(-b^+, \nu) - (i/\nu\delta)\psi'_-(-b^+, \nu) + \frac{i}{(2\pi)^{1/2}\nu\delta}\left[\frac{k_{1x} - \delta k_{1z}}{\nu - k_{1z}}\right] e^{ik_{1x}b} = 0, \tag{19c}$$

$$\psi_-(-b^-, \nu) + (i/\nu\delta)\psi'_-(-b^-, \nu) - \frac{i}{(2\pi)^{1/2}\nu\delta}\left[\frac{k_{1x} + \delta k_{1z}}{\nu - k_{1z}}\right] e^{ik_{1x}b} = 0 \tag{19d}$$

where b^+ and b^- denote the upper and lower sides of the plates at $x = \pm b$ and the prime denotes differentiation with respect to x.

The solution of Eq. (17) satisfying the radiation condition, can be formally written as

$$\psi(x, \nu) = \begin{cases} A(\nu)e^{-\varkappa x}, & x > b \\ B(\nu)e^{-\varkappa x} + C(\nu)e^{\varkappa x}, & -b < x < b \\ D(\nu)e^{\varkappa x}, & x < -b \end{cases} \tag{20}$$

where: $A(\nu)$, $B(\nu)$, $C(\nu)$ and $D(\nu)$ are unknowns. In terms of Eq. (18a), the values of $\psi(x, \nu)$ at the two plates can easily be written as:

$$\psi_+(b, \nu) + \psi_-(b^+, \nu) = A(\nu)e^{-\varkappa b}, \tag{21a}$$

$$\psi_+(b, \nu) + \psi_-(b^-, \nu) = B(\nu)e^{-\varkappa b} + C(\nu)e^{\varkappa b}, \tag{21b}$$

$$\psi_+(-b, \nu) + \psi_-(-b^+, \nu) = B(\nu)e^{\varkappa b} + C(\nu)e^{\varkappa b}, \tag{21c}$$

$$\psi_+(-b, \nu) + \psi_-(-b^-, \nu) = D(\nu)e^{-\varkappa b}, \tag{21d}$$

$$\psi'_+(b, \nu) + \psi'_-(b^+, \nu) = -\varkappa A(\nu)e^{-\varkappa b}, \tag{22a}$$

$$\psi'_+(b, \nu) + \psi'_-(b^-, \nu) = -\varkappa B(\nu)e^{-\varkappa b} + \varkappa C(\nu)e^{\varkappa b}, \tag{22b}$$

$$\psi'_+(-b, \nu) + \psi'_-(-b^+, \nu) = -\varkappa B(\nu)e^{\varkappa b} + \varkappa C(\nu)e^{-\varkappa b}, \tag{22c}$$

$$\psi'_+(-b, \nu) + \psi'_-(-b^-, \nu) = \varkappa D(\nu)e^{-\varkappa b}, \tag{22d}$$

where:

$$\psi_+(\pm b^+, \nu) = \psi_+(\pm b^-, \nu) = \psi_+(\pm b, \nu).$$

From Eqs. (21) and (22) we have

$$A(\nu) = \left[J_-(b,\nu) - \frac{1}{\varkappa} J'_-(b,\nu)\right] e^{\varkappa b} - \left[J_-(-b,\nu) - \frac{1}{\varkappa} J'_-(-b,\nu)\right] e^{-\varkappa b}, \tag{23a}$$

$$B(\nu) = -\left[J_-(-b,\nu) - \frac{1}{\varkappa} J'_-(-b,\nu)\right] e^{-\varkappa b}, \tag{23b}$$

$$C(\nu) = -\left[J_-(b,\nu) + \frac{1}{\varkappa} J'_-(b,\nu)\right] e^{-\varkappa b}, \tag{23c}$$

$$D(\nu) = \left[J_-(-b,\nu) + \frac{1}{\varkappa} J'_-(b,\nu)\right] e^{\varkappa b} - \left[J_-(b,\nu) + \frac{1}{\varkappa} J'_-(b,\nu)\right] e^{-\varkappa b}. \tag{23d}$$

In writing Eqs. (23) we have used

$$J_-(b,\nu) = \frac{1}{2}\left[\psi_-(b^+,\nu) - \psi_-(b^-,\nu)\right], \tag{24a}$$

$$J_-(-b,\nu) = \frac{1}{2}\left[\psi_-(-b^-,\nu) - \psi_-(-b^+,\nu)\right], \tag{24b}$$

$$J'_-(b,\nu) = \frac{1}{2}\left[\psi'_-(b^+,\nu) - \psi'_-(b^-,\nu)\right], \tag{24c}$$

$$J'_-(-b,\nu) = \frac{1}{2}\left[\psi'_-(-b^-,\nu) - \psi'_-(-b^+,\nu)\right]. \tag{24d}$$

Substitution of Eqs. (23a) and (24c) in Eq. (22a) yields

$$\psi'_+(b,\nu) + \frac{1}{2}\left[\psi'_-(b^+,\nu) + \psi'_-(b^-,\nu)\right]$$
$$= -\varkappa J_-(b,\nu) + \varkappa e^{-2\varkappa b}\left[J_-(-b,\nu) - \frac{1}{\varkappa} J'_-(-b,\nu)\right]. \tag{25}$$

Subtraction of Eq. (19a) from Eq. (19b) gives

$$\psi'_-(b^+,\nu) + \psi'_-(b^-,\nu) = -2i\nu\delta J_-(b,\nu) + \frac{k_{1x} e^{-ik_{1x}b}}{(2\pi)^{1/2}(\nu - k_{1z})}. \tag{26}$$

Making use of Eq. (26) in Eq. (25), we have

$$\psi'_+(b,\nu) + J_-(b,\nu)\left[\varkappa - i\nu\delta\right] + \frac{k_{1x} e^{-ik_{1x}b}}{(2\pi)^{1/2}(\nu - k_{1z})} = \varkappa e^{-2\varkappa b}\left[J_-(-b,\nu) - \frac{1}{\varkappa} J'_-(-b,\nu)\right]. \tag{27}$$

In a similar way, we can derive the following equations:

$$\psi'_+(-b,\nu) - J_-(b,\nu)\left[\varkappa - i\nu\delta\right] + \frac{k_{1x} e^{ik_{1x}b}}{(2\pi)^{1/2}(\nu - k_{1z})}$$
$$= -\varkappa e^{-2\varkappa b}\left[J_-(b,\nu) + \frac{1}{\varkappa} J'_-(b,\nu)\right], \tag{28}$$

$$\psi_+(b,\nu)+J'_-(b,\nu)\left[\frac{1}{\varkappa}+\frac{i}{\nu\delta}\right]+\frac{ik_{1z}e^{-ik_{1x}b}}{\nu(2\pi)^{1/2}(\nu-k_{1z})}$$

$$=-e^{-2\varkappa b}\left[J_-(-b,\nu)-\frac{1}{\varkappa}J'_-(-b,\nu)\right], \tag{29}$$

$$\psi_+(-b,\nu)-J'_-(-b,\nu)\left[\frac{1}{\varkappa}+\frac{i}{\nu\delta}\right]+\frac{ik_{1z}e^{ik_{1x}b}}{\nu(2\pi)^{1/2}(\nu-k_{1z})}$$

$$=-e^{-2\varkappa b}\left[J_-(b,\nu)+\frac{1}{\varkappa}J'_-(b,\nu)\right]. \tag{30}$$

Now subtraction and addition of Eqs. (27) and (28), respectively, gives

$$D'_+(\nu)-\frac{2ik_{1x}\sin(k_{1x}b)}{(2\pi)^{1/2}(\nu-k_{1z})}=i\nu\delta S_-(\nu)-2\varkappa^2 bS_-(\nu)G(\nu)+R'_-(\nu)e^{-2\varkappa b}, \tag{31}$$

$$S'_+(\nu)+\frac{2k_{1x}\cos(k_{1x}b)}{(2\pi)^{1/2}(\nu-k_{1z})}=i\nu\delta D_-(\nu)-2\varkappa D_-(\nu)L(\nu)-T'_-(\nu)e^{-2\varkappa b}. \tag{32}$$

Again, subtracting and adding Eqs. (29) and (30), respectively, yields

$$P_+(\nu)+\frac{2k_{1z}\sin(k_{1x}b)}{\nu(2\pi)^{1/2}(\nu-k_{1z})}=(-i/\nu\delta)T'_-(\nu)-2bT'_-(\nu)G(\nu)+D_-(\nu)e^{-2\varkappa b}, \tag{33}$$

$$Q_+(\nu)+\frac{2ik_{1z}\cos(k_{1x}b)}{\nu(2\pi)^{1/2}(\nu-k_{1z})}=(-i/\nu\delta)R'_-(\nu)-(2/\varkappa)R'_-(\nu)L(\nu)-S_-(\nu)e^{-2\varkappa b}. \tag{34}$$

In writing Eqs. (31)–(34), we have used:

$$\left.\begin{aligned}
D'_+(\nu)&=\psi'_+(b,\nu)-\psi'_+(-b,\nu),\\
S'_+(\nu)&=\psi'_+(b,\nu)+\psi'_+(-b,\nu),\\
P_+(\nu)&=\psi_+(b,\nu)-\psi_+(-b,\nu),\\
Q_+(\nu)&=\psi_+(b,\nu)+\psi_+(-b,\nu),\\
S_-(\nu)&=J_-(b,\nu)+J_-(-b,\nu),\\
D_-(\nu)&=J_-(b,\nu)-J_-(-b,\nu),\\
R'_-(\nu)&=J'_-(b,\nu)-J'_-(-b,\nu),\\
T'_-(\nu)&=J'_-(b,\nu)+J'_-(-b,\nu),
\end{aligned}\right\} \tag{35}$$

$$G(\nu)=\frac{e^{-\varkappa b}\sinh\varkappa b}{\varkappa b}, \tag{36}$$

$$L(\nu)=e^{-\varkappa b}\cosh\varkappa b. \tag{37}$$

The functions $G(\nu)$ and $L(\nu)$ may be factorized as

$$G(\nu)=G_+(\nu)G_-(\nu)=G_+(\nu)G_+(-\nu), \tag{38a}$$

$$L(\nu)=L_+(\nu)L_-(\nu)=L_+(\nu)L_+(-\nu) \tag{38b}$$

where:

$$G_+(\nu) = \left[\frac{\sin k_{1xz}b}{k_{1xz}b}\right]^{1/2} \exp\left\{\frac{ib\nu}{\pi}[1 - C_1 + \ln(2\pi/k_{1xz}b) + i\pi/2]\right\}$$

$$\times \exp\left\{\frac{ib\varkappa}{\pi}\ln((\nu-\varkappa)/k_{1xz})\right\} \prod_{\substack{n=2\\ \text{even}}}^{\infty} (1 + \nu/i\varkappa_n)e^{i2\nu b/n\pi},$$

$$L_+(\nu) = \left[\cos k_{1xz}b\right]^{1/2} \exp\left\{\frac{ib\nu}{\pi}[1 - C_1 + \ln(\pi/2k_{1xz}b) + i\pi/2]\right\}$$

$$\times \exp\left\{\frac{ib\varkappa}{\pi}\ln((\nu-\varkappa)/k_{1xz})\right\} \prod_{\substack{n=2\\ \text{even}}}^{\infty} (1 + \nu/i\varkappa_n)e^{i2\nu b/n\pi},$$

$\varkappa_n = [(n\pi/2b)^2 - k_{1xz}^2]^{1/2}$, $C_1 = 0.5772 \ldots$ is Euler's constant.

Equations (31)–(34) are the desired Wiener–Hopf equations to be solved for the unknowns A, B, C and D. Their solution is given in the next Section.

4. Simplification of Wiener–Hopf equations

Let us first concentrate on the solution of Eq. (31). Rewrite it as

$$\frac{D'_+(\nu)}{(\nu+k_{1xz})G_+(\nu)} - i\delta U_+(\nu) - V_+(\nu) - \frac{2ik_{1x}\sin(k_{1x}b)}{(2\pi)^{1/2}(\nu-k_{1z})}$$
$$\times\left[\frac{1}{(\nu+k_{1xz})G_+(\nu)} - \frac{1}{(k_{1z}+k_{1xz})G_+(k_{1z})}\right]$$
$$= i\delta U_-(\nu) - V_-(\nu) + \frac{2ik_{1x}\sin(k_{1x}b)}{(2\pi)^{1/2}(\nu-k_{1z})(k_{1z}+k_{1xz})G_+(k_{1z})}$$
$$-2(\nu-k_{1xz})bS_-(\nu)G_-(\nu) \qquad (38c)$$

where the splitting technique of Noble [19] has been used

$$\varkappa = (\nu^2 - k_{1xz}^2)^{1/2} = (\nu+k_{1xz})^{1/2}(\nu-k_{1xz})^{1/2},$$

$$\frac{S_-(\nu)}{(\nu+k_{1xz})G_+(\nu)} = U_+(\nu) + U_-(\nu),$$

$$\frac{R'_-(\nu)e^{-2\varkappa b}}{(\nu+k_{1xz})G_+(\nu)} = V_+(\nu) + V_-(\nu).$$

We note that all the terms on the left-hand side are "+" functions, while all the terms on the right-hand side are "−" functions. The left-hand side of Eq. (38c) is analytic in the domain $-\mathrm{Im}(k_{1xz}) < \mathrm{Im}(\nu)$ and the right-hand side is analytic in the domain $\mathrm{Im}(\nu) < \mathrm{Im}(k_{1z})$, and thus both expressions define an entire function because of the strip common to both domains. With the help of the edge condition, it may be shown that both sides of Eq. (38c) are equal to zero and the resultant equations are valid for all v. In particular, equating the right-hand side of Eq. (38c) to zero gives

$$S_{-}(v)=\frac{iv\delta U_{-}(v)+V_{-}(v)}{2b(v-k_{1xz})G_{-}(v)}+\frac{(v-k_{1xz})^{-1}ik_{1x}\sin(k_{1x}b)}{b(2\pi)^{1/2}(v-k_{1z})(k_{1xz}+k_{1z})G_{+}(k_{1z})G_{-}(v)}. \tag{39}$$

In a similar fashion, Eqs. (32), (33) and (34) can be shown to give

$$D_{-}(v)=\frac{i\delta X_{-}(v)-Y_{-}(v)}{2(v-k_{1xz})^{1/2}L_{-}(v)}-\frac{(v-k_{1xz})^{-1/2}k_{1x}\cos(k_{1x}b)}{(2\pi)^{1/2}(v-k_{1z})(k_{1xz}+k_{1z})^{1/2}L_{+}(k_{1z})L_{-}(v)}, \tag{40}$$

$$T'_{-}(v)=\frac{(-i/\delta)E_{-}(v)+F_{-}(v)}{2bG_{-}(v)}+\frac{\sin(k_{1x}b)}{(2\pi)^{1/2}vG_{+}(0)G_{-}(v)b}$$
$$-\frac{\sin(k_{1x}b)}{(2\pi)^{1/2}(v-k_{1z})G_{+}(k_{1z})G_{-}(v)b}, \tag{41}$$

$$R'_{-}(v)=-\frac{(i/\delta)W_{-}(v)+I_{-}(v)}{2(v-k_{1xz})^{-1/2}L_{-}(v)}+\frac{i(v-k_{1xz})^{1/2}(k_{1xz})^{1/2}\cos(k_{1x}b)}{(2\pi)^{1/2}vL_{+}(0)L_{-}(v)}$$
$$-\frac{i(v-k_{1xz})^{1/2}(k_{1z}+k_{1xz})^{1/2}\cos(k_{1x}b)}{(2\pi)^{1/2}(v-k_{1z})L_{+}(k_{1z})L_{-}(v)} \tag{42}$$

where:

$$\frac{D_{-}(v)}{(v+k_{1xz})^{1/2}L_{+}(v)}=X_{+}(v)+X_{-}(v),\quad \frac{T'_{-}(v)}{G_{+}(v)}=E_{+}(v)+E_{-}(v),$$

$$\frac{T'_{-}(v)e^{-2\varkappa b}}{(v+k_{1xz})^{1/2}L_{+}(v)}=Y_{+}(v)+Y_{-}(v),\quad \frac{D_{-}(v)e^{-2\varkappa b}}{G_{+}(v)}=F_{+}(v)+F_{-}(v),$$

$$\frac{R'_{-}(v)(v+k_{1xz})^{1/2}}{L_{+}(v)}=W_{+}(v)+W_{-}(v),$$

$$\frac{S_{-}(v)e^{-2\varkappa b}(v+k_{1xz})^{1/2}}{L_{+}(v)}=I_{+}(v)+I_{-}(v).$$

From Eqs. (18b), (20), (23) and (35) we can write

$$Q_{1y}^{\text{sca}}(x,z)=-\frac{1}{(2\pi)^{1/2}}\int_{-\infty+i\tau}^{\infty+i\tau}[S_{-}(v)\cosh\varkappa x+D_{-}(v)\sinh\varkappa x$$
$$+\frac{1}{\varkappa}R'_{-}(v)\cosh\varkappa x+\frac{1}{\varkappa}T'_{-}(v)\sinh\varkappa x]e^{-\varkappa b-ivz}\,dv, \tag{43}$$

where: $-\mathrm{Im}(k_{1xz})<\mathrm{Im}(v)<\mathrm{Im}(k_{1z})$.

5. Field within the waveguide

The transmitted field inside the waveguide can be calculated from Eq. (43). For negative z, we enclose the contour of integration in the upper half-plane. The integrand has simple poles at: (i) $v = k_{1xz}$, k_{1z} and $v = 0$, (ii) $v = i\varkappa_n$ ($n = 0, 2, 4, \ldots$) in $S_-(v)$ and $R'_-(v)$ corresponding to the equation: $G_-(v) = 0$, (iii) $v = i\varkappa_n$ ($n = 1, 3, 5, \ldots$) in $D_-(v)$ and $T'_-(v)$ corresponding to $L_-(v) = 0$. Evaluating the residues, we have

$$
\begin{aligned}
Q_{1y}^{\mathrm{sca}}(x,z) = {} & -e^{-i(k_{1x}x+k_{1z}z)} + \frac{\sin(k_{1x}b)e^{-ik_{1xz}z}}{k_{1x}bG_-(k_{1xz})G_+(k_{1z})} \\
& + \left\{ \sum_{\substack{n=1\\ \text{odd}}}^{\infty} \frac{(-1)2k_{1x}b\cos(k_{1x}b)(k_{1xz}+i\varkappa_n)^{1/2}}{n\pi(k_{1xz}+k_{1z})^{1/2}(i\varkappa_n-k_{1z})L_+(k_{1z})L'_-(i\varkappa_n)} \right. \\
& + \sum_{\substack{n=2\\ \text{even}}}^{\infty} \frac{k_{1x}\sin(k_{1x}b)}{b(k_{1xz}+k_{1z})(k_{1xz}-i\varkappa_n)(k_{1z}-i\varkappa_n)G_+(k_{1z})G'_-(i\varkappa_n)} \\
& + \sum_{\substack{n=1\\ \text{odd}}}^{\infty} \frac{\sin(k_{1x}b)}{n\pi G'_-(i\varkappa_n)}\left[\frac{1}{(k_{1z}-i\varkappa_n)G_+(k_{1z})} + \frac{i}{\varkappa_n G_+(0)}\right] \\
& \left. + \sum_{\substack{n=2\\ \text{even}}}^{\infty} \frac{\cos(k_{1x}b)}{bL'_-(i\varkappa_n)}\left[\frac{(k_{1z}-i\varkappa_n)^{-1}}{L_+(k_{1z})} + \frac{i}{\varkappa_n L_+(0)}\right]\right\} \cos(n\pi(x-b)/2b)e^{\varkappa_n z} \\
& - e^{i(k_{1x}x-k_{1z}z)} + \frac{i\delta U_-(k_{1xz})+V_-(k_{1xz})}{2bG_-(k_{1xz})}e^{-ik_{1xz}z} \\
& + \frac{1}{2i}\left\{\frac{\cos(k_{1x}b)\cos(k_{1xz}x)}{\cos(k_{1xz}b)} + \frac{\sin(k_{1x}b)\sin(k_{1xz}x)}{\sin(k_{1xz}b)}\right\} \\
& + [M_-(i\varkappa_n)+N_-(i\varkappa_n)+H_-(i\varkappa_n)+O_-(i\varkappa_n)]\cos(n\pi(x-b)/2b)e^{\varkappa_n z},
\end{aligned}
$$

where:

$$
M_-(i\varkappa_n) = \frac{i\delta X_-(i\varkappa_n) - Y_-(i\varkappa_n)}{2(i\varkappa_n - k_{1xz})^{1/2}L'_-(i\varkappa_n)},
$$

$$
N_-(i\varkappa_n) = -\frac{(i/\delta)E_-(i\varkappa_n) - F_-(i\varkappa_n)}{2bG'_-(i\varkappa_n)},
$$

$$
H_-(i\varkappa_n) = \frac{i\delta U_-(i\varkappa_n) + V_-(i\varkappa_n)}{2b(i\varkappa_n - k_{1xz})G'_-(i\varkappa_n)},
$$

$$O_-(i\varkappa_n) = -\left\{\frac{(i/\delta)W_-(i\varkappa_n)+I_-(i\varkappa_n)}{2L'_-(i\varkappa_n)}\right\}(i\varkappa_n - k_{1xz})^{1/2},$$

$$L'_-(i\varkappa_n) = \left.\frac{\mathrm{d}L_-(v)}{\mathrm{d}v}\right|_{v=i\varkappa_n}, \quad G'_-(i\varkappa_n) = \left.\frac{\mathrm{d}G_-(v)}{\mathrm{d}v}\right|_{v=i\varkappa_n}.$$

6. Conclusions

We have studied a canonical diffraction problem in a homogeneous biisotropic medium. To summarize the preceding analysis, we make the following remarks:

i) The analysis for the right-handed Beltrami field Q_2 is similar to that of its left-handed counterpart Q_1.

ii) We recall that

$$Q_{1y}(x,y,z) = Q_{1y}(x,z)\exp(-ik_y y), \tag{44}$$

and it is clear from Eqs. (6) and (44) that after obtaining $Q_{1y}(x,y,z)$ the remaining field quantities $Q_{1x}(x,y,z)$ and $Q_{1z}(x,y,z)$ can be calculated.

iii) A major use of the presented analysis shall be for antennas operating in hazardous environments, for instance, in the decontamination chambers of hospitals in which textiles and apparatuses are routinely sterilized using highly toxic organic gases. Antennas and bodies coated with chiral materials offer another use for the presented analysis.

In nature chiral media occur as the stereo-isomers of organic chemistry which reveal circular birefringence or optical activity at optical frequencies.

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