

# Optimization of energetic parameters of passively Q-switched lasers

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The paper presents optimization of energy and peak power of the pulse of laser with a slow passive Q-switch. The excited states absorption (ESA) in absorber is included in an optimization process. Using the laser model described it is possible to determine optimal values of unsaturated absorber transmission and reflectivity of the output mirror as a function of the normalized initial gain coefficient  $z$  and parameter  $\alpha$ , characterizing the ratio of the absorption cross-section of nonlinear absorber to the emission cross-section of the laser medium. To optimize the passively Q-switched laser the charts and simple analytical formulas, presented in the paper, can be applied. As an example, optimization of energy and peak power of the pulse of Nd:YAG laser, with  $\text{Cr}^{4+}$ :YAG crystal as passive Q-switch, pumped by 100 W matrix of laser diodes is presented. It has been shown that for laser energy optimization, ESA slightly changes the value of optimal unsaturated absorber transmission and optimal reflectivity of the output mirror of the laser.

## 1. Introduction

The development of materials engineering of solid-state laser media and nonlinear absorbers in the 1990's made it possible to construct simple, efficient, and reliable pulsed Q-switched lasers. At present  $\text{Cr}^{4+}$ :YAG,  $\text{Cr}^{4+}$ :GSGG,  $\text{Cr}^{4+}$ : $\text{Mg}_2\text{SiO}_4$  crystals are commonly used as Q-switches. They are characterized by large absorption cross-section and low saturation energy, at a wavelength of 1.064  $\mu\text{m}$ , and in comparison with dyes and colour centre crystals, previously used in lasers, they have high photochemical and thermal stability and high damage threshold. Due to these properties they are applicable both to continuously and pulsed pumped lasers. Application of the passive Q-switch eliminates the special high-voltage controllers of active modulators (Pockels cell, Kerr cell, and acousto-optic modulators). Due to their use laser design can be simplified and laser dimensions can be reduced, especially when diode pumping is applied. A scheme of passively Q-switched laser is shown in Fig. 1.

Theoretical models of generation process of a laser with a nonlinear absorber have been developing for almost 40 years. They are focused on maximization of the laser pulse energy or peak power, or minimization of pulse duration as a function of the output mirror transmission and unsaturated absorber transmission. Special attention should be paid to work of Degnan [1]. This paper presents the method of maximization of pulse energy as a function of two parameters: the normalized

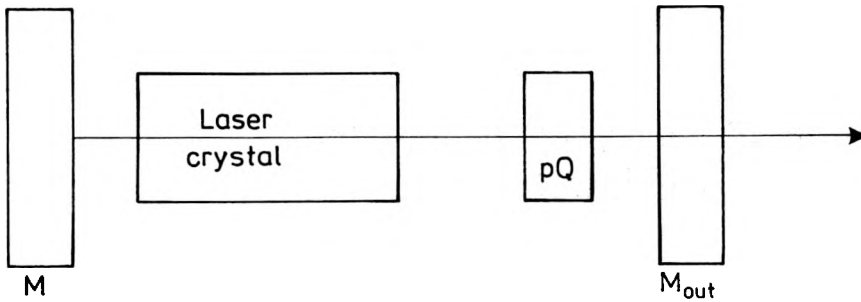


Fig. 1. Optical scheme of passively Q-switched laser with passive Q-switch (pQ); M is the rear mirror,  $M_{out}$  is the output mirror.

round-trip gain and the parameter  $\alpha$  characterizing the ratio of absorption cross-section of the nonlinear absorber to the emission cross-section of the laser medium. This model describes the so-called slow absorber in which lifetime of the excited levels is longer than a generated pulse duration. This assumption is valid for the absorbers used in practice, whose lifetimes of excited level are of the order of single microseconds (*e.g.*,  $\text{Cr}^{4+}:\text{YAG}$  – about  $4 \mu\text{s}$ ). However, the Degnan's model does not include absorption from the excited states that causes considerable losses in laser.

Influence of the ESA on the laser optimization is discussed in [2]. The papers [1] and [2] present a lot of characteristics of a passively Q-switched laser obtained for several  $\alpha$  values. These characteristics can be applied to optimization of a laser with a slow passive Q-switch for which parameter  $\alpha$  is higher than 10 (ideal passive Q-switch) or takes values near  $\alpha = 1.5, 2, 3, 5$ .

The present paper shows optimization of pulse energy and pulse peak power of a laser with slow passive Q-switch with ESA using the normalized variables proposed by Degnan. Moreover, the parameter  $\alpha$  has been modified so as to make its value dependent on magnification  $M$  determining the ratio of the cross-section of generated laser beam in the laser medium to the cross-section of the laser beam in the nonlinear absorber. Using the present model of a passively Q-switched laser one can easily determine the unsaturated absorber transmission and the reflectivity of the output mirror which maximize the output energy or maximize the peak power of laser as a function of a normalized gain coefficient. The charts presented in the paper can be applied for optimization too.

## 2. Theoretical model of a passively Q-switched laser

The laser with a slow passive Q-switch, the scheme of which is presented in Fig. 1, can be described by the set of kinetic equations (Eqs. (1a)–(e)). In comparison with the equations applied in [1], [2], the round-trip gain and losses were substituted by the adequate coefficients of amplification and losses. The photon flux was substituted by the radiation intensity  $I$ . In addition the laser beam magnification  $M$  in the resonator was considered. The equations are of the following form:

$$\frac{dI(t)}{dt} = \frac{2I}{t_r} I(t) \left( k(t) - \frac{l_s}{l} \rho_g(t) - \frac{l_s}{l} \rho_e(t) - \frac{1}{2l} \ln \frac{1}{R} - \rho_D \right), \quad (1a)$$

$$\frac{dk(t)}{dt} = -\frac{k(t)I(t)}{E_a}, \quad (1b)$$

$$\frac{d\rho_g(t)}{dt} = -M \frac{I(t)\rho_g(t)}{E_g}, \quad (1c)$$

$$\frac{d\rho_e(t)}{dt} = -M \frac{I(t)\rho_e(t)}{E_e}, \quad (1d)$$

$$n_g(t) + n_e(t) = n_0 \quad (1e)$$

where:  $I(t)$  – the radiation intensity [ $\text{W}/\text{cm}^2$ ],  $k(t) = \sigma n(t)$  – the gain coefficient in a laser medium [ $1/\text{cm}$ ],  $\sigma$  – the emission cross-section,  $n$  – the population inversion density in a laser medium,  $\sigma_g, \sigma_e$  – the absorption cross-sections from the ground level and the excited absorber level, respectively,  $n_g(t), n_e(t)$  – the populations of the absorption centres on the ground level and excited level, respectively,  $n_0$  – the total concentrations of the absorption centres,  $\rho_g(t) = \sigma_g n_g(t), \rho_e(t) = \sigma_e n_e(t)$  – the loss coefficients resulting from the ground state absorption (GSA) and the excited state absorption, respectively,  $l_s, l$  – the lengths of the nonlinear absorber and the laser medium, respectively,  $t_r = 2l'/c$  – the round-trip transit time of light in the resonator,  $l'$  – the optical length of the resonator,  $c$  – the speed of light,  $h\nu$  – the radiation photon energy,  $\rho_D$  – the dissipative loss coefficient with no transmissive losses of the resonator and the losses resulting from absorption in the absorber,  $R$  – the reflectivity of the output mirror,

$$M = \frac{S_a}{S_{na}} = \frac{\text{cross-section area of the laser beam in the active medium}}{\text{cross-section area of the laser beam in the absorber}},$$

$$E_a = \frac{h\nu}{\gamma\sigma} \text{ – saturation energy for an active medium, } \gamma = 1 \text{ – four-level medium, } \gamma = 2, \text{ – three-level medium,}$$

$$E_g = \frac{h\nu}{\sigma_g} \text{ – saturation energy for GSA, } E_e = \frac{h\nu}{\sigma_e} \text{ – saturation energy for ESA.}$$

The initial conditions are:

$$I(0) = I_0,$$

$$k(0) = k_i,$$

$$\rho_g(0) = \rho_{gi} = \sigma_g n_0, \quad (1f)$$

$$\rho_e(0) = 0,$$

$$n_g(0) = n_0,$$

where:  $k_i$  – the initial gain coefficient,  $\rho_{gi}$  – the absorber's absorption coefficient for a weak signal. Equations (1c), (1d), (1e) describe the model of a four-level nonlinear absorber with ESA (Fig. 2).

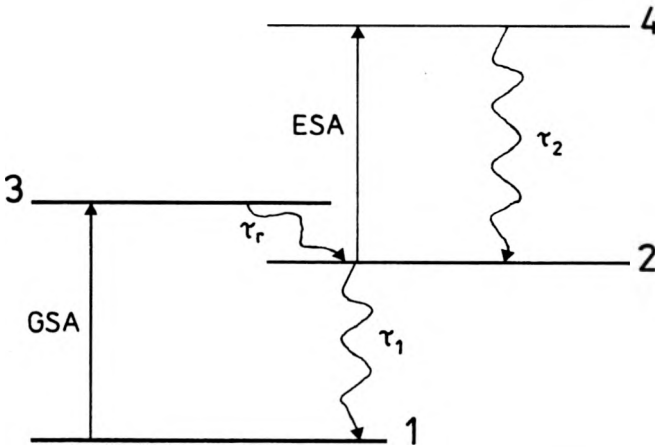


Fig. 2. Four-level model of a nonlinear absorber with ESA:  $\Delta t \ll \tau_1$  is the slow absorber,  $\Delta t \gg \tau_r, \tau_2$ ,  $\Delta t$  – the time of the pulse generation,  $\tau_1, \tau_r, \tau_2$  are the lifetimes on the 2nd, 3rd and 4th levels, respectively.

The set of Equations (1) has no analytical solutions. However, integrating these equations, it is possible to obtain algebraic expressions determining energy, peak power, and pulse duration. Before formulation of these expressions we analyse the dynamics of the changes of laser losses resulting from absorption process in Q-switch that occur during pulse generation. Dividing Eqs. (1c) by (1d) and (1b) by (1c) and next integrating them, we have:

$$\rho_g(t) = \rho_{gi} \left( \frac{k(t)}{k_i} \right)^\alpha, \quad (2)$$

$$\rho_e(t) = \frac{\rho_{gi}}{\text{FOM}} \left[ 1 - \left( \frac{k(t)}{k_i} \right)^\alpha \right], \quad (3)$$

$$\alpha = M \frac{\sigma_g}{\gamma \sigma} \quad (4)$$

$$\text{FOM} = \frac{\sigma_g}{\sigma_e} = \frac{E_e}{E_g}, \quad (5)$$

$$T_0 = \exp(-\rho_{gi} l_s) \quad (6)$$

where:  $T_0$  – the unsaturated absorber transmission,  $\alpha$  – the parameter dependent on selection of the two media: the laser medium and the nonlinear absorber medium as well as resonator geometry, FOM – the figure of merit factor for the nonlinear absorber (the better the absorber, the lower its absorption from the excited states).

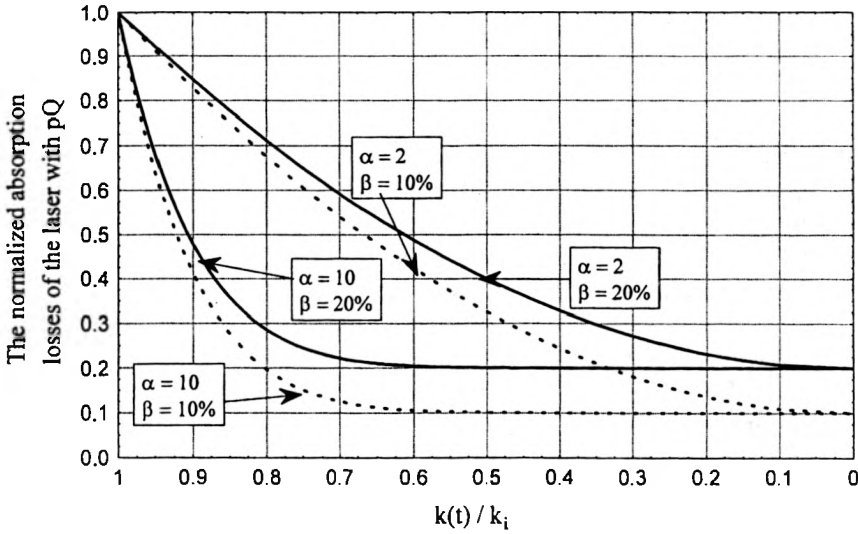


Fig. 3. Dynamics of changes of the absorption losses during pulse generation for  $\alpha = 2, 10$  and  $\beta = 10\%, 20\%$  ( $\beta = 1/\text{FOM}$ ).

It results from Equations (1) and (3) that during pulse generation the laser losses caused by GSA decrease but those caused by ESA increase. The higher the parameter  $\alpha$ , the higher the dynamics of changes. This is shown in Fig. 3 illustrating a change of the normalized total losses introduced by Q-switch during pulse generation. According to (2) and (3) these losses are expressed by the relationship

$$\frac{\rho_a(t) + \rho_e(t)}{\rho_{gi}} = \left(1 - \frac{1}{\text{FOM}}\right) \left(\frac{k(t)}{k_i}\right)^\alpha + \frac{1}{\text{FOM}}. \tag{7}$$

The gain coefficient  $k(t)$  has been assumed to decrease during generation from the initial value  $k_i$  to zero. It results from formula (7) and the charts presented in Fig. 3 that the rate of absorber bleaching increases with the value of parameter  $\alpha$ . The advantageous effect of the magnification  $M$  results from the fact that decreasing the volume of the laser beam in the absorber, in comparison with the generation volume, diminishes the part of energy stored in the laser medium that is lost for absorber saturation. It should be noticed that at the moment of absorber bleaching the losses resulting from ESA still exist. The higher the FOM of the absorber used in the laser, the lower these losses are.

The energy of a generation pulse of passively Q-switched laser is expressed by

$$E = \frac{S_a l'}{c t_r} \ln \frac{1}{R} \int_0^\infty I(t) dt = \frac{h\nu S_a}{2\gamma\sigma} \ln\left(\frac{1}{R}\right) \ln\left(\frac{k_i}{k_f}\right) \tag{8}$$

where  $k_f$  is the final gain coefficient for  $k(t) = k_f, I(t) = 0$ .

According to Equation (1a) for  $t = 0$  the initial gain coefficient  $k_i$  is equal to

$$k_i = \rho_{gi} + \frac{l_s}{l} + \frac{1}{2l} \ln\left(\frac{1}{R}\right) + \rho_D \quad (9)$$

By measuring the energetic characteristics of the laser, with no Q-switch, as a function of pumping energy for various output mirror transmittances, it can be possible to determine the value of initial gain coefficient  $k_i$  for the given pump energy and dissipative loss coefficient  $\rho_D$  using the Findlay-Clay method [3]. In the Q-switched laser, losses resulting from the unsaturated absorber transmission (the first component of the sum in Eq. (9)) and transmission of the output mirrors (the second component of the sum) have to fulfil Eq. (9). Dividing Eq. (1a) by (1b) and next integrating it with the use of relationships (2) and (3) we obtain the transcendental equation determining the value of the final gain coefficient  $k_f$ , so we can have the value of pulse generation energy from (8)

$$k_i - k_f - \left( \rho_{TD} + \frac{l_s}{l} \frac{\rho_{gi}}{\text{FOM}} \right) \ln\left(\frac{k_i}{k_f}\right) - \frac{l_s}{l} \frac{\rho_{gi}}{\alpha \gamma_s} \left[ 1 - \left(\frac{k_f}{k_i}\right)^\alpha \right] = 0 \quad (10)$$

where:

$$\gamma_s = \frac{\text{FOM}}{\text{FOM} - 1}, \quad \rho_{TD} = \rho_T + \rho_D = \frac{1}{2l} \ln\left(\frac{1}{R}\right) + \rho_D. \quad (11)$$

The pulse intensity attains its maximum  $I_{\max}$  when the condition of  $dI(t)/dt = 0$  is fulfilled (then  $k(t) = k_t$ ). The peak power of the generation pulse is expressed by the relationship which is obtained dividing Eq. (1a) by (1b) and next integrating it and substituting  $k(t) = k_t$

$$P = \frac{h\nu S_a l}{\gamma \sigma t_r} \ln\left(\frac{1}{R}\right) \left\{ k_i - k_t + \left( \frac{l_s}{l} \frac{\rho_{gi}}{\text{FOM}} + \rho_{TD} \right) \ln\left(\frac{k_t}{k_i}\right) - \frac{l_s}{l} \frac{\rho_{gi}}{\alpha \gamma_s} \left[ 1 - \left(\frac{k_t}{k_i}\right)^\alpha \right] \right\}. \quad (12)$$

Dividing Eq. (1a) by (1b) and using the condition  $dI(t)/dk = 0$  for  $k = k_t$  we obtain the following equation enabling determination of  $k_t$

$$\frac{\rho_{TD} + \frac{l_s}{l} \frac{\rho_{gi}}{\text{FOM}}}{k_i} + \frac{l_s}{l} \frac{\rho_{gi}}{k_i \gamma_s} \left(\frac{k_t}{k_i}\right)^\alpha = \frac{k_t}{k_i}. \quad (13)$$

Using expressions (8) and (12) the value of pulse duration  $\Delta t$  can be determined

$$\Delta t \simeq \frac{E}{P}. \quad (14)$$

### 3. Optimization of passively Q-switched laser

The present analysis of a passively Q-switched laser is aimed at determination of simple relations and the smallest possible number of graphic characteristics of

a laser to obtain optimal values of the output mirror transmission and the unsaturated absorber transmission which maximizes output energy or pulse peak power. The following variables, which are normalized to the total losses coefficient including dissipative losses and the losses resulting from ESA, will be applied:

$$z = \frac{k_i}{\rho}, \tag{15}$$

$$z_s = \frac{l_s}{l} \frac{1}{\gamma_s} \frac{\rho_{gi}}{\rho}, \tag{16}$$

$$x = \frac{\frac{1}{2l} \ln \left( \frac{1}{R} \right)}{\rho}, \tag{17}$$

$$\kappa = \ln \left( \frac{k_i}{k_f} \right), \tag{18}$$

$$z_t = \frac{k_t}{\rho}, \tag{19}$$

where:  $\rho = \rho_D + \rho_{em}$ ,  $\rho_{em} = \frac{l_s}{l} \frac{\rho_{gi}}{\text{FOM}}$ .

For variables thus defined, Eqs. (8), (9), (10), (13) and (12) used for optimization of passively Q-switched laser take the form:

– initial condition of a generation process

$$z - x - z_s = 1, \tag{20}$$

– final gain coefficient

$$\Phi(x, z_s) = \alpha z (1 - \exp(-\kappa) - \kappa) - z_s (1 - \exp(-\alpha \kappa) - \alpha \kappa) = 0, \tag{21}$$

– gain coefficient  $k_t$

$$\frac{x+1}{z} + \frac{z_s}{z} y^\alpha = y, \quad y = \frac{k_t}{k_i} = \frac{z_t}{z}, \tag{22}$$

– pulse energy

$$E = E_0 \kappa (z - z_s - 1), \quad E_0 = \frac{h\nu S_a \rho l}{\sigma \gamma}, \tag{23}$$

– pulse peak power

$$P = P_0 x z \left[ 1 - y + \frac{x+1}{z} \ln y - \frac{1}{\alpha} \frac{z_s}{z} (1 - y^\alpha) \right],$$

$$P_0 = \frac{2h\nu S_a l^2 \rho^2}{\sigma \gamma t_r}. \tag{24}$$

**3.1. Optimization of pulse generation energy**

For optimization of pulse generation energy of Q-switched laser, the values of  $(z_{s(opt)})$  and  $(x_{opt})$  variables, which maximize the value of expression (23), should be determined for the given normalized initial gain coefficient  $z$ . In order to do that the Lagrange's multipliers method can be applied [1]:

$$\begin{aligned} \frac{dE(z_s, \kappa)}{dz_s} + \lambda \frac{d\Phi(z_s, \kappa)}{dz_s} &= 0, \\ \frac{dE(z_s, \kappa)}{d\kappa} + \lambda \frac{d\Phi(z_s, \kappa)}{d\kappa} &= 0 \end{aligned} \tag{25}$$

where  $\lambda$  is the Lagrange multiplier.

Substituting relationships (21) and (23) into (25) and using the initial condition given in (20) we obtain the following equation:

$$z(\alpha, \kappa) = \frac{(1 - e^{-\alpha\kappa} - \alpha\kappa)^2}{(1 - e^{-\alpha\kappa} - \alpha\kappa)(1 - e^{-\alpha\kappa} - \alpha\kappa e^{-\kappa}) - \alpha(1 - e^{-\kappa} - \kappa)(1 - e^{-\alpha\kappa} - \alpha\kappa e^{-\alpha\kappa})} \tag{26}$$

It enables us to determine, for the given value  $z$ , the value  $\kappa_{opt}$  maximizing the energy of the generated pulse. Graphical solutions of the above equation as a function of  $z$  and for several values of  $\alpha$  are shown in Fig. 4. They were obtained by calculating from (26) the values of  $z$  as a function of the variable  $\kappa$ .

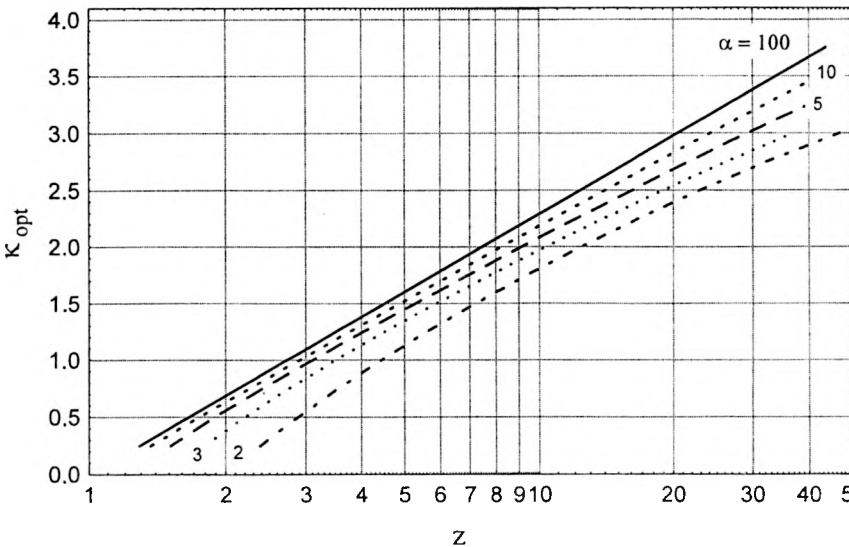


Fig. 4. Value of parameter  $\kappa_{opt}$  maximizing pulse energy as a function of the normalized initial gain coefficient  $z$ .

It results from Fig. 4 that for the given value of the normalized initial gain coefficient  $z$ , the higher the parameter  $\alpha$ , the higher the optimal value  $\kappa_{opt}$  is. It can be seen from the definition of parameter  $\kappa$  (18) that an increase in parameter  $\alpha$  causes



a decrease in the final amplification coefficient and the same increase in generation energy.

The lower the final gain coefficient  $k_f$  is, the greater part of energy stored in the laser medium is generated in a laser pulse. It results from the definition of  $\alpha$  (Eq. (4)) that limited increase in the value of this parameter, due to selection of absorbing medium for the laser medium, can be extended by applying an adequate configuration of a resonator, ensuring the value of magnification  $M$  higher than unity. The passive Q-switch should be situated in a resonator in a place where generated laser beam is of minimal diameter. This conclusion results also from the analysis of the dynamics of changes in absorption losses presented in Fig. 3.

Substituting the value of  $\kappa_{opt}$ , determined from (26) or from Fig. 4, for  $\kappa$  into Eq. (21) we have the expression for  $z_{s(opt)}$  and next for  $x_{opt}$  taking Eq. (20):

$$\frac{z_{s(opt)}}{z} = \frac{(1 - \exp(-\kappa_{opt}) - \kappa_{opt})\alpha}{1 - \exp(-\alpha\kappa_{opt}) - \alpha\kappa_{opt}}, \tag{27}$$

$$\frac{x_{opt}}{z} = 1 - \frac{z_{s(opt)}}{z} - \frac{1}{z}. \tag{28}$$

The parameters  $z_{s(opt)}$  and  $x_{opt}$  maximizing the energy of generated pulse are presented in Figs. 5 and 6 as a function of normalized initial gain coefficient  $z$ .

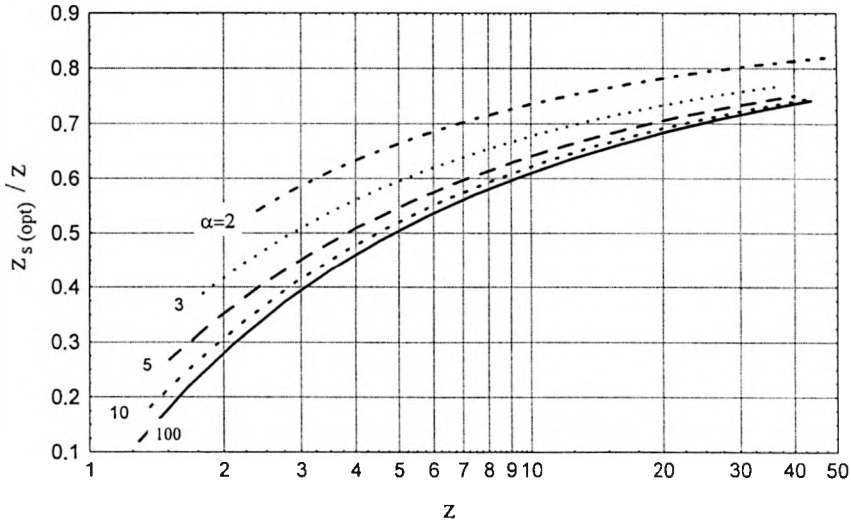


Fig. 5. Dependence of  $z_{s(opt)}/z$  parameter maximizing the energy of a laser pulse on the normalized initial gain coefficient  $z$ .

Using the definitions of variables (6), (15)–(17) we obtain the following relationships determining both the length of the nonlinear absorber  $l_{s(opt)}$  (the initial transmission  $T_{0(opt)}$ ), and reflectivity of the output mirror of the laser  $R_{opt}$  that maximize energy of a generated pulse:

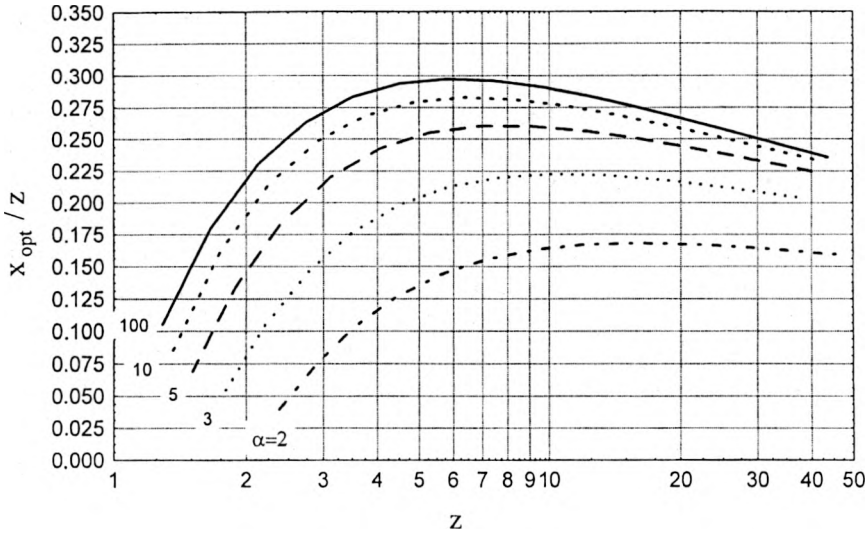


Fig. 6. Dependence of  $x_{opt}/z$  parameter maximizing the energy of a laser pulse on the normalized initial gain coefficient  $z$ .

$$\frac{z_{s(opt)}}{z} = \left(1 - \frac{1}{FOM}\right) \frac{l_{s(opt)} \rho_{gi}}{lk_i}, \quad T_{0(opt)} = \exp(-l_{s(opt)} \rho_{gi}), \tag{29}$$

$$\frac{x_{opt}}{z} = \frac{\frac{1}{2l} \ln\left(\frac{1}{R_{opt}}\right)}{k_i}. \tag{30}$$

It results from the characteristics presented in Figs. 5 and 6, as well as from relationships (29) and (30) that for the given values of the initial normalized gain coefficient  $z$ , together with an increase in parameter  $\alpha$ , both the value of optimal unsaturated absorber transmissions and the value of optimal transmission of the laser output mirror increase. The pulse energy of the energetically-optimized, passively Q-switched laser, is calculated from relationship (23)

$$E = E_0 z \kappa_{opt} \frac{x_{opt}}{z}. \tag{31}$$

It follows from the characteristics presented in Figs. 4 and 6 and relationship (31) that an increase in parameter  $\alpha$  ensures high efficiency of pulse generation. The higher the parameter  $\alpha$ , the higher the  $\kappa_{opt}$  and  $x_{opt}$ , and the higher the value of energy of a generated pulse.

The peak power of pulse generation is calculated from relationship (24)

$$P = P_0 z^2 \frac{x_{opt}}{z} \left[ 1 - y + \left(\frac{x_{opt}}{z} + \frac{1}{z}\right) \ln y - \frac{1}{\alpha} \frac{z_{s(opt)}}{z} (1 - y^\alpha) \right] \tag{32}$$

where  $y$  should be substituted by the value determined from relationship (22). This relationship for  $\alpha \gg 1$  is simplified to the form

$$y = \frac{x_{opt}}{z} + \frac{1}{z} \tag{33}$$

In recapitulation, the procedure of energetic optimization of Q-switched laser with application of the characteristics presented is as follows:

1. The initial gain coefficient  $k_i$  and dissipative loss coefficient  $\rho_D$  is determined by the Findlay-Clay method.

2. For the pair consisting of the laser medium and the nonlinear absorber we determine the value of parameter  $\alpha$  considering magnification  $M$  of a laser beam in a resonator.

3. We calculate the value of the normalized gain coefficient  $z$  from relationship (15), substituting  $\rho_{em} = 0$  (FOM =  $\infty$ ) because the length of a nonlinear absorber is not known yet.

4. The values of parameters  $\frac{z_{s(opt)}}{z}$  and  $\frac{x_{opt}}{z}$  are taken from Figs. 5 and 6.

5. We calculate the value of optimal length of the nonlinear absorber  $l_{s(opt)}$ , optimal value of the unsaturated absorber transmissions  $T_{0(opt)}$ , and the value of optimal reflectivity of the output mirror of a laser  $R_{opt}$  from relationships (39) and (30).

6. If we want to take into account the ESA phenomenon in a nonlinear absorber the optimization procedure should be repeated from the point 3, taking  $\rho_{em} = \frac{l_{s(opt)} \rho_{gi}}{l \text{ FOM}}$ . The procedure should be repeated till the moment the value of optimal length of a nonlinear absorber  $l_{s(opt)}$  is invariable and has required accuracy.

### 3.2. Optimization of peak power of generated pulse

Optimization of the peak power of a generated pulse consists in determination of the initial value of unsaturated absorber transmissions and transmission of the output mirror that maximize expression (24). Using expressions (20) and (22), relationship (24) for the pulse peak power can be presented as a function of parameter  $y$

$$P = P_0 z^2 \left( \frac{y - y^\alpha}{1 - y^\alpha} - \frac{1}{z} \right) \left[ 1 - y + \frac{y - y^\alpha}{1 - y^\alpha} \ln y - \frac{1}{\alpha} (1 - y) \right] \tag{34}$$

The peak power of the pulse reaches its maximum value for  $y = y_{opt}$  which is the solution of the equation  $dP/dy = 0$ . It has the following form:

$$z = \frac{B'}{A'B + AB'} \tag{35}$$

where:

$$A = \frac{y - y^\alpha}{1 - y^\alpha}$$

$$\begin{aligned}
 B &= 1 - y + A \ln y - \frac{1}{\alpha}(1 - y), \\
 A' &= \frac{1 - y^\alpha - \alpha y^{\alpha-1}(1 - y)}{(1 - y^\alpha)^2}, \\
 B' &= A' \ln y + A \frac{1}{y} + \frac{1}{\alpha} - 1.
 \end{aligned}
 \tag{35a}$$

Graphical solutions of Eq. (35) as a function of the normalized initial gain coefficient  $z$  are presented in Fig. 7. Equation (35) is simplified when the condition  $y^\alpha \ll 1$  is fulfilled, then

$$z = \frac{\ln y + \frac{1}{\alpha}}{2y \left( \ln y + \frac{1}{\alpha} \right) + 1 - y - \frac{1}{\alpha}}.
 \tag{36}$$

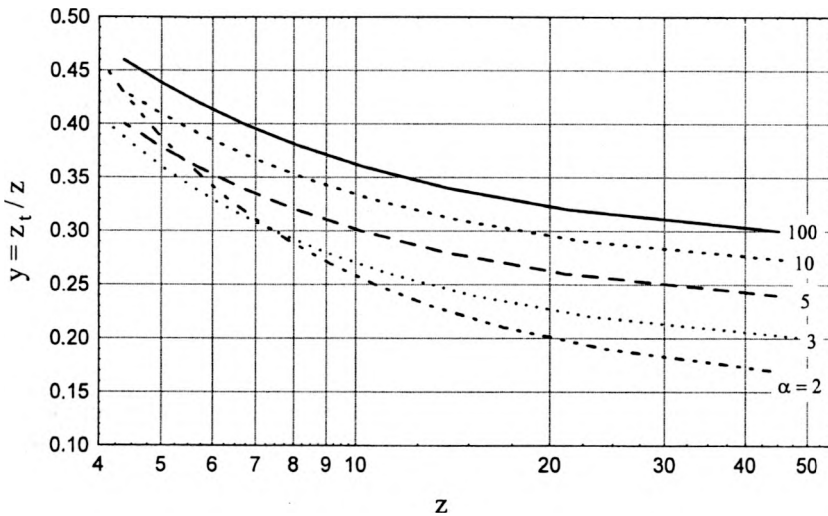


Fig. 7. Characteristics of  $y_{opt}$  maximizing the peak power of a pulse as a function of the normalized initial gain coefficient  $z$ .

Because with an increase in the normalized initial gain coefficient  $z$ ,  $y$  ( $y < 1$ ) decreases, so with an increase in parameter  $\alpha$  the region of variable  $z$  extends, in which approximation (36) can be applied. The above mentioned approximation can be applied for  $z > 40$  when  $\alpha = 3$ , for  $z > 4$  when  $\alpha = 5$ , for  $z > 1.5$  when  $\alpha = 10$ , and for  $z > 1$  when  $\alpha = \infty$ . In the passively Q-switched laser systems, used in practice, the above conditions expressed in Eqs. (36) are easy to fulfil, mainly due to an increase in parameter  $\alpha$ , both by selection of the pair: active medium and nonlinear absorber medium, as well as by an increase in magnification  $M$  of the laser beam in the resonator. Determining the parameters  $z_{s(opt)}$ ,  $x_{opt}$  from Eqs. (20) and (22)

which maximize the peak power of a laser we have:

$$\frac{x_{opt}}{z} = \frac{y_{opt} - y_{opt}^\alpha}{1 - y_{opt}^\alpha} - \frac{1}{z}, \tag{37}$$

$$\frac{z_{s(opt)}}{z} = 1 - \frac{x_{opt}}{z} - \frac{1}{z}. \tag{38}$$

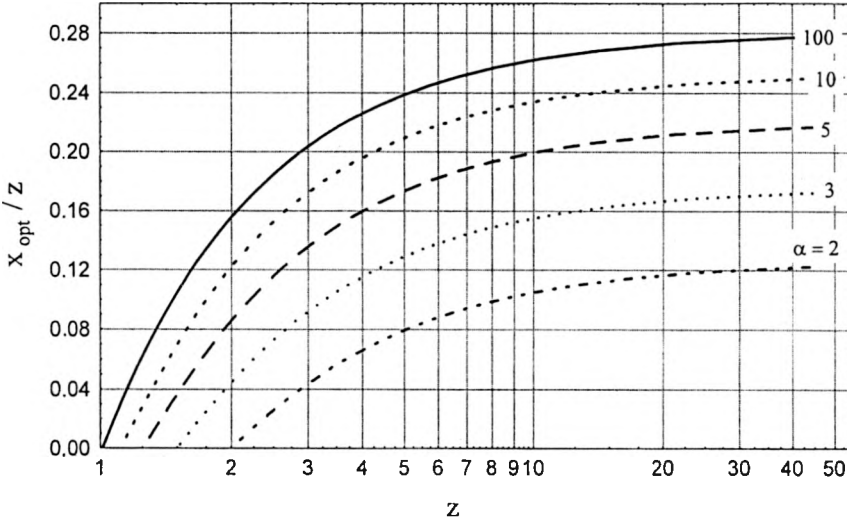


Fig. 8. Characteristics optimizing transmission of the output mirror of laser with a passive Q-switch, when the laser is pulse peak power maximized.

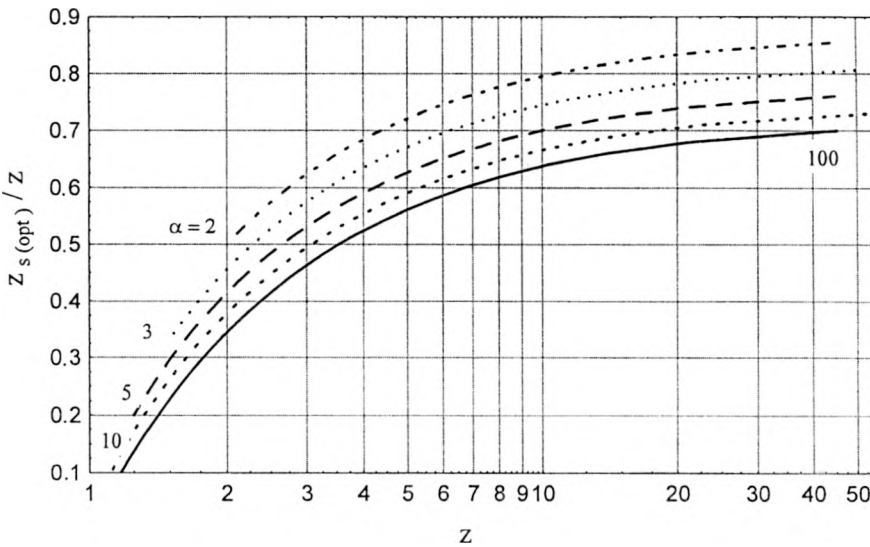


Fig. 9. Characteristics optimizing unsaturated transmission of the nonlinear absorber, when the laser is pulse peak power maximized.

The charts of the above relationships as a function of the normalized initial gain coefficient  $z$  are presented in Figs. 8 and 9. From these characteristics for the given values  $z$  and  $\alpha$  we can determine the values  $z_{s(\text{opt})}/z$  and  $x_{\text{opt}}/z$ . Next, using Eqs. (29) and (30) we calculate the values of optimal unsaturated absorber transmissions  $T_{0(\text{opt})}$  and optimal reflectivity of the output mirror of the laser  $R_{\text{opt}}$  that maximize the peak power of a generated pulse.

The procedure for optimizing the peak power of a Q-switched laser is the same as the procedure of pulse energy optimization presented earlier. Instead of Figs. 5 and 6, Figs. 8 and 9 should be considered.

We calculate the value of a peak power from relationships (34) substituting the value  $y_{\text{opt}}$  determined from Fig. 7 instead of  $y$ .

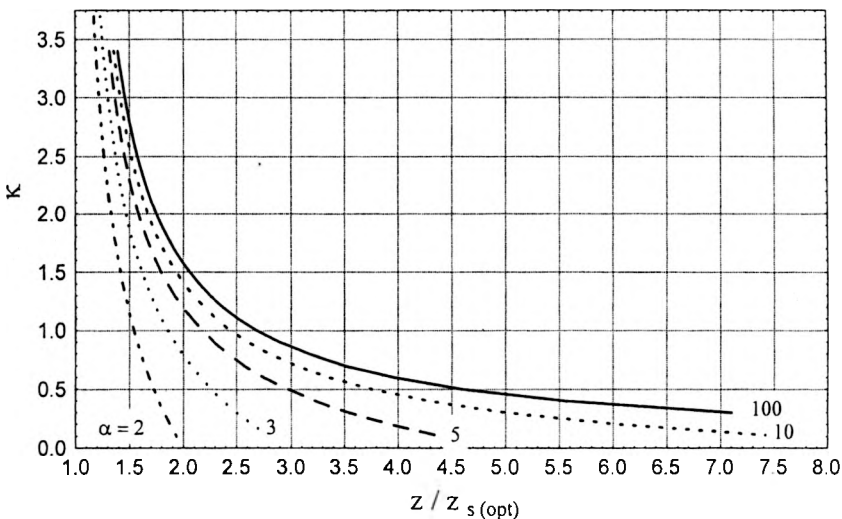


Fig. 10. Characteristics of the parameter  $\kappa$  for passively Q-switched laser, when the laser is pulse peak power maximized.

We calculate the value of pulse energy from relationship (31), substituting the value determined from (21) instead of  $\kappa$  and  $z_{s(\text{opt})}$  instead of  $z_s$ . The dependence of  $\kappa$ , as a function of  $z/z_{s(\text{opt})}$ , for a passively Q-switched, peak power-optimized laser is shown in Fig. 10. The value of generated pulse duration can be determined from relationship (14)

#### 4. Influence of ESA on optimal parameters of passively Q-switched laser

It has already been mentioned in Section 2 that the ESA is a source of additional losses in a passively Q-switched laser and a factor worsening energy parameters of a generated pulse. These losses occur during pulse generation. It cannot be directly concluded from the present analysis how this phenomenon influences optimal values of unsaturated absorber transmissions and reflectivity of the laser's output mirror. This problem is described using, as an example, Nd:YAG laser with the  $\text{Cr}^{4+}$ :YAG

crystal as a passive Q-switch, pumped by 100 W laser diodes array of the SDL-3225 type. For this laser, using  $\text{LiNbO}_3$  crystal Pockels cell, generation of pulses with energy of 2.3 mJ for multi-mode output and 1.7 mJ for single-mode output was obtained. However, optimizing the laser with  $\text{Cr}^{4+}:\text{YAG}$  Q-switch in respect of energy, the pulses of energy of 1.2 mJ and duration of 18 ns for the resonator length of 10 cm, were obtained. For this laser the unsaturated absorber transmission  $T_0$  was equal to 66% and reflectivity of the output mirror  $R$  was 75%. The values of these parameters, optimal in respect of pulse energy, calculated according to the procedure presented in Subject. 3.1, are  $T_{0(\text{opt})} = 66.8\%$ ,  $R_{\text{opt}} = 74.6\%$ . The change of these values in the experiment caused decrease in the pulse energy. This confirms correctness of optimization of the laser being tested. The following values were taken for optimization:  $\sigma = 6.5 \cdot 10^{-19} \text{ cm}^2$ ,  $h\nu = 1.87 \cdot 10^{-16} \text{ mJ}$ ,  $l = 1 \text{ cm}$ ,  $\gamma = 1$ ,  $k_i = 0.564 \text{ cm}^{-1}$ ,  $\sigma_g = 3 \cdot 10^{-18} \text{ cm}^2$ ,  $\sigma_e = 2.1 \cdot 10^{-19} \text{ cm}^2$ ,  $\rho_{gt} = 1.868 \text{ cm}^{-1}$ ,  $\text{FOM} = 14.3$ ,  $\beta = 0.07$ ,  $\gamma_s = 1.075$ ,  $M = 1$ ,  $\alpha = 4.615$ ,  $S_a = 0.013 \text{ cm}^2$ ,  $\rho_D = 0.025 \text{ cm}^{-1}$ ,  $t_r = 6.6 \cdot 10^{-10} \text{ ns}$ .

The successive figures show optimal parameters of the laser under investigation, both in respect of energy and peak power of a pulse, for various and theoretically possible values of FOM of the nonlinear absorber ( $\beta = \text{FOM}^{-1} = 0, 0.07, 0.14, 0.28$ ). The characteristics presented below have been obtained using the procedures described in the previous section.

According to the analysis presented in Section 2, the ESA process causes a decrease in both energy and peak power of a generated pulse and its longer duration. It can be seen from Figs. 11 and 12 that optimization with respect to peak power causes mainly the shortening of the pulse duration.

Figure 13 presents mutual relationship of optimal parameters of a nonlinear absorber and the output mirror. There can be seen a very slight influence of ESA on

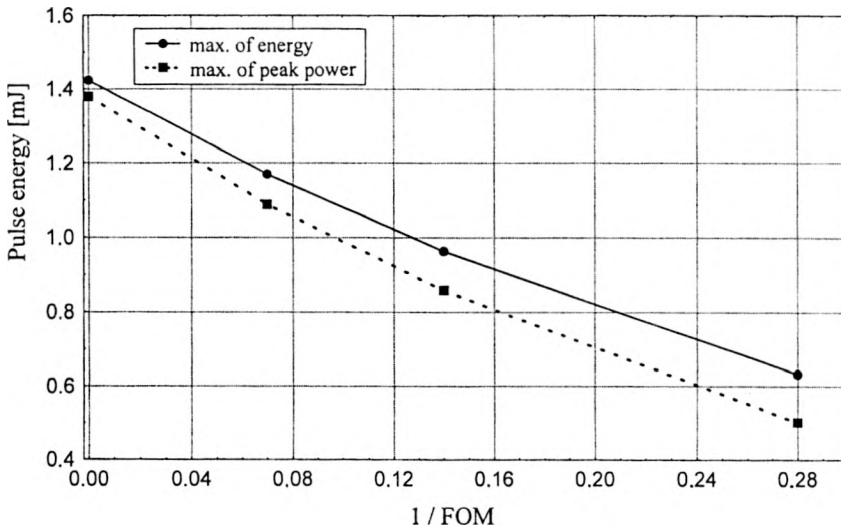


Fig. 11. Change in generation energy of the Nd:YAG laser depending on ESA.

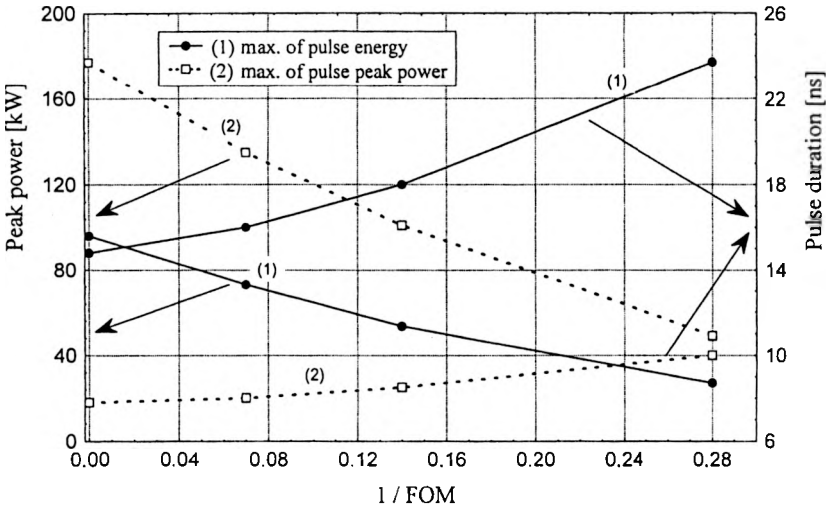


Fig. 12. Characteristics of peak power of a pulse and its duration as a function of ESA.

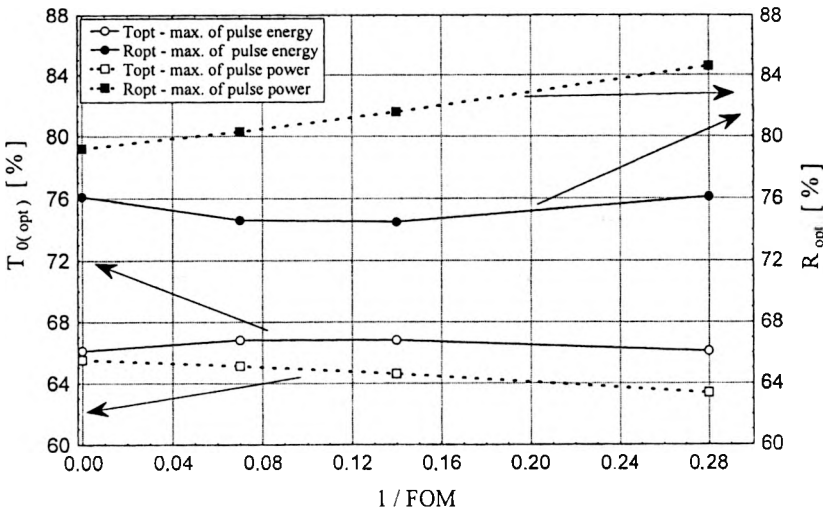


Fig. 13. Dependence of optimal values  $T_{0(opt)}$  and  $R_{opt}$  on ESA for energetic and peak-power optimization of Nd:YAG,  $Cr^{4+}$ :YAG, passively based Q-switched laser.

these parameters in the case of energy optimization. For energy optimization, the change in  $\beta$ , within the range of 0–0.28 causes the change in  $T_{0(opt)}$  and  $R_{opt}$  within the range of 0.7% and for the peak power optimization within the range of 5.5%. Of course, according to initial condition (9) or (20), with an increase in  $T_{0(opt)}$  optimal reflectivity  $R_{opt}$  decreases and vice versa. It follows from Fig. 13 that the ESA phenomenon can be neglected when the laser is energy maximized. It has significant influence only on energy and temporal parameters of a generated pulse.



## 5. Conclusions

Graphical and analytical procedures presented in this paper enable us to optimize a passively Q-switched laser, both in respect of maximal generation energy, as well as maximal peak power of a pulse. The optimization described requires experimental determination, for the given laser, of such parameters as the initial gain coefficient for the given energy  $k_i$ , coefficient of dissipative losses  $\rho_D$ , and magnification  $M$ . It also requires knowledge of physical parameters of a laser medium and a nonlinear absorber medium, which enable determination of FOM and parameters  $\alpha$ . The FOM parameter of a nonlinear absorber influences mainly the energy-time characteristic of a generation process. An increase in efficiency of the optimized laser with a Q-switch can be obtained mainly using the resonator with magnification  $M > 1$ , situating a nonlinear absorber in the waist plane of a laser beam (large  $\alpha$ ) and application of a nonlinear absorber characterized by low ESA (high values of FOM). The problem presented in the paper is the subject of experimental investigations carried out within the frame of the ordered grant PBZ-023-10.

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