Optica Applicata, Vol. XXX, No. 2–3, 2000

Equivalence relations for the enhanced polar Kerr effect at normal incidence

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An enhancement of the polar Kerr effect in the magneto-optical (MO) readout system is achieved by specific dielectric thin-film coatings. The MO medium of refractive index n and MO constant q, overcoated by a dielectric thin film, is equivalent in normal reflection to an uncoated MO medium of parameters N and Q. Following this reasoning we present simple relations for equivalent parameters N and Q of different systems. Behaviour of parameters N and Q is illustrated by numerical examples. The use of equivalent parameters allows an easier comparison among different MO systems.

1. Introduction

The polar Kerr effect is largely used for optical readout of magnetically stored information. High-quality magneto-optical materials having a perpendicular magnetization have been developed. Kerr rotation angles of such materials, however, are of the order of 0.1°, resulting in an insufficient signal-to-noise ratio in the readout system. An enhancement of the polar Kerr effect is achieved by specific dielectric thin-film coatings [1]–[5]. This enhancement is caused when a polarized light beam experiences multiple reflections and interferences in the dielectric thin layers. It depends on the refractive indices and thicknesses of the MO and dielectric materials.

In this paper, we present simple equivalence relations for different MO systems. These relations allow a better understanding and an easier comparison among the MO systems.

2. Characteristics of magneto-optical media

The dielectric tensor of the MO medium is described as

$$\tilde{\varepsilon} = n^2 \begin{matrix} 1 & -jq & 0 \\ jq & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \tag{1}$$

where n denotes a complex refractive index in the unmagnetized state of the medium, q is the complex MO constant which is proportional to the magnetization that is assumed to be in the z direction, and $j = (-1)^{1/2}$.

The normal incident polar Kerr effect in the MO medium is characterized by the response for right (+) and left (−) circularly polarized light. The complex refractive indices that correspond to (+) and (−) polarizations are given by [2], [4]

$$n_{\pm} \simeq n(1 \pm q/2), \quad |q| \ll 1. \tag{2}$$

If we denote by A_p, A_s, and B_p, B_s the electric-field amplitudes of normal incident and reflected p and s modes, the 2×2 extended Jones matrix of the MO medium is introduced as follows [2], [6]:

$$\begin{pmatrix} B_s \\ B_p \end{pmatrix} = \begin{bmatrix} -r & k \\ k & r \end{bmatrix} \begin{pmatrix} A_s \\ A_p \end{pmatrix} \tag{3}$$

where:

$$r = (1/2)(r_{+} + r_{-}), \quad k = (j/2)(r_{+} - r_{-}), \tag{4}$$

$r_{\pm}$ represents the reflection coefficient of the MO medium for (±) polarization, $r_{\pm} = (n_{\pm} - n_0)/(n_{\pm} + n_0)$, and n_0 is the refractive index of the ambient medium, $n_0 = 1$.

If we denote for an s-polarized incident wave $\rho = -k/r$, the Kerr rotation angle θ_{K} is given by [2]

$$\theta_{\mathrm{K}} \simeq -\mathrm{Real}(\rho). \tag{5}$$

A figure of merit (FOM) has been defined as [1], [4], [7]

$$\mathrm{FOM} = (R\theta_{\mathrm{K}}^2)^{1/2} \tag{6}$$

where R is the power reflectivity, $R = |r|^2 + |k|^2$.

3. Equivalence relations for dielectric coated magneto-optical substrates

Let us consider a thin dielectric film of refractive index n_1 and thickness h_1 that is coated on the surface of the MO material. The $r_{\pm}$ reflection coefficient for (±) polarization is determined by relation

$$r_{\pm} = (r_{10} + r_{\pm 1} X_1^2)/(1 + r_{10} r_{\pm 1} X_1^2) \tag{7}$$

where: $r_{10} = (n_1 - n_0)/(n_1 + n_0)$, $r_{\pm 1} = (n_{\pm} - n_1)/(n_{\pm} + n_1)$, and $X_1^2 = \exp(-j2\beta)$, with $2\beta = 4\pi n_1 h_1/\lambda$, and λ being the light wavelength in vacuum. Numerical examples for θ_{K} and FOM variation against 2β are shown in Fig. 1 for three values of n_1, in the case of the MO substrate of parameters $n = 3.1 - j3.5$ and $q = -0.052 - j0.025$ [1]. One can see that the greater the refractive index n_1 is, the greater the maximum values of θ_{K} and FOM are.

The thin-film coated MO substrate is equivalent in normal reflection to an uncoated MO material of refractive indices $N_{\pm}$ which are determined by relation

$$N_{\pm} = n_1(1 + r_{\pm 1} X_1^2)/(1 - r_{\pm 1} X_1^2). \tag{8}$$

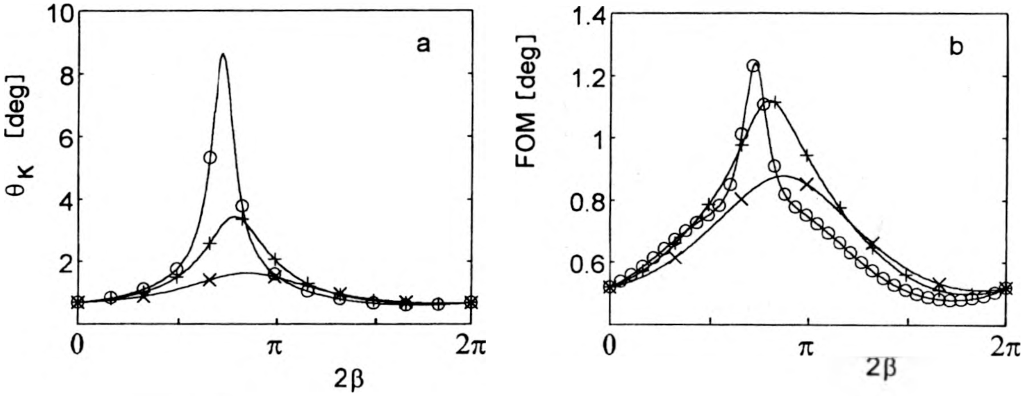

Fig. 1. Variation of θ_K (**a**) and FOM (**b**) against 2β for a dielectric coated MO substrate of parameters $n = 3.1-j3.5$ and $q = -0.052-j0.025$, at three values of n_1: $n_1 = 1.5$ (×), $n_1 = 2$ (+), and $n_1 = 2.5$ (o).

Similarly to Eq. (1), we can write

$$N_{\pm} = N(1 \pm Q/2). \tag{9}$$

Thus, the equivalent parameters N and Q are determined by relations:

$$N = (N_{+}+N_{-})/2, \quad Q = 2(N_{+}-N_{-})/(N_{+}+N_{-}). \tag{10}$$

One obtains

$$N = n_1[n+n_1+X_1^2(n-n_1)]/[n+n_1-X_1^2(n-n_1)], \tag{11a}$$

$$Q = 4nn_1qX_1^2/[(n+n_1)^2+X_1^4(n-n_1)^2]. \tag{11b}$$

For a quarter-wave-thick dielectric layer, $2\beta = \pi$, $X_1^2 = -1$, $N = n_1^2/n$, and $Q = -q$. Considering that $n = n_r+jn_i$, $q = q_r+jq_i$, and similarly $N = N_r+jN_i$, and $Q = Q_r+jQ_i$, one obtains the following relations for the real and imaginary parts of N and Q:

$$N_r = 2n_1^2n_r/G_n, \tag{12a}$$

$$N_i = (n_1/G_n)[2n_1n_1\cos 2\beta-(|n|^2-n_1^2)\sin 2\beta], \tag{12b}$$

$$Q_r = 2(n_1/G_q)\{2n_1|n|^2q_r\cos 2\beta+[n_rq_i(|n|^2+n_1^2)-n_iq_r(|n|^2-n_1^2)]\sin 2\beta\}, \tag{12c}$$

$$Q_i = 2(n_1/G_q)\{2n_1|n|^2q_i\cos 2\beta-[n_rq_r(|n|^2+n_1^2)+n_iq_i(|n|^2-n_1^2)]\sin 2\beta\} \tag{12d}$$

where

$$G_n = |n|^2+n_1^2-2n_1n_i\sin 2\beta-(|n|^2-n_1^2)\cos 2\beta, \tag{13a}$$

$$G_q = 4n_1^2n_r^2+[(|n|^2-n_1^2)\sin 2\beta-2n_1n_i\cos 2\beta]^2. \tag{13b}$$

Variations of N_r against N_i and variations of Q_r against Q_i are shown in Fig. 2 **a**,**b** for three values of n_1. On either curve, 2β rises counterclockwise from 0 (the point O) to 2π, as indicated by arrows. The curves in Fig. 2**a** representing N_r against N_i

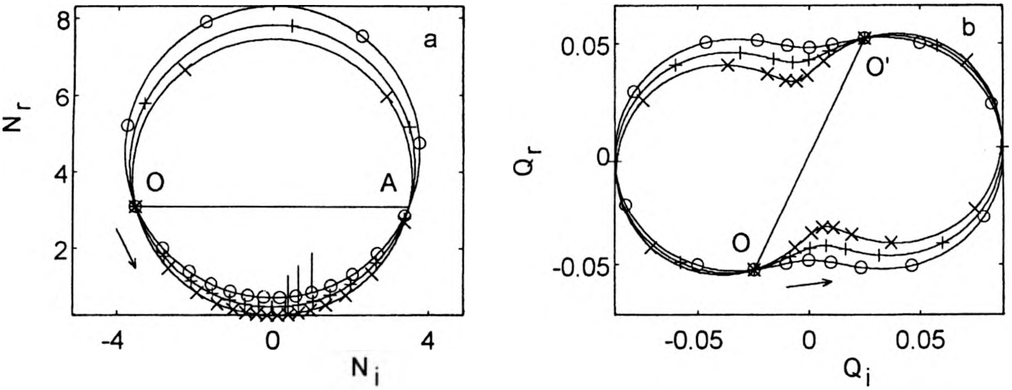

Fig. 2. Variation of N_r against N_i (a) and variation of Q_r against Q_i (b) for dielectric coated MO substrate of parameters as in Fig. 1, at three values of n_1: $n_1 = 1.5$ (×), $n_1 = 2$ (+), and $n_1 = 2.5$ (○). The arrows indicate the counterclockwise rise of 2β from 0 (the point O) to 2π. The marking signs specify the values of 2β in steps of 20°. In figure a, the curves from the circles that cross each other at points O and A; the values of $2\beta = \pi$ that correspond to quarter-wave-thick dielectric films are indicated by small bars of equal lengths. In figure b, the curves resemble the ovals of Cassini [8]; they cross each other at points O and O'.

form circles. They cross each other at points O and A that correspond to $N = n$ and $N = n^*$, respectively, where the asterisk specifies the complex conjugate. Thus, the line OA is parallel to the N_i axis. The circles have radii $R_c = [(|n|^2 - n_1^2)^2 + 4n_1^2 n_i^2]^{1/2}(2n_r)$ and are centered on the N_r axis at $C = (|n|^2 + n_1^2)/(2n_r)$.

The curves in Fig. 2b representing Q_r against Q_i resemble the ovals of Cassini [8]. They cross each other at points O and O' that correspond to $Q = q$ (at $2\beta = 0$) and $Q = -q$ (at $2\beta = \pi$), respectively. If $2\beta_0 < \pi$, the values of Q_r and Q_i at $2\beta_0$ are equal and have opposite sign to their values at $2\beta = 2\beta_0 + \pi$.

From Equations (4), (7), and (9) one obtains

$$\rho \simeq -jn_0 NQ/(N^2 - n_0^2). \tag{14}$$

Then, using Eq. (5) gives

$$\theta_K \simeq n_0[N_i Q_r(|N|^2 + n_0^2) - N_r Q_i(|N|^2 - n_0^2)]/[(|N|^2 - n_0^2)^2 + 4n_0^2 N_i^2]. \tag{15}$$

The maximum value of θ_K is attained approximately when $N_i = 0$ and $|N|^2$ is minimum. From Fig. 2a one can see that the points at $N_i = 0$ are distant from the points at $2\beta = \pi$, being marked by small vertical bars of equal lengths, and the greater the n_1 is, the greater the distance becomes. From Eq. (12b) one obtains at $N_i = 0$ the approximate location of the maximum value of θ_K

$$\tan 2\beta_m \simeq 2n_1 n_i/(|n|^2 - n_1^2). \tag{16}$$

Then, the maximum value of θ_K is determined approximately by relation

$$\theta_{Km} \simeq -n_0 N_{rm} Q_{im}/(N_{rm}^2 - n_0^2) \tag{17}$$

where N_{rm} and Q_{im} are the values of N_r and Q_i corresponding to $2\beta_m$. In Figure 1**a**, locations and maximum values at $n_1 = 1.5$, 2, and 2.5 are $2\beta_m = 155.1°$, 143.3°, and 131.8°, and $\theta_{Km} = 1.632°$, 3.445°, and 8.645°, respectively. Equations (16) and (17) give at $n_1 = 1.5$, 2, and 2.5 the approximate values $2\beta_m \simeq 151.8°$, 141.9°, and 131.7°, and $\theta_{Km} \simeq 1.627°$, 3.436°, and 8.634°, respectively. Approximations are quite good.

Thus, for any dielectric coated MO substrate one can determine an equivalent uncoated MO medium of refractive indices $N_\pm$ given by Eq. (9), and parameters N and Q given by Eq. (10). Then, using Eqs. (16) and (17) gives approximately the location $2\beta_m$ and the maximum value θ_{Km}.

Following this procedure one can determine the overall equivalent parameters N and Q of other MO systems, as for example, for dielectric overcoated magneto-optical thin films on glass substrates [2], [4].

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Received January 3, 2000