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Inverse problem in scatterometry of rough surfaces

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The aim of the paper is to examine one of variant inverse problems constituting the basis for surface roughness measurements by light scattering methods. This problem consists in determining the scatterer form based on measured distribution of scattered light intensity. A solution of the inverse problem amounts to determination of the optical signal from a measured squared modulus of the Fourier transform of this signal. The paper presents methods for solving the inverse problem. For that purpose, the initial modification of the light wave on the surface measured was carried out so that the complex light amplitude was described by accordingly attenuated Hermitian function. The method presented can be used for analysis of other, similar problems.

Keywords: light scattering, inverse problem, surface roughness.

1. Introduction

The light scattering methods assume great importance in surface metrology, especially in optical surface metrology [1]–[3]. Configuration and dimensions of surface roughness in these methods are assessed by analysis of an electromagnetic field of light waves scattered by rough surfaces. The theoretical basis for the surface roughness assessment by these methods stems from the theory of optical imaging [4].

Analyzing the optical imaging process, from the viewpoint of surface metrology, two main problems can be distinguished: the so-called direct problem and the so-called inverse problem. A direct problem consists in determination of the electromagnetic field distribution of light waves scattered by the rough surface on the basis of the function $z = f(x_s, y_s)$ describing the configuration of the surface roughness in the illuminated area of surface. Generally, it is assumed in this case that the surface at each point has the same optical properties. Furthermore, it is presumed that the method of its illumination is known. The inverse problem is solved with similar assumptions. It consists in determination of the function $z = f(x_s, y_s)$ describing the surface roughness, from the measured distribution of light waves scattered by the rough surface.

Thus, from the viewpoint of surface metrology, the inverse problem assumes greater importance since the solution to this problem makes it possible to take full advantage of the light scattering phenomenon in the surface roughness assessment.

Regarding a great number of possible situations there are many variant inverse problems [5].

To solve the inverse problem, the respective mathematical models describing the rough surface and the light scattering phenomenon are used. The models assumed should make allowance for conditions related to the configuration and dimensions of the surface roughness, the method of its illumination and to the position and configuration of the area where the scattered field is to be determined. If the inverse problem applies to surfaces described by random functions, then often the mere formulation of this problem undergoes modification. In this case, the problem consists in determination of statistical characteristics of the random function describing the rough surface based on statistical features of the scattered light [6], [7]. Mostly mathematical models based on the scalar diffraction theory find application concerning the scattered light phenomena [8]–[10].

The subject of this work is one of the variant inverse problems. This variant inverse problem is of great importance in practical terms, because it often concerns methods of the surface roughness assessment based on Fourier optical transform [11]–[13]. The paper presents a general method of solution to this variant inverse problem.

2. Inverse problem

The work [5] gives a review of the extensive, general classification of inverse problems occurring in optics. Many problems mentioned here are related to widely acknowledged issues of diffraction and scattering of electromagnetic waves.

Two methods of solving the inverse problems are known. The first consists in determination of formulas or algorithms expressing the inverse transformations. The application of the inverse transformations is strictly related to mathematical problems

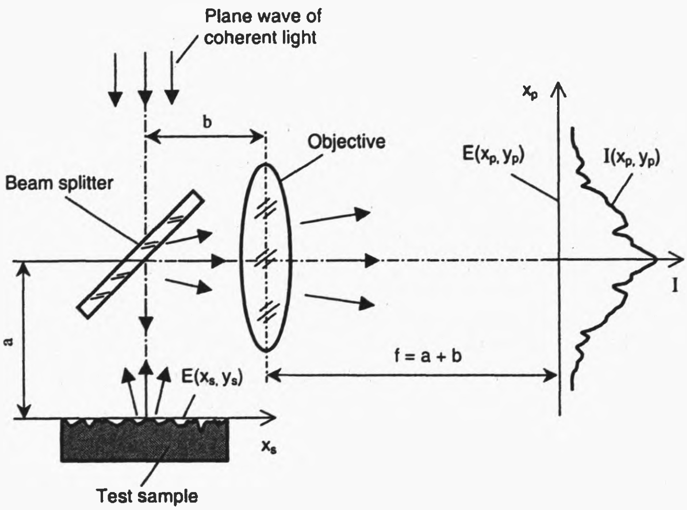

Fig. 1. Set-up for analyzing the inverse problem.

with the existence, the unambiguousness and the stability of solutions obtained [5]. The second method of solving the inverse problems consists in searching for a proper model of the scatterer that is able to satisfy the direct transformations. The parameters of this model are selected based on the experimental data. Once again, this way of solution to the inverse problem amounts to the direct problem. Solving the inverse problem by this method, the numerical analysis using the iterative algorithms finds application in many cases [14].

Let us consider the reflection of the coherent light plane wave with constant amplitude and wavelength λ, from the rough surface $z = f(x_s, y_s)$, the amplitude reflection coefficient ρ_A of which is equal to unity at each point of the surface. Let us assume that an analysis of light scattered by the rough surface is made by means of the lens with focal lengths f. Let the primary focal plane of the lens coincide with the plane tangent to the rough surface of the object, which is shown in Fig. 1. It results from the diffraction model of light scattering that in such a case the light intensity $I(x_p, y_p)$, measured in the secondary focal plane of the lens is proportional to the squared modules of the Fourier transform of the complex light amplitude $E(x_s, y_s)$, in the primary focal plane

$$I(x_p, y_p) = \frac{1}{(\lambda f)^2}\left|\Im[E(x_s, y_s)]\right|^2 = \frac{1}{(\lambda f)^2}\left|\Im\{\exp[i\Delta\varphi(x_s, y_s)]\}\right|^2$$

$$= \frac{1}{(\lambda f)^2}\left|\Im\left\{\exp\left[i\frac{4\pi}{\lambda}f(x_s, y_s)\right]\right\}\right|^2 = \left|E(x_p, y_p)\right|^2 \qquad (1)$$

where λ is the light wavelength, f – the focal length of the lens, symbol $\Im$ stands for the Fourier transformation, i is the imaginary unit, $\Delta\varphi(x_s, y_s)$ – the function describing changes in light-wave phases caused by surface roughness, $f(x_s, y_s)$ – the function describing the height of surface irregularities, whereas $E(x_p, y_p)$ – the complex light amplitude in the secondary focal plane.

It results from Eq. (1) that in a given case the inverse problem consists in reconstruction of the complex light amplitude $E(x_s, y_s)$ from measured values of the light intensity $I(x_p, y_p)$. Considering the consecutive sides of Eq. (1) one can easily observe that the essence of the inverse problem amounts to the determination of the function $\Delta\varphi(x_s, y_s)$ describing changes in the light-wave phases or the function $f(x_s, y_s)$ describing the rough surface. This is done on the basis of measured values of the light intensity $I(x_p, y_p)$.

The most serious difficulties which arose when solving the inverse problems resulted from the fact that photoelectric detectors respond to the light intensity, but not to the complex light amplitude. The light intensity $I(x_p, y_p)$ is proportional to a squared modulus of the complex light amplitude $E(x_p, y_p)$. It results from Eq. (1) that information on the light wave phase during measurements of the light intensity is being lost.

Is it possible to determine the complex light amplitude $E(x_s, y_s)$ based on intensity measurements of light reflected from the rough surface and measured in the secondary focal plane of the lens and the function $\Delta\varphi(x_s, y_s)$, determining changes in the light wave or the function $f(x_s, y_s)$ describing the height of surface irregularities? In the case under consideration this question points out the essence of the inverse problem. Therefore, the inverse problem from the mathematical viewpoint amounts to the determination of the function $E(x_s, y_s)$ based on the measured values of the squared modulus of Fourier transform for this function. Unfortunately, it is generally impossible to determine the function knowing only the squared modulus of Fourier transform. Thus, the general answer to the question put earlier is negative. Regardless of the above, there is a continuous search for methods of solving to the inverse problem for selected classes of function. One of such methods is presented below.

3. Solution of the inverse problem

The most serious difficulties which emerged when solving the inverse problems resulted from the fact that measuring the light intensity leads to the loss of phase information. From the mathematical viewpoint, there are two reasons for losing the information on the phase function $E(x_p, y_p)$. One of them is that a modulus of this function does not include information on which component – sinusoidal, cosinusoidal or their combination – the intensity measured at a given point of the plane (x_p, y_p) responds to. The second reason for losing the information on the phase function $E(x_p, y_p)$ results from the fact that the modulus of the complex light amplitude $E(x_p, y_p)$ and all the more its square does not include information on the sign of respective components. This double indeterminacy applies to the parity and the sign of components. It makes the reconstruction of the function $E(x_s, y_s)$ impossible as a result of the synthesis of harmonics obtained by inverse Fourier transform of function $|E(x_p, y_p)|$.

In general case, one cannot determine the function $E(x_s, y_s)$ from the squared modulus of its Fourier transform. However, there are known methods of solving this problem based on the initial modification of the transformed function [15], [16]. To solve the inverse problem it is necessary to transform initially the function $E(x_s, y_s)$ by means of some operator M into the new function $E_N(x_s, y_s)$, for which there exists an unambiguous relation between it and the squared modulus of its Fourier transform. By measuring the intensity of scattered light, the squared modulus of Fourier transform for the function $E_N(x_s, y_s)$ can be determined. The inverse Fourier transformation of the square root of the measured light intensity allows this function to restore. Then using the inverse transform in relation to operator M, the primary function $E(x_s, y_s)$ is obtained. The way of reconstructing the real-valued function $E(x_s, y_s)$, describing the amplitude objects, has been given in [16]. Application of similar procedures for phase objects has been proposed in [17].

It is most important in the proposed solution to the inverse problem that operator M enabling initial modification of the function $E(x_s, y_s)$ should be determined. With this end in view, the following question should be put: in what way the function $E(x_s, y_s)$ should be transformed into the new function $E_N(x_s, y_s)$, which could be determined if only the squared modulus of its Fourier transform were known? The answer to this question can be as follows: the function $E(x_s, y_s)$ should be transformed so that the new function $E_N(x_s, y_s)$ had real and non-negative Fourier transform. Obtaining such a transformation is a solution to the inverse problem.

3.1. Modification of the complex amplitude

It is worth considering before trying to find the applicable transformation of the function $E(x_s, y_s)$ that according to the above assumptions this function is complex in a given case. It describes a wave with constant amplitude whereas phase modulated through the rough surface. If $E_N(x_s, y_s)$ obtained as a result of transformation of the function $E(x_s, y_s)$ should have a real Fourier transform, then it should be a Hermitian function. The real part of Hermitian function is even and its imaginary part is odd [18]. Thus, operator M in demand should initially transform the complex amplitude $E_N(x_s, y_s)$ in Hermitian function $E_H(x_s, y_s)$.

The complex light amplitude in the plane (x_s, y_s) is Hermitian function if the real functions $f(x_s, y_s)$ and $\Delta\varphi(x_s, y_s)$, describing the roughness height and changes in the phase of light waves reflected from the rough surface, respectively, are odd. Therefore, the operator M should form other functions $f_n(x_s, y_s)$ and $\Delta\varphi_n(x_s, y_s)$ which are odd from the functions $f(x_s, y_s)$ or $\Delta\varphi(x_s, y_s)$.

In general, the above mentioned modification of the function $f(x_s, y_s)$ is difficult to bring about, because changes in the surface being tested are required. This modification of the function $f(x_s, y_s)$ is possible and quite easy if only the function satisfied three conditions specified below. Firstly, this function must be a periodic one. Then there exist such numbers $\Lambda_x > 0$ and $\Lambda_y > 0$ that satisfy the following relation:

$$f(x_s + \Lambda_x, y_s + \Lambda_y) = f(x_s, y_s) \tag{2}$$

where symbols Λ_x and Λ_y denote the shortest periods of the function $f(x_s, y_s)$, determined along axes x_s and y_s, respectively. Secondly, the function $f(x_s, y_s)$ must be even, *i.e.*, must have the symmetry of the first kind satisfying the following condition:

$$f(x_s, y_s) = f(-x_s, -y_s). \tag{3}$$

Thirdly, this function must have the symmetry of the third kind, given by the relation

$$f\left(x_s + \frac{1}{2}\Lambda_x, y_s + \frac{1}{2}\Lambda_y\right) = -f(x_s, y_s). \tag{4}$$

The rough surface – described by the function $f(x_s, y_s)$, satisfying the conditions determined by relations (2), (3), (4) – after shifting along axis x_s by half period Λ_x and along axis y_s by half period Λ_y will have the symmetry of the second kind, *i.e.*, will be described by the odd function $f_n(x_s, y_s)$. Modification of the function $f(x_s, y_s)$, in this case, consists in its shift in relation to the coordinate system $0x_sy_s$.

In general, more advantageous is the modification of the function $\Delta\varphi(x_s, y_s)$ describing changes in the phase of light waves reflected from the rough surface. Since the light wave can be transformed by means of various optical systems, the modification of the function $\Delta\varphi(x_s, y_s)$ is obtainable without any change in the function $f(x_s, y_s)$. To obtain the odd function $\Delta\varphi_n(x_s, y_s)$ from the function $\Delta\varphi(x_s, y_s)$, the following operations should be applied. First, the function $\Delta\varphi(x_s, y_s)$ is to be shifted so that it is definite in the coordinate system $0x_sy_s$ only for non-negative values of both arguments. Afterwards, this shifted function is complemented symmetrically in relation to the origin of coordinates forming the even function, and then the complemented part is multiplied by -1. With this end in view, image multiplication techniques and optical systems realizing Hilbert transforms could be used.

Regardless of that which of the above methods transforming the function $f(x_s, y_s)$ or $\Delta\varphi(x_s, y_s)$ will be applied, the complex amplitude of the reflected light will be described by Hermitian function $E_H(x_s, y_s)$.

Fourier transform of the complex amplitude $E_H(x_s, y_s)$ gives a real function, which in general may assume positive, null and negative values. Therefore another question arises: in what way the complex light amplitude $E_H(x_s, y_s)$ should be additionally modified to devoid its Fourier transform of negative values? In general, one cannot obtain a satisfactory answer to this question. However, if the function describing the complex light amplitude $E_H(x_s, y_s)$ is a function with a finite spectrum, it is possible to find the way of its transformation, as postulated in the question. This way is presented below.

The functions with finite spectra have non-zero Fourier transforms in a limited frequency interval [18]. If the complex amplitude $E_H(x_s, y_s)$, in the primary focal plane of the lens is described by a function with a finite spectrum, then Fourier transform $F_H(f_s, f_s)$ of this function will satisfy the following condition:

$$F_H(f_x, f_y) = \Im[E_H(x_s, y_s)] = 0 \quad \text{for} \quad |f_{xy}| > f_g \tag{5}$$

where

$$f_{xy} = \pm\sqrt{f_x^2 + f_y^2}, \tag{6}$$

while f_x and f_y are the spatial frequencies determined along the x_s and y_s axes, respectively, symbol $\Im$ denotes Fourier transform, f_{xy} is the spatial frequency determined along an arbitrary direction in the plane (x_s, y_s), f_g is the spatial Nyquist frequency ($f_g > 0$).

If the Hermitian function $E_H(x_s, y_s)$ is complex analytic function with the finite spectrum, its real part $\mathrm{Re}\{E_H(x_s, y_s)\}$ will reach the extremum at the origin of coordinates $0x_sy_s$, which is given by the relation

$$\mathrm{Re}\{E_H(0, 0)\} = \int_{-\infty}^{\infty}\int F_H(f_x, f_y)\mathrm{d}f_x\mathrm{d}f_y = \int_{-f_g}^{f_g}\int F_H(f_x, f_y)\mathrm{d}f_x\mathrm{d}f_y \tag{7}$$

where Re denotes the real part, while $F_H(f_s, f_s)$ is Fourier transform of the function $E_H(x_s, y_s)$. In a zero-point neighborhood, that is

$$|\rho_s| \le \frac{\pi}{2f_g} \tag{8}$$

where

$$\rho_s = \pm\sqrt{x_s^2 + y_s^2}, \tag{9}$$

the real part of the function $E_H(x_s, y_s)$, with the assumptions accepted above, satisfies the following relations:

$$|\mathrm{Re}E_H(x_s, y_s)| \ge \cos(f_g\rho_s)\left|\int_{-f_g}^{f_g}\int F_H(f_x, f_y)\mathrm{d}f_x\mathrm{d}f_y\right|, \tag{10}$$

$$|\mathrm{Re}E_H(x_s, y_s)| \ge \cos(f_g\rho_s)|\mathrm{Re}E_H(0, 0)|. \tag{11}$$

Let us find the way of transforming the function $E_H(x_s, y_s)$ into an other Hermitian function $E_N(x_s, y_s)$ with the finite spectrum, the Fourier transform of which is non-negative. Fourier transform of such a new function $E_N(x_s, y_s)$, besides satisfying Eq. (5), must be greater than zero or equal to zero for $|f_{xy}| \le f_g$. Since Hermitian function $E_N(x_s, y_s)$ with the finite spectrum had Fourier transform $F_N(f_x, f_y)$ without any negative values, some additional conditions should be imposed on this function, as follows:

$$\mathrm{Re}E_N(x_s, y_s) = 1 \quad \text{for} \quad |\rho_s| \le \frac{\pi}{2f_g}, \tag{12}$$

$$|\mathrm{Re}E_N(x_s, y_s)| \le \inf\left\{1, \frac{1}{C_r|\rho_s|}\right\} \quad \text{for} \quad |\rho_s| > \frac{\pi}{2f_g}, \tag{13}$$

and

$$|\mathrm{Im}E_N(x_s, y_s)| \le \inf\left\{1, \frac{1}{C_r|\rho_s|}\right\} \quad \text{for} \quad |\rho_s| > \frac{\pi}{2f_g}, \tag{14}$$

where C_r is a carefully selected constant coefficient, while Re and Im are the symbols of real and imaginary parts.

This means that additional modification of the complex amplitude $E_H(x_s, y_s)$ on the rough surface – designed to obtain Hermitian function with non-negative Fourier transform – consists of two operations. The first operation expressed by Eq. (12),

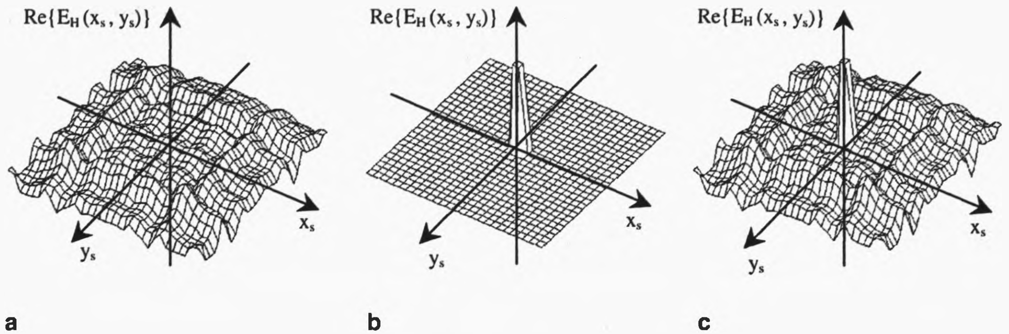

Fig. 2. Plots showing the supplement of Re{ $E_H(x_s, y_s)$} by unitary pulse: the function before modification (**a**), unitary pulse (**b**), the function after modification (**c**).

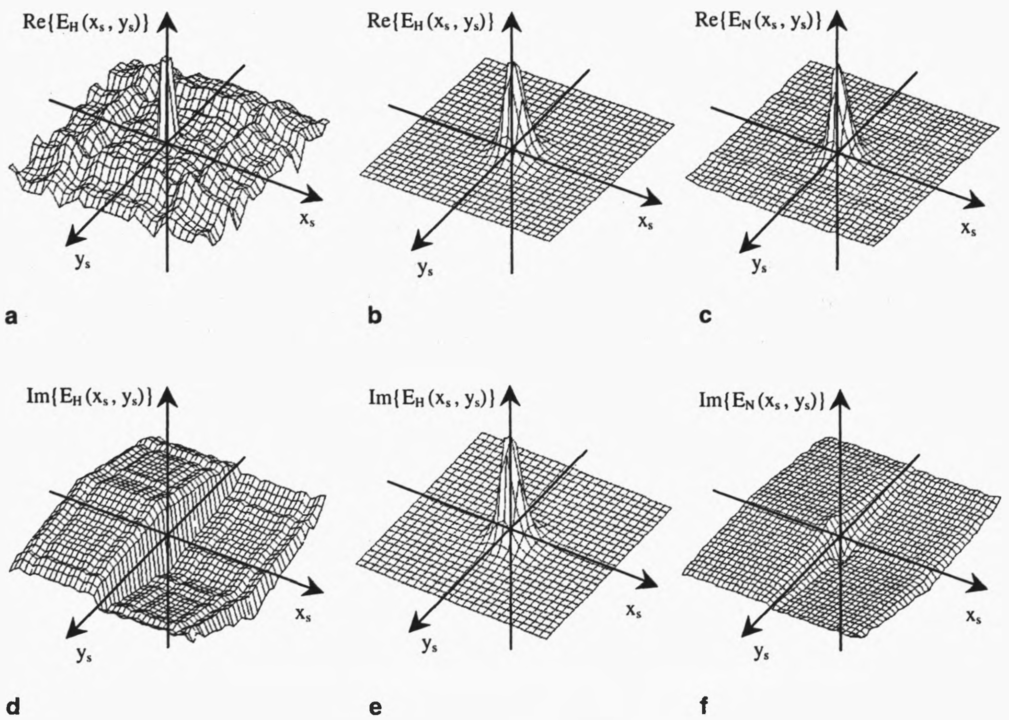

Fig. 3. Plots showing attenuation of complex amplitude $E_H(x_s, y_s)$: function Re{ $E_H(x_s, y_s)$} before attenuation (**a**), attenuation function (**b**), function Re{ $E_N(x_s, y_s)$} after attenuation (**c**), function Im{ $E_H(x_s, y_s)$} before attenuation (**d**), attenuation function (**e**), function Im{ $E_N(x_s, y_s)$} after attenuation (**f**).

includes the replacement of previous values of real parts of the function $E_H(x_s, y_s)$, in a close neighborhood of the origin of coordinates $0x_sy_s$, for values equal to 1. This can be interpreted as the replacement of the real part of the function $E_H(x_s, y_s)$, at the origin of coordinates for the unitary pulse. This operation is presented in Fig. 2. The second operation, described by inequalities (13) and (14), consists in such an attenuation of the function $E_H(x_s, y_s)$, at respective points of the plane (x_s, y_s), which is proportional to the distance of a given point from the origin of coordinates. This operation could be easily performed multiplying the real part and the imaginary part of the complex amplitude $E_H(x_s, y_s)$ by the factor $1/(C_r\,|\rho_s|)$, which describes half of a hyperboloid of two sheets, as shown in Fig. 3.

3.2. Analysis of attenuation coefficient

The present modification of the complex amplitude $E_H(x_s, y_s)$ will be effective if the attenuation coefficient C_r is correctly selected. The values of the coefficient C_r should be selected so that any Hermitian function $E_N(x_s, y_s)$ with the finite spectrum, satisfying conditions (12)–(14), has non-negative Fourier transform. This is equivalent to the requirement that for all f_{xy}, included in the closed interval $\langle -f_g, f_g\rangle$, the real and imaginary parts of this function should satisfy the following inequalities:

$$\int_0^\infty\!\!\int \mathrm{Re}[E_N(x_s, y_s)]\cos(\rho_s f_{xy})\mathrm{d}x_s\mathrm{d}y_s \geq 0, \tag{15}$$

$$\int_0^\infty\!\!\int \mathrm{Re}[E_N(x_s, y_s)]\cos(\rho_s f_{xy})\mathrm{d}x_s\mathrm{d}y_s - \int_0^\infty\!\!\int \mathrm{Im}[E_N(x_s, y_s)]\sin(\rho_s f_{xy})\mathrm{d}x_s\mathrm{d}y_s \geq 0. \tag{16}$$

In work [19], inequalities (15) and (16) were analyzed to determine the admissible values of the coefficient C_r. The analysis of inequality (15) leads to a conclusion that attenuation coefficient C_r should satisfy the condition

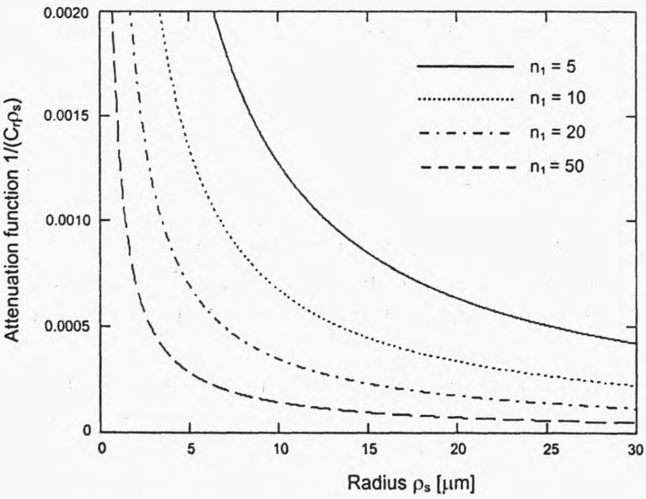

Fig. 4. Plots of attenuation function $1/(C_r\rho_s)$.

$$C_r \geq \frac{2f_g(2n_1 + 1)}{\frac{\pi}{2} - 1} \tag{17}$$

where n_1 is an integer depending on the spatial Nyquist frequency f_g and the maximum value of the radius ρ_s. The condition resulting from the analysis of inequality (16) is stronger than from (17) and takes the following form:

$$C_r \geq \frac{f_g(8n_1 + 5)}{\frac{\pi}{2} - 1}. \tag{18}$$

Figure 4 shows the graphs of the attenuation function $1/(C_r\rho_s)$, assuming that the attenuation coefficient C_r was calculated from Eq. (18) for the spatial Nyquist frequency $f_g = 1000$ 1/mm.

3.3. Retrieval of phase and surface irregularities

It follows from the above discussion that consequently to the proposed transformations the complex amplitude $E_N(x_s, y_s)$ becomes Hermitian function the Fourier transform of which is the real function not including any negative values. Its analysis allows the light-wave phase and the height of surface irregularities to restore. Let us then analyze the properties and relations of this transform with the function $f(x_s, y_s)$ describing the surface irregularities. The light intensity $I(x_p, y_p)$ in the secondary focal plane of the lens presented in Fig. 1 is proportional to a squared modulus of the complex amplitude in the plane (x_s, y_s), which results from Eq. (1). If the complex amplitude $E(x_s, y_s)$ had previously been transformed so that the complex amplitude $E_N(x_s, y_s)$ had arose in the plane (x_s, y_s), the square root of the measured light intensity $I(x_p, y_p)$ would have been proportional to the modulus of the complex amplitude $E_N(x_s, y_s)$ and the transform alone as well. This is illustrated by the following relation:

$$\begin{aligned}\sqrt{I(x_p, y_p)} &= |E(x_p, y_p)| = \frac{1}{\lambda f}|\Im[E_N(x_s, y_s)]| \\ &= E(x_p, y_p) = \frac{1}{\lambda f}\Im[E_N(x_s, y_s)]\end{aligned} \tag{19}$$

where λ is the light wavelength, f is the focal length of the lens, symbol $\Im$ stands for the Fourier transform, $E(x_p, y_p)$ is the complex amplitude in the secondary focal plane of the lens, whereas $E_N(x_s, y_s)$ is the complex amplitude after making all the above transformations.

The complex amplitude $E(x_p, y_p)$ obtained as a result of Fourier transformation of Hermitian function $E_N(x_s, y_s)$ includes only the real part. The even part of the function $E(x_p, y_p)$ is proportional to Fourier transform of the real part of the function $E_N(x_s, y_s)$. However, the odd part of the function $E(x_p, y_p)$ is proportional to Fourier transform of the imaginary part of the function $E_N(x_s, y_s)$. By denoting the even part of the function

$E(x_p, y_p)$ by the symbol $e(x_p, y_p)$, while the odd by the symbol $o(x_p, y_p)$, the following is obtained:

$$E(x_p, y_p) = \frac{1}{\lambda f} \Im[E_N(x_s, y_s)] = e(x_p, y_p) + o(x_p, y_p) \tag{20}$$

where

$$e(x_p, y_p) = \frac{1}{\lambda f} \Im[\mathrm{Re}E_N(x_s, y_s)], \tag{21}$$

$$o(x_p, y_p) = \frac{1}{\lambda f} \Im[\mathrm{Im}E_N(x_s, y_s)]. \tag{22}$$

To retrieve the function describing surface irregularities, the light intensity should be measured at respective points of the plane (x_p, y_p). For each pair of the measured values $I(x_p, y_p)$ and $I(-x_p, -y_p)$ the system of equations is to be solved:

$$\left.\begin{aligned} \sqrt{I(x_p, y_p)} &= E(x_p, y_p) = e(x_p, y_p) + o(x_p, y_p), \\ \sqrt{I(-x_p, -y_p)} &= E(-x_p, -y_p) = e(x_p, y_p) - o(x_p, y_p). \end{aligned}\right\} \tag{23}$$

It allows the values of the functions $e(x_p, y_p)$ and $o(x_p, y_p)$ to be determined at respective points of the plane (x_p, y_p), according to the following formulas:

$$e(x_p, y_p) = \frac{1}{2}[\sqrt{I(x_p, y_p)} + \sqrt{I(-x_p, -y_p)}], \tag{24}$$

$$o(x_p, y_p) = \frac{1}{2}[\sqrt{I(x_p, y_p)} + \sqrt{I(-x_p, -y_p)}]. \tag{25}$$

Then transforming Eqs. (21) and (22) and applying the inverse Fourier transform denoted by $\Im^{-1}[\ ]$, the following is obtained:

$$\mathrm{Re}E_N(x_s, y_s) = \Im^{-1}[(\lambda f)e(x_p, y_p)], \tag{26}$$

$$\mathrm{Im}E_N(x_s, y_s) = \Im^{-1}[(\lambda f)o(x_p, y_p)], \tag{27}$$

where λ is the light wavelength and f is the focal length of the lens.

Equations (26) and (27) make it possible to determine the real and imaginary parts of the function $E_N(x_s, y_s)$ obtained as a result of the modification described above. To retrieve the initial values of the complex amplitude $E(x_s, y_s)$, the inverse transformation should be successively applied to the function $E_N(x_s, y_s)$ in relation to these transformations that were applied to the complex amplitude $E(x_s, y_s)$ during its initial modification.

First, the unitary pulse should be superseded to this end – within the argument $|\rho_s| \le \pi/2f_g$, for the function $\mathrm{Re}\{E_N(x_s, y_s)\}$ – by the primary values of the function

$\mathrm{Re}\{E_H(x_s, y_s)\}$. Unfortunately, to be precise this is not possible directly because the primary values of the function $\mathrm{Re}\{E_H(x_s, y_s)\}$ are unknown. Consequently, the loss of information on the values of the function $\mathrm{Re}\{E_H(x_s, y_s)\}$ in the neighborhood of the origin of coordinates will occur. Fortunately, the area where information will be lost is small. The following procedure is recommended to avoid this problem. To obtain a unitary pulse in the process of modifying the complex amplitude, the multiplication or other linear operation should be applied. However, the inverse operation, which was applied during the modification, should be used for reconstruction of the complex amplitude.

The next step in the process of reconstructing the function $E(x_s, y_s)$ is the multiplication of the values $\mathrm{Re}\{E_N(x_s, y_s)\}$ and $\mathrm{Im}\{E_N(x_s, y_s)\}$, calculated from Eqs. (26) and (27), by the factor $C_r|\rho_s|$. It enables obtaining the values $\mathrm{Re}\{E_H(x_s, y_s)\}$ and $\mathrm{Im}\{E_H(x_s, y_s)\}$ at respective points of the plane (x_s, y_s):

$$\mathrm{Re}\, E_H(x_s, y_s) = C_r|\rho_s|\,\mathrm{Re}\, E_N(x_s, y_s) \quad \text{for} \quad |\rho_s| > \frac{\pi}{2f_g}, \tag{28}$$

$$\mathrm{Im}\, E_H(x_s, y_s) = C_r|\rho_s|\,\mathrm{Im}\, E_N(x_s, y_s) \quad \text{for} \quad |\rho_s| > \frac{\pi}{2f_g}. \tag{29}$$

Then, on the basis of these values the following functions are reproduced: the function $\Delta\varphi(x_s, y_s)$ describing changes in the light wave phase, and the function $f(x_s, y_s)$ describing the surface irregularities. To this end, the values $\mathrm{Re}\{E_H(x_s, y_s)\}$ and $\mathrm{Im}\{E_H(x_s, y_s)\}$ obtained from Eqs. (28) and (29) are subject to the following transformations:

$$\Delta\varphi_n(x_s, y_s) = \arccos[\mathrm{Re}\, E_H(x_s, y_s)] \quad \text{for} \quad |\rho_s| > \frac{\pi}{2f_g}, \tag{30}$$

$$\Delta\varphi_n(x_s, y_s) = \arcsin[\mathrm{Im}\, E_H(x_s, y_s)] \quad \text{for} \quad |\rho_s| > \frac{\pi}{2f_g} \tag{31}$$

where $\Delta\varphi_n(x_s, y_s)$ denotes the odd function describing changes to the phase. Then, the function $\Delta\varphi_n(x_s, y_s)$ should be limited so that it is determined only for the non-negative values x_s, y_s. The function $\Delta\varphi(x_s, y_s)$ is determined after applying the inverse translation to that which was made at the stage of modification. At last the function $f(x_s, y_s)$ describing the surface irregularities is retrieved

$$f(x_s, y_s) = \frac{\lambda}{4\pi}\,\Delta\varphi(x_s, y_s). \tag{32}$$

Symbol λ in formula (32), as before, stands for the light wavelength.

Some problems are connected with transformation of relations (30) and (31) described above since the arcsines and arccosines of the functions, in general, are many-valued functions. It is possible to avoid problems arising from this fact on condition that the range of changes to the function $\Delta\varphi(x_s, y_s)$ will be limited. Consequently, acceptable changes to the function $f(x_s, y_s)$ describing the surface irregularities within ±$\lambda/4$ are also limited. However, there are possibilities to overcome the above problems, and thereby to extend the range of changes in the surface irregularities, on condition that adequate analytical algorithms are applied. An example of such an algorithm along with its metrological analysis is described in [20]. This algorithm is applied to the analysis of interferograms obtained during the tests on periodic vibration, where the identical mathematical problem has occured.

4. Conclusions

The present way of solving the inverse problem in scatterometry of the rough surfaces consists in initial modification of the complex amplitude in the object plane. It is rather complicated and involves a series of limitations discussed above. Its advantage is universality. It can be applied in practice, not only to testing on precise machined surface, but also in those fields of investigations and measurements where problems with retrieving phase information have occurred. The methods proposed have been verified during the model testing, the results of which are to be published.

References

[1] Whitehouse D.J., *Handbook of Surface Metrology*, Institute of Physics Publishing, Bristol and Philadelphia 1994.

[2] Bennett J.M., Mattsson L., *Introduction to Surface Roughness and Scattering*, Optical Society of America, Washington, D.C., 1989.

[3] Stover J.C., *Optical Scattering: Measurement and Analysis*, McGraw-Hill, Inc., New York 1990.

[4] Jóźwicki R., *Theory of Optical Imaging*, (in Polish), [Ed.] PWN, Warszawa 1988.

[5] Baltes H.P. [Ed.], *Inverse Source Problems in Optics*, Springer-Verlag, Berlin, Heidelberg, New York 1978.

[6] Harvey J.E., Proc. SPIE **107** (1977), 41.

[7] Marx E., Vorburger T.V., Appl. Opt. **29** (1990), 3613.

[8] Beckmann P., Progress in Optics **VI** (1967), 53.

[9] Bass F.G., Fuks I.M., *Wave Scattering from Statistically Rough Surfaces*, Pergamon, Oxford 1979.

[10] Ogilvy J.A., *Theory of Wave Scattering from Random Rough Surfaces*, Adam Hilger, Bristol, Philadelphia and New York 1991.

[11] Goodman J.W., *Introduction to Fourier Optics*, McGraw-Hill Book Co., London 1968.

[12] Thwaite E.G., Wear **57** (1979), 71.

[13] Thwaite E.G., Annals of the CIRP **29** (1980), 419; ibidem **31** (1982), 463.

[14] Fienup J.R., Opt. Eng. **19** (1980), 297; Fienup J.R., J. Opt. Soc. Am. A **4** (1987), 118.

[15] Fienup J.R., Appl. Opt. **21** (1982), 2758.

[16] Vasilenko G.I., Maslov A.V., Belinskij A.N., [In] *Optical Processing of Images*, (in Russian), [Eds.] S.B. Gurevič, G.A. Gavrilov, Nauka, Leningrad 1985, p. 12.

[17] Łukianowicz C., Proc. SPIE **4517** (2001), 120.

[18] Bracewell R., *The Fourier Transform and its Applications*, McGraw-Hill, Inc., New York 1965.

[19] Łukianowicz C., *Principles of Surface Roughness Measurements by Light Scattering Methods*, (in Polish), [Ed.] Technical University of Koszalin Publ., Koszalin 2001.

[20] Łukianowicz T., Łukianowicz C., Proc. SPIE **2340** (1994), 415.

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