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Multiple omnidirectional reflection bands from half-wave layered periodic structures

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The paper is concerned with one-dimensional multilayers that exhibit multiple bands of omnidirectional reflection, which means complete reflection regardless of incidence angle and polarization. The typical quarter-wave constraint is replaced by the stipulation that a pair of adjacent layers should have a total optical thickness of one half of wavelength. Simple approximate analytic expressions are derived for the multiple bandgaps of the half-wave periodic structures.

Keywords: thin films, multilayers, filters, bandgaps, 1D photonic crystals.

1. Introduction

One-dimensional periodic structures with alternating layers of low and high refractive indices can exhibit omnidirectional reflection (ODR), which means a high reflection at any polarization and any incidence angle over a certain spectral range [1–7]. In contrast with the creation of three-dimensional photonic bandgaps [8], the production of these ODR stacks is more feasible, the periodic layered structures, often called interference filters, being long applied to polarizing beam splitters and highly reflecting mirrors [9]. Most of the ODR studies for the 1D structures were concerned with a single, the lowest bandgap, and with alternating layers of optical thickness equal to one quarter of the design light wavelength λ_0. Researches on highly reflecting coatings showed that replacing the quarter-wavelength constraint with the stipulation that a pair of adjacent layers should have a total optical thickness of one half of λ_0 produces higher design flexibility [10].

In this paper we examine how the proportion of the high- and low-index materials in one half-wave pair may be varied to get multiple ODR bands. Numerical examples are given showing that for certain pairs of layer refractive indices and fractional half-wave optical thicknesses, five ODR bands may be obtained. Simple approximate expressions for the multiple bandgaps of the half-wave periodic structures are given. All materials are assumed to be linear, homogeneous, nonabsorbent, and with no optical activity.

2. Notation for half-wave layered pairs

Consider the 1D structure of alternating films with geometric thicknesses d_i and refractive indices n_i (i = 1, 2) which is periodic on the y-axis with period a. We assume for simplicity the structure is in air ($n_0 = 1$) and $n_1 < n_2$. We take the layer thicknesses [10]:

$$d_1 = \frac{(0.5 + \sigma)\lambda_0}{2n_1}, \tag{1a}$$

$$d_2 = \frac{(0.5 - \sigma)\lambda_0}{2n_2} \tag{1b}$$

where σ is subunitary and $\sigma = 0$ reproduces the traditional quarter-wave design. Using the constraint of periodicity $d_1 + d_2 = a$ gives

$$\frac{d_1}{a} = \left[\frac{1 + n_1(1-\chi)}{n_2\chi}\right]^{-1}, \tag{2a}$$

$$\frac{d_2}{a} = \left[\frac{1 + n_2\chi}{n_1(1-\chi)}\right]^{-1} \tag{2b}$$

where $\chi = 0.5 + \sigma$.

3. Projected band structures

The ODR from the periodic layered stacks is conveniently examined by using the projected band structures [2]. Consider the light is incident in the x-z plane. It can be either s- or p-polarized, with the electric-field vector perpendicular or parallel to the incidence plane, respectively. The frequency bands are computed numerically by solving Maxwell's equations in the periodic medium. We used a simple MATLAB algorithm [11] that is based on the plane wave expansion method. Figure 1 illustrates two examples, for $\chi = 0.5$ (the quarter-wave limit) and $\chi = 0.15$, when $n_1 = 1.7$ and $n_2 = 3.4$. The frequencies are in units of $2\pi c/a$ and the parallel wave vector component k_x in units of $2\pi/a$ (c is the light velocity in vacuum). Because the medium is periodic in y and homogeneous in the x-z plane, one may assume [2] the wave vector $\mathbf{k}$ of components $k_z = 0$, $k_x \geq 0$, and k_y restricted to the interval $[0, \pi/a]$. The triangular region above the lines $\omega = k_x$ contains the bands for all possible incidence angles in the ambient medium [2]. The white regions represent the bandgaps. When $\chi = 0.5$,

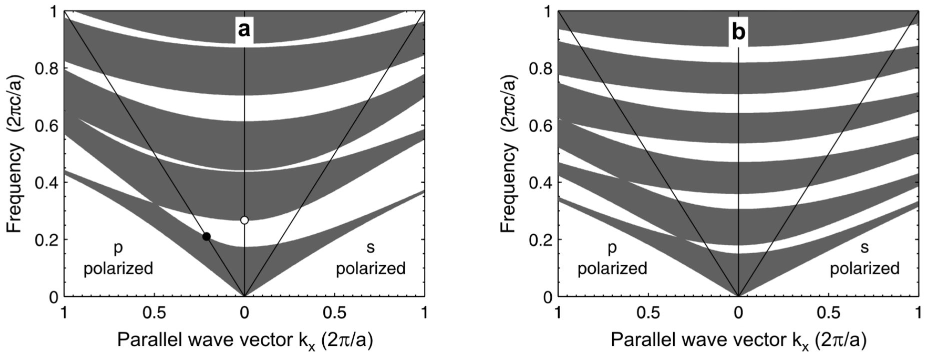

Fig. 1. Projected band structures for the periodic layered stack with $n_1 = 1.7$ and $n_2 = 3.4$, in air ($n_0 = 1$), when: **a** – $\chi = 0.5$ (the quarter-wave limit) and **b** – $\chi = 0.15$. Electromagnetic modes exist only in the shaded regions: *s*-polarized modes are plotted to the right of the origin, and *p*-polarized modes to the left. Diagonal lines are $\omega = k_x$, with frequencies in units of $2\pi c/a$ and parallel wave vectors in units of $2\pi/a$.

the lowest ODR band ($m = 1$) lies from the filled circle at $\omega a/(2\pi c) = 0.21$, to the open circle at $\omega a/(2\pi c) = 0.27$ [2]. When $\chi = 0.15$, there are five ODR bands. Figure 1 shows also some Brewster crossing points [2, 7] of the *p* bands outside the triangular region: when $\chi = 0.5$, there are two points at $k_x = 0.5$ and 1, and when $\chi = 0.15$, three points at $k_x = 0.31m$, where $m = 1–3$. The coordinates (k_x, ω) of the Brewster crossing points expressed in units of Fig. 1 are given by relations:

$$k_x = \frac{m}{2}\frac{n_1 + (n_2 - n_1)\chi}{n_2 - (n_2 - n_1)\chi}, \tag{3a}$$

$$\omega = k_x\left(n_1^{-2} + n_2^{-2}\right)^{1/2}. \tag{3b}$$

When $\chi = 0.5$ (the quarter-wave limit), Eq. (3a) gives $k_x = m/2$ regardless of layer refractive indices (when $n_0 = 1$). From Eq. (3b) one can see that, if $\left(n_1^{-2} + n_2^{-2}\right)^{1/2} < 1$, then $\omega < k_x$, and all the Brewster crossing points lie outside the triangular region, regardless of χ, as shown in Fig. 1.

4. Relations for multiple bandgaps

Like in the case of quarter-wave periodic structures, when approximate analytic expressions have been derived [2–7], similar useful relations can be obtained in the case of half-wave periodic structures. Thus, the centers of the multiple bandgaps expressed in $2\pi c/a$ units are

$$\omega_0 \approx \frac{m}{2}\,\frac{n_1 + (n_2 - n_1)\chi}{n_1 n_2}, \tag{4a}$$

at normal incidence, and

$$\omega_p \approx \frac{m}{2}\,\frac{n_1 + (n_2 - n_1)\chi}{n_1\zeta_2 + (n_2\zeta_1 - n_1\zeta_2)\chi}, \tag{4b}$$

at the limit π/2 of the incidence angle and p-polarization, where $m = 1, 2, 3,\ldots$, and $\zeta_i = (n_i^2 - 1)^{1/2}$, with $i = 1, 2$. Further, one obtains the full widths of the multiple bandgaps

$$\Delta_0 \approx \frac{4\omega_0\gamma_0(n_2 - n_1)}{\pi m(n_2 + n_1)} \tag{5a}$$

at normal incidence, and

$$\Delta_p \approx \frac{4\omega_p\gamma_p\left|n_2^2\zeta_1 - n_1^2\zeta_2\right|}{\pi m\left(n_2^2\zeta_1 + n_1^2\zeta_2\right)} \tag{5b}$$

at oblique incidence for p-polarization and π/2 incidence angle. Notations γ_0 and γ_p are different for even and odd m,

$$\gamma_{0,\,p} = \sin\alpha_{0,\,p} \quad \text{when} \quad m = 2j, \tag{6a}$$

$$\gamma_{0,\,p} = \cos\alpha_{0,\,p} \quad \text{when} \quad m = 2j - 1 \quad \text{with} \quad j = 1, 2,\ldots \tag{6b}$$

where

$$\alpha_0 = \pi m(\chi - 0.5), \tag{6c}$$

$$\alpha_p = \frac{\pi m(\chi - 0.5 + G)}{1 + 4(\chi - 0.5)G} \tag{6d}$$

with $G = 0.5(n_2\zeta_1 - n_1\zeta_2)/(n_2\zeta_1 + n_1\zeta_2)$. For high average refractive indices, $(n_1 + n_2)/2 > 2$, approximations (4) and (5) are within 0.5% of the exact values. The full widths of the multiple ODR bands result in

$$\Delta_{\mathrm{ODR}} = \omega_0 - \omega_p + \frac{\Delta_0 + \Delta_p}{2} > 0. \tag{7}$$

5. Numerical examples

Further we give several examples that illustrate the more simple and flexible search and map of the data for the multiple ODR bands when the half-wave constraint is applied. Let us consider the same pair of layer refractive indices like in Fig. 1, but with χ varied from 0.1 to 0.9. Results are shown in Fig. 2. The lowest ODR band is the largest in the quarter-wave case, at $\chi = 0.5$ [4]. Multiple ODR bands appear especially when $\chi < 0.5$: the smaller the value of χ, the greater the number of ODR bands.

At $\chi = 0.5$, in the quarter-wave case, the even-order bandgaps ($m = 2, 4, \ldots$) are missing – $\gamma_0 = 0$ in Eq. (6a) – and the spectral response is periodic. In general, when $\chi < 0.5$, the spectral response of the half-wave stack is not periodic, as shown in Fig. 3. The centers of the bandgaps are almost periodic and the periodicity, which is different at normal and oblique incidences, can be determined with Eq. (4).

Pairs of layer refractive indices allowing multiple ODR bands can be mapped at different values of χ, as shown in Fig. 4. At either order m, only bandgaps for more

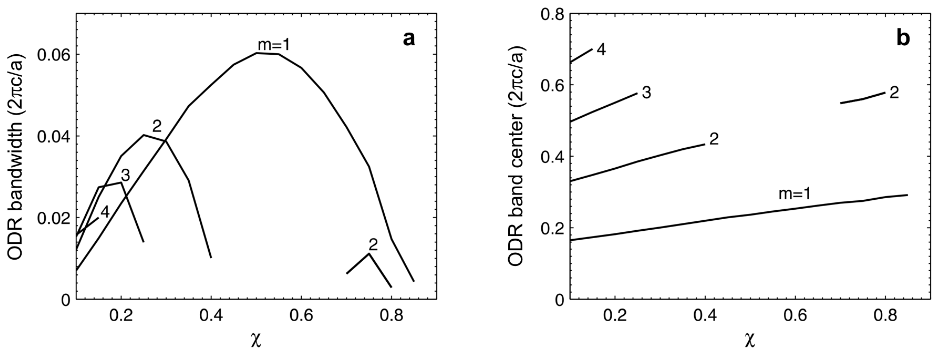

Fig. 2. Parameters of the ODR bands: **a** – the full width and **b** – the center, both in units of $2\pi c/a$, against the fraction χ, at the orders specified on the graphs. χ varies from 0.1 to 0.9 in increments of 0.05 ($n_0 = 1$, $n_1 = 1.7$ and $n_2 = 3.4$).

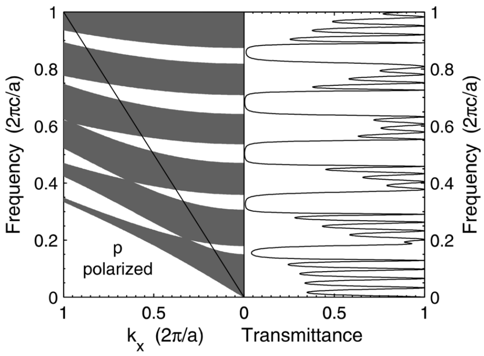

Fig. 3. Projected band structure for p-polarized modes (on the left side) and the respective transmittance through five periods in air at normal incidence (on the right side) with the same data like in Fig. 1**b**.

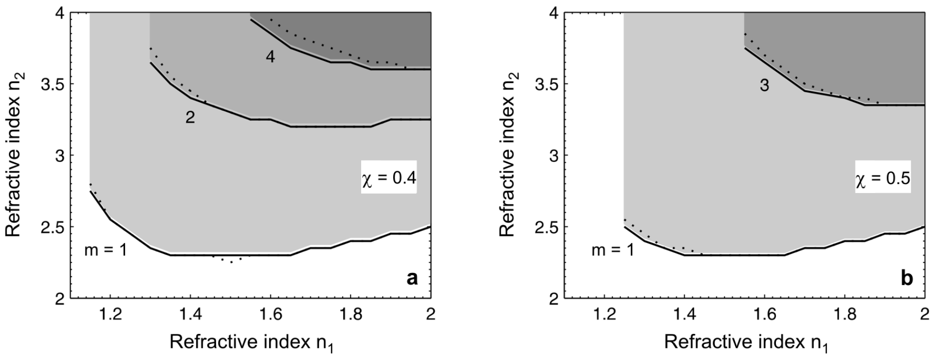

Fig. 4. Pairs (n_1, n_2) of layer refractive indices (shaded areas) allowing ODR at the orders specified on the graphs, when: **a** – χ = 0.4 and **b** – χ = 0.5. Solid curves represent the minimum values of the refractive index n_2 required to attain the ODR at the specified order. Dotted curves represent approximations (4)–(7). Refractive indices were varied in increments of 0.05.

than three pairs of refractive indices were considered. Thus, in Fig. 4**a** the order $m = 3$ is missing. One can see that the higher the bandgap's order, the greater the minimum values of n_2 and the smaller the shaded area for the pairs of allowed refractive indices. Approximate relations (4)–(7) may be useful for checking if the orders of the multiple bandgaps were correctly attributed.

6. Summary

Most of the studies referring to the ODR have been concerned with a single, the lowest, ODR band for quarter-wave thick alternating layers. This paper was devoted to multiple ODR bands. The typical quarter-wave constraint was replaced by a half-wave one: a pair of adjacent layers should have a total optical thickness of one half of wavelength. It was found that with the concept described above a higher design flexibility is available. Emphasis was placed on a more simple and systematic mapping of the data for the multiple ODR bands. Like in case of quarter-wave designs, simple approximate analytic expressions were presented for the multiple bandgaps of the half-wave layered periodic structures.

References

[1] HO K.M., CHAN C.T., SOUKOULIS C.M., *Existence of a photonic gap in periodic dielectric structures*, Physical Review Letters **65**(25), 1990, pp. 3152–5.

[2] WINN J.N., FINK Y., FAN S., JOANNOPOULOS J.D., *Omnidirectional reflection from a one-dimensional photonic crystal*, Optics Letters **23**(20), 1998, pp. 1573–5.

[3] YABLONOVITCH E., *Engineered omnidirectional external-reflectivity spectra from one-dimensional layered interference filters*, Optics Letters **23**(21), 1998, pp. 1648–9.

[4] SOUTHWELL W.H., *Omnidirectional mirror design with quarter-wave dielectric stacks*, Applied Optics **38**(25), 1999, pp. 5464–7.

[5] CHIGRIN D.N., LAVRINENKO A.V., YAROTSKY D.A., GAPONENKO S., *Observation of total omnidirectional reflection from a one-dimensional dielectric lattice*, Applied Physics A: Materials Science and Processing **68**(1), 1999, pp. 25–8.

[6] ABDULHALIM I., *Reflective polarization conversion Fabry-Perot resonator using omnidirectional mirror of periodic anisotropic stack,* Optics Communications **215**(4-6), 2003, pp. 225–30.

[7] COJOCARU E., *Omnidirectional reflection from multilayers with alternating birefringent and isotropic thin films*, Journal of Modern Optics **50**(9), 2003, pp. 1455–67.

[8] JOANNOPOULOS J.D., MEADE R. D., WINN J.N., *Photonic Crystals: Molding the Flow of Light*, Princeton University Press, Princeton 1995.

[9] MACLEOD H.A., *Thin-Film Optical Filters*, Macmillan, New York 1986.

[10] THOMSEN M., WU Z.L., *Polarizing and reflective coatings based on half-wave layer pairs*, Applied Optics **36**(1), 1997, pp. 307–13.

[11] GUO S., ALBIN S., *Simple plane wave implementation for photonic crystal calculations*, Optics Express **11**(2), 2003, pp. 167–75.

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