

Fuzzy clustering with spatial constraints for image thresholding

YONG YANG*, CHONGXUN ZHENG, PAN LIN

Key Laboratory of Biomedical Information Engineering of Education Ministry,
Institute of Biomedical Engineering, Xi'an Jiaotong University, Xi'an 710049, P.R. China

*Corresponding author: Yong Yang, greatyyy765@sohu.com

Image thresholding plays an important role in image segmentation. This paper presents a novel fuzzy clustering based image thresholding technique, which incorporates the spatial neighborhood information into the standard fuzzy c -means (FCM) clustering algorithm. The prior spatial constraint, which is defined as weight in this paper, is inspired by the k -nearest neighbor (k -NN) algorithm and is modified from two aspects in order to improve the performance of image thresholding. The algorithm is initialized by a fast FCM algorithm, in which the iteration is carried out with the statistical gray level histogram of image instead of the conventional whole data of image; therefore its convergence is fast. Extensive experiment results and both qualitative and quantitative comparative studies with several existing methods on the thresholding of some synthetic and real images illustrate the effectiveness and robustness of the proposed algorithm.

Keywords: image thresholding, fuzzy c -means, k -nearest neighbor, fuzzy thresholding.

1. Introduction

Image thresholding is an important technique for image segmentation based on the assumption that objects can be distinguished and extracted from the background by their gray levels. The output of the thresholding operation is a binary image whose gray level 0 (black) indicates the foreground and gray level 255 (white) indicates the background, and *vice versa*. Many thresholding methods have been developed and detailed surveys can be found in references [1–3]. In general, threshold selection can be categorized into two classes, local methods and global methods. The global thresholding methods segment an entire image with a single threshold using the gray level histogram of image, while the local methods partition the given image into a number of sub-images and select a threshold for each of the sub-images. The global thresholding techniques are easy to implement and computationally less involved, therefore they are superior to local methods in terms of many real image processing

applications. The global thresholding methods select the threshold based on different criteria, such as Otsu's method [4], minimum error thresholding [5], and entropic method which was first proposed by PUN [6] and then modified and extended by KAPUR *et al.* [7], *etc.*

OTSU [4] selects optimal thresholds by maximizing the between-class variance of gray values. KITTNER and ILLINGWORTH [5] assume that the gray values of object and background are normally distributed. In their methods, threshold was chosen by a minimum error rate scheme for resultant classes. KAPUR *et al.* [7] proposed a thresholding method by maximizing the entropy of the histogram of gray levels of object and background. Generally, all these conventional one-dimensional (1D) histogram thresholding techniques work well when the two consecutive gray levels of the image are distinct. However, all the above 1D thresholding techniques did not combine the spatial information and the gray-level information of the pixels into the process for image segmentation. This drawback will lead to serious misclassification in the case of image thresholding, since the data in the image are inherently correlated. In addition, when the image is corrupted by noise and other artifacts the performance of these thresholding techniques will be poor or even fail. To compensate this drawback, ABUTALEB [8] extended 1D method to 2D thresholding method by considering the joint entropy of two random variables, namely, the image gray value and the average gray value, but it is very time consuming. BRINK [9] refined Abutaleb's method and later CHEN *et al.* [10] improved Brink's method and proposed a fast two-stage approach to search for the optimal threshold. GONG *et al.* [11] proposed a recursive algorithm for 2D entropic thresholding to further reduce the computation complexity. However, all these methods are still more complex than 1D entropic method proposed by KAPUR *et al.* [7].

Another important issue for image thresholding is that in real life situations a number of images are ambiguous and usually have indistinguishable histogram. In these cases, it is not easy for the above classical thresholding techniques to find a criterion of similarity or closeness for thresholding. Since the fuzzy set theory was introduced, it has become a powerful tool to tackle this difficulty in image thresholding. Fuzzy set theory has been successfully applied to image thresholding to partition the image space into meaningful regions [12–14], and detailed information about its applications to image processing and pattern recognition can be seen in reference [15]. In [12], JAWAHAR *et al.* proposed several different fuzzy thresholding schemes based on fuzzy *c*-means (FCM) clustering algorithm. In [13], CHENG *et al.* introduced the concept of fuzziness into the maximum entropy technique to select threshold values. More recently, ZHAO *et al.* [14] presented a more straightforward solution in the search for fuzzy thresholding parameters by exploiting the relationship between the fuzzy *c*-partition and the probability partition. However, all the three above-mentioned algorithms still do not include the contextual information on image thresholding.

In this paper, we proposed a new global image thresholding technique named spatially weighted fuzzy c -means (SWFCM) algorithm. It is formulated by incorporating the spatial neighboring information into the standard FCM algorithm. The prior spatial constraint defined as weight in the paper plays a key role in this algorithm, which is inspired by the k -nearest neighbor (k -NN) [16] pattern classifier and then modified from two aspects to improve the performance of image thresholding. The method is a 1D thresholding approach. Since the algorithm is initialized by a fast FCM algorithm, the method is as fast as the conventional 1D techniques. Moreover, due to considering the neighborhood information, the method is more tolerant to noise.

The rest of this paper is organized as follows. Section 2 describes the fast FCM algorithm. The SWFCM clustering algorithm is presented in Section 3. Experimental results and comparisons are given in Section 4. Finally, some conclusions are drawn in Section 5.

2. Fast fuzzy c -means algorithm

The fuzzy c -means (FCM) algorithm is an iterative clustering method that produces an optimal c partition, which minimizes the weighted within group sum of squared error objective function $J_q(U, V)$ [17], with respect to U , a fuzzy c -partition of the data set, and with respect to V , a set of C prototypes:

$$J_q(U, V) = \sum_{k=1}^n \sum_{i=1}^c (u_{ik})^q d^2(x_k, v_i) \quad (1)$$

where $X = \{x_1, x_2, \dots, x_n\} \subseteq R^p$ is the data set in the p -dimensional vector space, n is the number of data items, c is the number of clusters with $2 \leq c < n$, u_{ik} is the degree of membership of x_k in the i -th cluster, q is a weighting exponent on each fuzzy membership, v_i is the prototype of the centre of cluster i , $d^2(x_k, v_i)$ is a distance measure between object x_k and cluster centre v_i . A solution of the object function J_q can be obtained via an iterative process, which is carried out as follows:

1. Set values for c , q and ε ;
2. Initialize the fuzzy partition matrix U ;
3. Set the loop counter $b = 0$;
4. Calculate the c cluster centers $\{v_i^{(b)}\}$ with $U^{(b)}$:

$$v_i^{(b)} = \frac{\sum_{k=1}^n [u_{ik}^{(b)}]^q x_k}{\sum_{k=1}^n [u_{ik}^{(b)}]^q}; \quad (2)$$

5. Calculate the membership $U^{(b+1)}$. For $k = 1, \dots, n$ calculate the following:

$$I_k = \left\{ i \mid 1 \leq i \leq c, d_{ik} = \|x_k - v_i\| = 0 \right\},$$

$$\tilde{I}_k = \{1, 2, \dots, c\} - I_k;$$

if $I_k = \phi$, then

$$u_{ik}^{(b+1)} = \frac{1}{\sum_{j=1}^c \left(\frac{d_{jk}}{d_{jk}} \right)^{2/(q-1)}} \quad (3a)$$

else

$$\text{if } i \in \tilde{I}_k, \text{ then } u_{ik}^{(b+1)} = 0. \quad (3b)$$

Additionally $\sum_{i \in I_k} u_{ik}^{(b+1)} = 1$;

6. If $\|U^{(b)} - U^{(b+1)}\| < \varepsilon$, stop; otherwise, set $b = b + 1$ and go to step 4.

Since FCM algorithm is an iterative operation, it is very time consuming, which makes the algorithm impractical for use in image segmentation. To cope with this problem, the statistical gray level histogram of image is applied to the algorithm. Define the non-negative integrate set $G = \{L_{\min}, L_{\min+1}, \dots, L_{\max}\}$ as gray level, where L_{\min} is the minimum gray level, L_{\max} is the maximum gray level, so the gray scale is $L_{\max} - L_{\min}$. For image size $S \times T$, at point (s, t) , $f(s, t)$ is the gray value with $0 \leq s \leq S - 1$, $0 \leq t \leq T - 1$. Let $\text{His}(g)$ denote the number of pixels having gray level g , $g \in G$. The statistical histogram function is as follows:

$$\text{His}(g) = \sum_{s=0}^{S-1} \sum_{t=0}^{T-1} \delta(f(s, t) - g) \quad (4)$$

where $\delta(0) = 1$ and $\delta(g \neq 0) = 0$. With the statistical gray level histogram $\text{His}(g)$, the new objective function of the fast FCM algorithm is now defined as:

$$J_q = \sum_{g=L_{\min}}^{L_{\max}} \sum_{i=1}^c (u_{ig})^q \text{His}(g) d^2(g, v_i) \quad (5)$$

where u_{ig} represents the membership degree of the gray level g to cluster i . This objective function can be minimized in a fashion similar to the standard FCM algorithm. The membership function and the cluster centers are now updated by:

$$u_{ig}^{(b)} = \frac{1}{\sum_{j=1}^c \left(\frac{d_{ig}}{d_{jg}} \right)^{2/(q-1)}}, \quad (6)$$

$$v_i^{(b+1)} = \frac{\sum_{g=L_{\min}}^{L_{\max}} [u_{ig}^{(b)}]^q \text{His}(g) g}{\sum_{g=L_{\min}}^{L_{\max}} [u_{ig}^{(b)}]^q \text{His}(g)}. \quad (7)$$

Since now the FCM algorithm only operates on the histogram of the image, it is faster than the conventional version, which processes the whole data. However, it is important to note that even if the fast FCM algorithm is faster than the standard FCM algorithm, the results of the two algorithms are identical.

3. Spatially weighted fuzzy *c*-means algorithm

The general principle of the technique presented in this paper is to incorporate the neighborhood information into the FCM algorithm. Since in the standard FCM algorithm for a pixel $x_k \in I$ where I is the image, the clustering x_k of with class i only depends on the membership value u_{ik} , if we consider a noisy image, FCM is noise sensitive because the clustering process is related only to gray levels independently of pixels. Considering the influence of the neighboring pixels on the central pixel, the fuzzy membership function given in Eq. (3) can be extended to:

$$u_{ik}^* = u_{ik} p_{ik} \quad (8)$$

where $k = 1, 2, \dots, n$ (n is the number of image data) and p_{ik} is the probability of data point k belonging to cluster i , which is the spatial constraint and further referred to as the weight in this paper, which can be determined by the following neighborhood model. Thus the objective function of SWFCM is changed as:

$$J_q(U^*, V^*) = \sum_{k=1}^n \sum_{i=1}^c (u_{ik}^*)^q d^2(x_k, v_i^*). \quad (9)$$

Similar to that of FCM algorithm, the degrees of membership u_{ik}^* and the cluster centers v_i^* are now updated via:

$$u_{ik}^{*(b)} = \frac{p_{ik}}{\sum_{j=1}^c \left(\frac{d_{ik}}{d_{jk}} \right)^{2/(q-1)}}, \quad (10)$$

$$v_i^{*(b+1)} = \frac{\sum_{k=1}^n [u_{ik}^{*(b)}]^q x_k}{\sum_{k=1}^n [u_{ik}^{*(b)}]^q}. \quad (11)$$

The core idea now is to define the auxiliary weight variable p_{ik} , which is a priori information to guide the outcome of the clustering process. This paper proposes a method for determining the weight based on the neighborhood information inspired by k -NN algorithm [16]

$$p_{ik} = \frac{\sum_{x_n \in N_k^i} 1/d^2(x_n, k)}{\sum_{x_n \in N_k} 1/d^2(x_n, k)} \quad (12)$$

where N_k is the data set of the nearest neighbors of central pixel k , and N_k^i is the subset of N_k composed of the data belonging to class i . In order to give an appropriate method to describe the probability of a data point belonging to any cluster, two improved implementations of the k -NN algorithm are introduced. First, Eq. (12) is extended by considering the potential function of each feature vector [18]

$$K(x, x_k) = \frac{1}{1 + \alpha \|x - x_k\|^2} \quad (13)$$

where α is a positive constant, and $\|x - x_k\|^2$ is the norm of the vector $(x - x_k)$. Then the potential is modified by assigning the proximity of feature vector to each prototype instead of the potential for feature vector to feature vector. Hence the new equation for the weight value is defined as

$$p_{ik} = \frac{\sum_{x_n \in N_k^i} 1/[1 + \alpha d^2(x_n, v_i)]}{\sum_{x_n \in N_k} 1/[1 + \alpha d^2(x_n, v_i)]} \quad (14)$$

where v_i is the prototype of cluster i . After the a priori weight is determined, a new iteration step starts with this auxiliary variable p_{ik} . To prevent the SWFCM from getting trapped in local minima, the SWFCM algorithm is initialized with the above fast FCM algorithm. Once the FCM is stopped, the SWFCM algorithm continues with the values for the prototypes and membership values obtained from the fast FCM algorithm. When the algorithm has converged, a defuzzification process then takes

place in order to convert the fuzzy partition matrix U to a crisp partition. A number of methods have been developed to defuzzify the partition matrix U , among which the maximum membership procedure is the most important. The procedure assigns object k to the class C with the highest membership:

$$C_k = \arg_i \left\{ \max(u_{ik}) \right\}, \quad i = 1, 2, \dots, c. \quad (15)$$

With this procedure, the fuzzy images are then converted to crisp image. For image thresholding, $c = 2$ in Eq. (15). We call this method soft thresholding scheme contrary to conventional hard threshold scheme, which has been proven to be associated with loss of structure details on thresholding [12]. Although JAWAHAR *et al.* [12] have proposed a fuzzy thresholding method with FCM algorithm by finding the hard threshold at the intersection of both membership distributions (see Eq. (10) in [12]), it is easily verified that this technique is almost equivalent to thresholding the image using the maximum membership procedure.

4. Experimental results

In this section, results of application of the SWFCM algorithm are presented. The performance of the method proposed is compared with those of fuzzy thresholding method introduced by JAWAHAR *et al.* [12] (see Section 3) and two well-known thresholding methods, including algorithms developed by OTSU [4] and KAPUR *et al.* [7]. For all cases, unless otherwise stated, the weighting exponent $q = 2.0$ and $\varepsilon = 0.0001$. We tried several values for α and found that a value of $\alpha = 1$ gives a convenient result. A 3×3 window of image pixels is considered in this paper, thus the spatial influence on the centre pixel is through its 8-neighborhood pixels. It should be noted that if the window size is larger, more computational time is needed and at the same time the results of the image will often be over-thresholded, which is concluded from the evaluations with different window sizes. Both 3×3 and 5×5 windows give almost the same results. All these algorithms are coded in Microsoft Visual C++ version 6.0 and are run on a 1.7 GHz Pentium IV personal computer with a memory of 256 MB. In all the experiments, we found that since the SWFCM algorithm is initialized by the fast FCM algorithm, the algorithm converges after several iterations and consumes within 1 second which is almost as fast as with the other three methods.

In the first example, we generate a synthetic image with gray levels 0 and 255 for background and foreground, respectively. The image was then corrupted by additive Gaussian noise such that the $\text{SNR} = 5$. Figure 1a is the original image and Fig. 1b is the degraded noisy image. Figure 1f shows the result of the method proposed. The results of Jawahar's method, Otsu's method and Kapur's method are displayed in Figs. 1c, d and e, respectively. The results show the our method is effective and outperforms the other methods in the noisy situation. The number of misclassified pixels for different thresholding methods is counted during the experiments and is

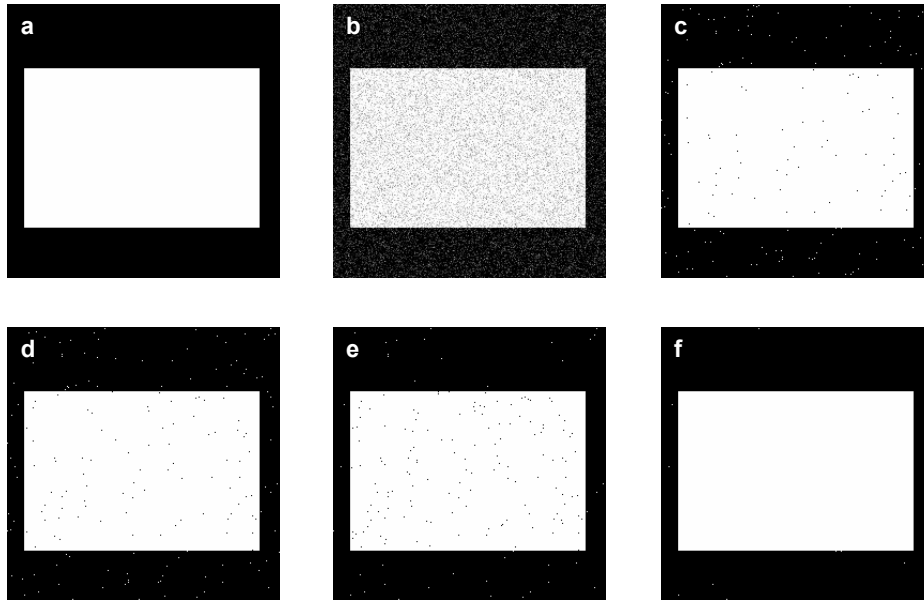


Fig. 1. Results of thresholding: the original image (a), noisy image with SNR = 5 (b), Jawahar's method (c), Otsu's method (d), Kapur's method (e), our method (f).

listed in Tab. 1. It can be seen that the total number of misclassification pixels for the method proposed is the least of the four different methods, and the total misclassification number for Jawahar's method, Otsu's method and Kapur's method is nearly the same, which is about 18 times that of the method presented here.

Table 1. Number of misclassified pixels with different methods.

Methods	Jawahar	Otsu	Kapur	Our method
Foreground	61	83	141	0
Background	118	96	39	10
Total	179	179	180	10

The test image in the second example, which is given in Fig. 2a, is obtained from MATLAB toolbox named *rice* blurred by 5% Gaussian noise. The results of thresholding by applying the different algorithms to the image are presented in Figs. 2b–e. From these images, we can see that Jawahar's method, Otsu's method and Kapur's method cannot correctly threshold the image in the case of noise, while our method can do this with the least errors in such a case, which shows that our method is more tolerant to noise.

The third example is a real T1-weighted magnetic resonance (MR) image as shown in Fig. 3a. Since the MR scanners usually produces normally distributed white noise [19], in order to extract the head from the background, the noise should be removed

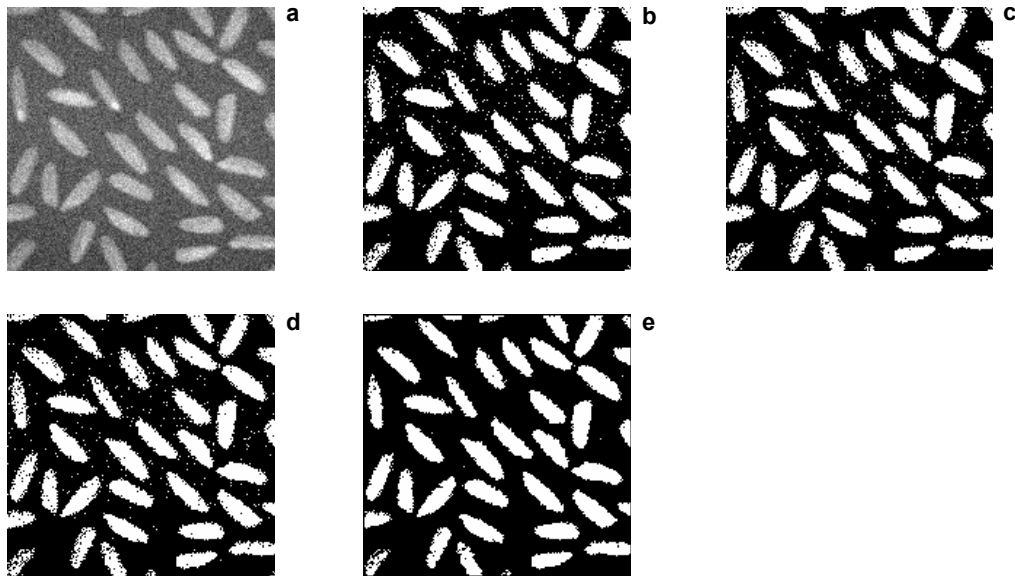


Fig. 2. Results of thresholding: the original image (a), Jawahar's method (b), Otsu's method (c), Kapur's method (d), our method (e).

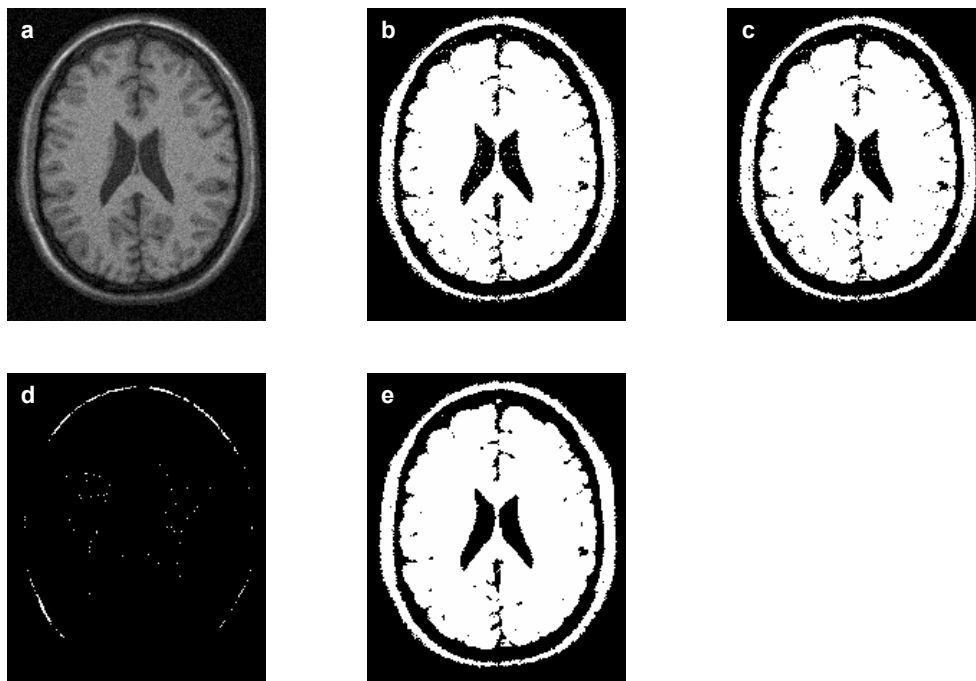


Fig. 3. Results of thresholding: the original image (a), Jawahar's method (b), Otsu's method (c), Kapur's method (d), our method (e).

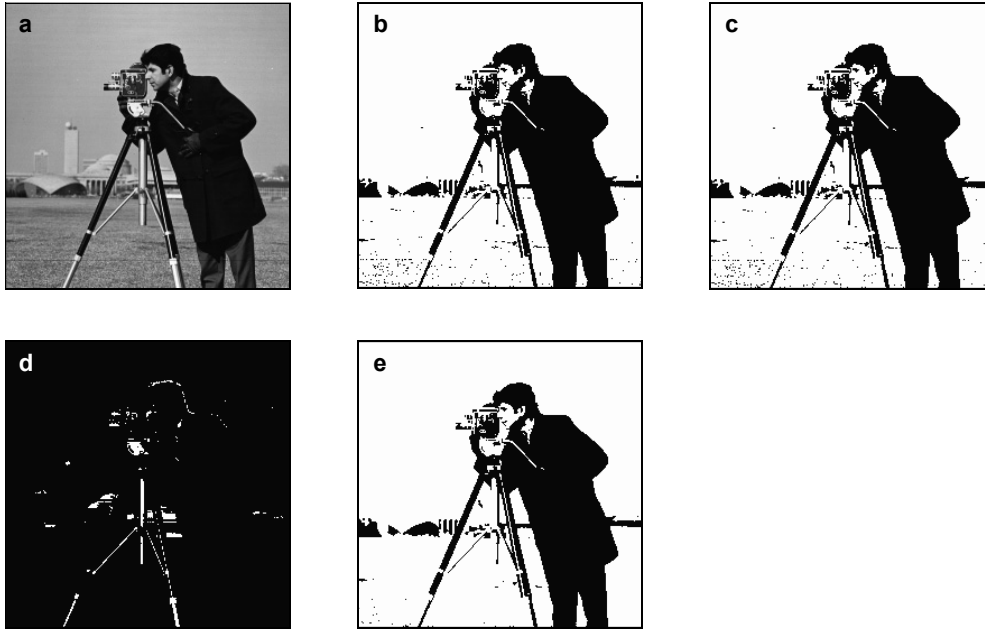


Fig. 4. Results of thresholding: the original image (a), Jawahar's method (b), Otsu's method (c), Kapur's method (d), our method (e).

firstly that is often the first stage for segmentation of MR images. The results of thresholding by applying the different algorithms to the image are presented in Figs. 3b–e. From these images, we can see that Kapur's method failed to threshold the image. Jawahar's method, Otsu's method and our method can well extract the head from the background, however with the first two methods some noise still exists especially in the cerebrospinal fluid (CSF) of the image.

In the last example, there is a famous standard test image named *camerman*, which is illustrated in Fig. 4a. The result of the method proposed is presented in Fig. 4e. The results for comparison are given in Fig. 4b–d. As can be seen, Jawahar's method and Otsu's method give almost the same result, while Kapur's method cannot accurately extract the object from the background as in the third example. However, it can be seen that our method performs best when segmenting the object from the background with the least spurious components and noise, particularly in the grass ground area.

For further quantitative evaluation of the performance of the algorithms and for the reason of no ground-truth information can be given for examples 2–4, the region nonuniformity (NU) measure [3] is employed here, which is defined as

$$\text{NU} = \frac{|F_T|}{|F_T + B_T|} \frac{\sigma_f^2}{\sigma^2} \quad (16)$$

Table 2. Thresholding evaluation ranking for examples 2–4 according to the NU measure.

Method	Jawahar	Otsu	Kapur	Proposed
Example 2	0.121	0.112	0.108	0.104
Example 3	0.049	0.047	0.578	0.045
Example 4	0.022	0.019	0.607	0.017

where F_T and B_T represent the background and foreground area pixels in the test image, the $|\dots|$ is the cardinality of the set, σ_f^2 represents the foreground variance, and σ^2 represents the variance of the whole image. According to [3], it is expected that a well-segmented image will have nonuniformity measure close to 0, while the worst case is $NU = 0$. In other words, the smaller the NU measure is, the better the performance of the thresholding algorithm, and *vice versa*. The NU measure values of the four algorithms for examples 2–4 are calculated and given in Tab. 2. It can be seen that the proposed thresholding scheme performs best in the four algorithms for examples 2–4 and attains the smallest NU measure values in all the cases.

5. Conclusions

We have presented a novel approach to image thresholding based on SWFCM algorithm. The algorithm is developed by incorporating the spatial constraints into the standard FCM algorithm. This method not only takes into account the advantage of the fuzzy framework, but also considers spatial relations between pixels. The weight plays a crucial role in this algorithm, which is inspired by k -NN algorithm and is modified from two aspects in order to improve its properties. The performance of our method is compared with those of Otsu's method, Kapur's method, and a fuzzy thresholding method proposed by JAWAHAR *et al.* [12]. Experiments with synthetic and real images show that SWFCM algorithm can effectively extract object from background. As the algorithm is initialized by the fast FCM algorithm, the presented approach is as fast as the conventional 1D techniques. Also, owing to the incorporation of spatial information, the SWFCM algorithm is less prone to noise. In fact, if the result of thresholding is an image with two gray values, the process can also be called bilevel segmentation. Future work will extend the algorithm to multi-level thresholding or segmentation.

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