Higher-order space charge field effects on the self-deflection of bright screening spatial solitons in two-photon photorefractive crystals

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We investigate the effects of higher-order space charge field on the self-deflection of bright screening spatial solitons due to two-photon photorefractive effects by a numerical method under steady-state conditions. The expression for an induced space charge electric field including higher-order space charge field terms is obtained. Numerical results indicate that bright screening solitons undergo self-deflection process during propagation, and the solitons always bend in the opposite direction of the \(c\) axis of the crystal. The self-deflection of bright screening solitons can experience considerable increase especially in the regime of high bias field strengths. Relevant examples are provided.

Keywords: non-linear optics, two-photon photorefractive effect, bright screening spatial solitons, self-deflection.

1. Introduction

During the last decade, the optical spatial solitons based on photorefractive effect have attracted much interest, for these photorefractive spatial solitons can be formed at low light intensity and are potentially useful for all-optical switching, beam steering, and optical interconnects. At present, three types of steady-state scalar solitons (screening solitons [1–3], photovoltaic solitons [4–7] and screening-photovoltaic solitons [8–10]) have been predicted theoretically and found experimentally.

The diffusion process introduces an asymmetric tilt in the light-induced photorefractive waveguide, which results in the self-deflection process of solitons [1]. Self-deflection was firstly found in bright screening solitons in bias photorefractive crystals [11, 12]. The self-deflection process was explained theoretically with first-order diffusion effect taken into account [13]. However, experimental results have shown that self-deflection can exceed the deflection predicted by theory, especially in the regime of high bias field strengths. To account for this discrepancy, SINGH et al. [14] investigated the effects that arise from the higher-order space charge field terms on
the evolution of bright screening solitons. Recently, LIU and HAO [15] and ZHANG et al. [16–18] investigated the higher-order space charge field effects on the evolution of bright screening-photovoltaic soliton, bright photovoltaic soliton, dark screening soliton, and dark photovoltaic soliton.

All of the above-mentioned solitons result from the single-photon photorefractive effect. Recently, CASTRO-CAMUS and MAGANA [19] provided a model of the two-photon photorefractive effect. Later, screening solitons [20], photovoltaic solitons [21] and screening-photovoltaic solitons [22] in two-photon photorefractive crystals have been predicted. On the other hand, incoherently coupled bright–bright, dark–dark, bright–dark, and grey–grey soliton pairs have been predicted [23–26] that result from the two-photon photorefractive effect. In this paper, we investigate the higher-order space charge field effects on the self-deflection of bright screening spatial solitons in two-photon photorefractive crystals through an approach similar to that presented in [14–18]. The induced space charge field in which those higher-order terms are included is obtained, a dynamical evolution equation is derived in which the effects that arise from these higher-order terms are considered. Our results show that bright screening solitons due to two-photon photorefractive effect possess a self-deflection procedure during propagation in the opposite direction of the crystal’s $c$ axis on the base of the first-order diffusion terms. Taking into account the higher-order space charge field, numerical results further indicate that the value of the spatial shift that is due to the first-order diffusion term alone is always smaller than that due to both the first-order diffusion term and the higher-order space charge field terms acting together. This behavior is similar to that of bright screening solitons due to single-photon photorefractive effect.

2. Theoretical model

We start with considering an optical beam that propagates in a biased photorefractive crystal with the two-photon photorefractive effect along the $z$ axis and is permitted to diffract only along the $x$ direction. The crystal is proposed here to be SBN:60 with its optical $c$ axis along the $x$ coordinate and is illuminated by the gating beam. Moreover, let us assume that the optical beam is linearly polarized along the $x$ direction. As usual, we express the optical field of the incident beam in terms of slowly varying envelope $\phi$, i.e., $E = E_{sc} = \hat{x}\phi(x, z)\exp(ikz)$, where $k = \frac{2\pi}{\lambda_0}n_e$, $n_e$ is the unperturbed extraordinary index of refraction, and $\lambda_0$ is the free-space wavelength. Under these conditions the optical beam satisfies the following envelope evolution equation:

$$i\phi_z + \frac{1}{2k}\phi_{xx} - \frac{k_0n_e^3r_{33}E_{sc}}{2}\phi = 0 \quad (1)$$

where $\phi_z = \partial\phi/\partial z$, $\phi_{xx} = \partial^2\phi/\partial x^2$, $r_{33}$ is the electro-optic coefficient, $E_{se} = E_{sc}\hat{x}$ is the space charge field in the crystals. Following Ref. [20], the space charge field in Eq. (1) can be obtained from the set of rate, current, and Poisson’s equations proposed
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by CASTRO-CAMUS and MAGANA [19] to describe the two-photon photorefractive effect. In the steady-state and under a strong bias field condition such that the photovoltaic field can be neglected, or in a non-photovoltaic crystal, these equations are [19, 20]:

\[
(s_1 I_1 + \beta_1)(N - N^+) - \gamma_1 n_1 N^+ - \gamma n N^+ = 0 \tag{2}
\]

\[
(s_1 I_1 + \beta_1) + (N - N^+) + \gamma_2 n(n_{01} - n_1) - \gamma_1 n_1 N^+ - (s_2 I_2 + \beta_2)n_1 = 0 \tag{3}
\]

\[
(s_2 I_2 + \beta_2)n_1 + \frac{1}{e} \frac{\partial J}{\partial x} - \gamma n N^+ - \gamma_2 n(n_{01} - n_1) = 0 \tag{4}
\]

\[
\varepsilon_0 \varepsilon_r \frac{\partial E_{sc}}{\partial x} = e(N^+ - n - n_1 - N_A) \tag{5}
\]

\[
J = e \mu n E_{sc} + eD \frac{\partial n}{\partial x} \tag{6}
\]

\[
\frac{\partial J}{\partial x} = 0 \quad \text{or} \quad J = \text{const} \tag{7}
\]

where \(N\) is the donor density, \(N^+\) is the ionized density, \(N_A\) is the acceptor or trap density, and \(n\) is the density of the electrons in the condition band (CB); \(n_1\) is the density of the electron in the intermediate state; \(n_{01}\) is the density of traps in the intermediate state; \(s_1\) and \(s_2\) are cross section; \(\beta_1\) and \(\beta_2\) are the thermoionization probability constants for the transitions of the value band (VB) to the allowed intermediate levels (IL) and IL-CB, respectively. \(\gamma_1, \gamma_2\) and \(\gamma_3\) are the recombination factors of the CB-VB, IL-VB, and CB-IL transitions, respectively; \(D\) is the diffusion coefficient; \(\mu\) and \(e\) are the electron mobility and charge, respectively; \(\varepsilon_0\) and \(\varepsilon_r\) are the vacuum and relative dielectric constants, respectively; \(J\) is the current density; \(I_1\) is the intensity of the gating beam, which can be considered as a constant; \(I_2\) is the intensity of the soliton beam. According to Poynting’s theorem, \(I_2\) can be expressed in terms of the \(\phi\), that is, \(I_2 = (n_e/2 \eta_0)|\phi|^2\) where \(\eta_0 = (\mu_0/\varepsilon_0)^{1/2}\). One can neglect the term \((n_{01} - n_1) \ll N^+\) with respect to the other terms. In this case, from Eqs. (2) and (3) we have

\[
n_1 = \frac{\gamma N^+ n}{s_2 I_2 + \beta_2} \tag{8}
\]

The substitution of Eq. (8) into Eq. (2) yields

\[
n = \frac{(s_1 I_1 + \beta_1)(s_2 I_2 + \beta_2)(N - N^+)}{\gamma N^+ (s_2 I_2 + \beta_2 + \gamma_1 N^+)} \tag{9}
\]
Under the approximation $n, n_1 \ll N^+, N_A$ yields

$$N^+ = N_A \left(1 + \frac{\varepsilon_0 \varepsilon_r}{eN_A} \frac{\partial E_{sc}}{\partial x}\right)$$  \hspace{1cm} (10)$$

In this case, from Eqs. (9) and (10) we have approximately [2]

$$n = \left(\frac{(s_1 I_1 + \beta_1)(s_2 I_2 + \beta_2)(N - N_A)}{\gamma N_A(s_2 I_2 + \beta_2 + \gamma_1 N_A)}\right) \frac{1}{1 + \frac{\varepsilon_0 \varepsilon_r}{eN_A} \frac{\partial E_{sc}}{\partial x}}$$  \hspace{1cm} (11)$$

According to Ref. [20], it can be assumed that the intensity of soliton beam attains asymptotically a constant value at infinity, that is, $I_2(x \to \pm \infty, z) = I_{2\infty}$. In these regions with uniform illumination, the space charge is also independent of $x$, namely, $E_{sc}(x \to \pm \infty, z) = E_0$. If the spatial extent of the soliton beam is much less than the $x$-width $W$ of the photorefractive medium, $E_0$ is approximately given by $\pm V/W$, where $V$ is the applied bias voltage. From Eq. (11) the free-electron density $n_\infty$ at $x \to \pm \infty$ can be given by

$$n_\infty = \left(\frac{(s_1 I_1 + \beta_1)(s_2 I_{2\infty} + \beta_2)(N - N_A)}{\gamma N_A(s_2 I_{2\infty} + \beta_2 + \gamma_1 N_A)}\right)$$  \hspace{1cm} (12)$$

Equation (7) indicates that the current density $J$ is constant everywhere and therefore $J = J_{\infty}$. Thus from Eq. (6) we have

$$e\mu n_\infty E_0 = e\mu n E_{sc} + eD \frac{\partial n}{\partial x}$$  \hspace{1cm} (13)$$

Substituting Eqs. (11) and (12) into (13), we find

$$E_{sc} = E_0 \left(\frac{I_{2\infty} + I_{2d}}{I_{2\infty} + I_{2d} + \frac{\gamma_1 N_A}{s_2}}\right) \left(\frac{I_2 + I_{2d} + \frac{\gamma_1 N_A}{s_2}}{I_2 + I_{2d}}\right) \left(1 + \frac{\varepsilon_0 \varepsilon_r}{eN_A} \frac{\partial E_{sc}}{\partial x}\right) +$$

$$\frac{D\gamma_1 N_A}{\mu s_2(I_2 + I_{2d} + \frac{\gamma_1 N_A}{s_2})(I_2 + I_{2d})} \frac{\partial I_2}{\partial x} + \frac{D}{\mu} \frac{\varepsilon_0 \varepsilon_r}{eN_A} \frac{\partial^2 E_{sc}}{\partial x^2}$$  \hspace{1cm} (14)$$

where $I_{2d} = \beta_2/s_2$ is the dark irradiance intensity. It is similar to that given in [14].
Under strong bias conditions \( E_0 \) will be large enough, and therefore the drift component of the current in the medium will be dominant and moreover in typical photorefractive crystals the dimensionless quantity \( \left| \frac{\varepsilon_0 \varepsilon_r}{e N_A} \frac{\partial E_{sc}}{\partial x} \right| \ll 1 \). In this case, to
the first order, \( E_{sc} \) is approximately given by

\[
E_{sc} \approx E_{sc0} = E_0 \left( \frac{(I_{2\infty} + I_{2d}) \left( I_2 + I_{2d} + \frac{\gamma_1 N_A}{s_2} \right)}{(I_{2\infty} + I_{2d}) \left( I_2 + I_{2d} + \frac{\gamma_1 N_A}{s_2} \right)} \right)
\]

(15)

To study the effects arising from higher-order space charge field terms such as \( \frac{\partial E_{sc}}{\partial x} \) and \( \frac{\partial^2 E_{sc}}{\partial x^2} \) in Eq. (14), we now use the first-order solution of Eq. (14), i.e., Eq. (15), and in turn the other terms are obtained in an iterative fashion. By doing so, the perturbative solution of the space charge field \( E_{sc} \) reads as follows:

\[
E_{sc} = E_{sc0} + E_\delta + E_\delta_1 + E_\delta_2 + E_\delta_3
\]

(16)

where:

\[
E_\delta = -\frac{DY_1 N_A}{\mu s_2 \left( I_2 + I_{2d} + \frac{\gamma_1 N_A}{s_2} \right) \left( I_2 + I_{2d} \right)} \frac{\partial I_2}{\partial x}
\]

(17a)

\[
E_\delta_1 = -\frac{\varepsilon_0 \varepsilon_r}{e N_A} \left( \frac{E_0 (I_{2\infty} + I_{2d})}{(I_{2\infty} + I_{2d}) \left( I_2 + I_{2d} + \frac{\gamma_1 N_A}{s_2} \right) \left( I_2 + I_{2d} \right)} \right)^2 \frac{\partial I_2}{\partial x}
\]

(17b)

\[
E_\delta_2 = \frac{2 \varepsilon_0 \varepsilon_r}{\mu} \frac{2}{e N_A} \left( \frac{E_0 (I_{2\infty} + I_{2d})}{(I_{2\infty} + I_{2d}) \left( I_2 + I_{2d} + \frac{\gamma_1 N_A}{s_2} \right) \left( I_2 + I_{2d} \right)} \right)^2 \left( \frac{\partial I_2}{\partial x} \right)^2
\]

(17c)

\[
E_\delta_3 = -\frac{D}{\mu} \frac{\varepsilon_0 \varepsilon_r}{e N_A} \left( \frac{E_0 (I_{2\infty} + I_{2d})}{(I_{2\infty} + I_{2d}) \left( I_2 + I_{2d} + \frac{\gamma_1 N_A}{s_2} \right) \left( I_2 + I_{2d} \right)} \right) \frac{\partial^2 I_2}{\partial x^2}
\]

(17d)

It is important to note that Eq. (16) is valid as long as the perturbations \( E_\delta \) and \( E_\delta_i \) (\( i = 1, 2, 3 \)) are much smaller than the leading term of the space charge field \( E_{sc0} \).
Substituting Eq. (16) into Eq. (1), and adopting the following dimensionless coordinates and variables:

\[ s = x / x_0, \quad \xi = z / (k x_0^2), \quad U = (2 \eta_0 I_{2d} / n_e)^{-1/2} \phi, \quad x_0 \]

is an arbitrary spatial width. Under these conditions, the following dynamical evolution equation can be obtained

\[
i U_{\xi} + \frac{1}{2} U_{ss} - \beta \frac{1 + \rho}{1 + \sigma + \rho} \left( 1 + \frac{\sigma}{1 + |U|^2} \right) U + \delta \frac{\sigma(|U|^2)_s}{(1 + |U|^2 + \sigma)(1 + |U|^2)} U + \\
+ \delta_1 \frac{(1 + \rho)^2 \sigma(1 + \sigma + |U|^2)(|U|^2)_s}{(1 + \rho + \sigma)^2(1 + |U|^2)^3} U - \delta_2 \frac{(1 + \rho)\sigma(|U|^2)_s^2}{(1 + \rho + \sigma)(1 + |U|^2)^3} U + \\
+ \delta_3 \frac{(1 + \rho)\sigma(|U|^2)_{ss}}{(1 + \rho + \sigma)(1 + |U|^2)^2} U = 0 \tag{18}
\]

where \( \beta = (k_0 x_0)^2 (n_e^4 r_{33} / 2) E_0, \quad \delta = (k_0 x_0)^2 (n_e^4 r_{33} / 2)(K_B T / e x_0), \quad \delta_1 = \beta E_0 \tau, \quad \delta_2 = 2 \beta \tau \kappa, \quad \delta_3 = \beta \tau \kappa, \quad \tau = \frac{e_0 e_r}{e N_A} \frac{1}{x_0}, \quad \kappa = \frac{D}{\mu x_0} = \frac{K_B T}{e x_0}, \quad U_{\xi} = \frac{\partial U}{\partial \xi}, \quad U_{ss} = \frac{\partial^2 U}{\partial s^2}, \quad \rho = I_{2d} / I_{2d}. \)

In Equation (18), the term \( \delta \) represents the first-order diffusion process whereas \( \delta_1, \delta_2, \delta_3 \) are higher-order space charge field effects.

By considering only the drift nonlinearity (i.e., \( \beta \) term) and by entirely neglecting all the \( \delta \) perturbations, for bright screening solitons (\( \rho = 0 \)), from Eq. (18) we have

\[
i U_{\xi} + \frac{1}{2} U_{ss} - \frac{\beta}{1 + \sigma} \left( 1 + \frac{\sigma}{1 + |U|^2} \right) U = 0 \tag{19}
\]

The fundamental bright screening solitary solution can be derived from Eq. (19) by expressing the beam envelope \( U \) in the usual fashion: \( U = r^{1/2} y(s) \exp(i \nu \xi) \). Here, \( \nu \) represents a nonlinear shift of the propagation constant, \( y(s) \) is a normalized real function bounded between \( 0 \leq y(s) \leq 1 \). By integrating Eq. (19) under the boundary conditions: \( y(0) = 1, \quad \dot{y}(0) = 0, \quad y(s \to \pm \infty) = 0 \), we found that [19]

\[
\left( \frac{2 \beta \sigma}{1 + \sigma} \right)^{1/2} s = \pm \int_{y}^{1} \frac{r^{1/2} d\tilde{y}}{\ln(1 + r\tilde{y}^2) - \tilde{y}^2 \ln(1 + r)} \tag{20}
\]

The bright solitary beam profile can be obtained from Eq. (20) by a simple numerical integration.
3. The self-deflection of bright screening solitons due to two-photon photorefractive effects

3.1. The self-deflection on base of the first-order diffusion terms

We will now investigate the first-order diffusion effects on the evolution of bright screening solitons due to two-photon photorefractive effects. By assuming solitary wave solutions as input beam profiles, we solve Eq. (18) numerically ignoring all the higher-order space charge field terms $\delta_1, \delta_2, \delta_3$ by using a finite-difference method. As an example, let us consider SBN:60 a crystal with following parameters [14]:

- $\gamma_{33} = 237 \times 10^{-12}$ m/V,
- $N_d = 4 \times 10^{16}$ cm$^{-3}$,
- $\varepsilon_r = 880$,
- $\lambda_0 = 0.5$ $\mu$m,
- $x_0 = 25$ $\mu$m,
- $E_0 = 1 \times 10^5$ V/m,
- $r = 10$.

We find that $\beta = 34.5, \delta = 0.35$. By numerically solving Eq. (18), we obtain the intensity profile evolution of the bright screening in the two-photon photorefractive crystal as shown in Fig. 1a. The evolution of the spatial shift on the base of the first-order diffusion terms, denoted by $\Delta s$, which is defined as the distance between $s = 0$ and the position of the beam centre at $\xi$, is shown in Fig. 1b. The results show that the bright screening solitons experience approximately adiabatic self-deflection in the opposite direction of the $c$ axis of the crystal and the spatial shift moves on an approximately parabolic trajectory. Its behavior is similar to bright screening solitons based on single-photon photorefractive effects [14].

3.2. The self-deflection on base of the higher-order space charge field

Now, we investigate the effects that arise from the higher-order terms $\delta_1, \delta_2, \delta_3$ on the bright screening solitons. The parameters of the crystal being taken as above, we find moreover that $\delta_1 = 0.168, \delta_2 = 0.0035, \delta_3 = 0.0017$. It is obvious that the terms $\delta_2$ and $\delta_3$ are much smaller than $\delta$ and $\delta_1$, so we neglect the effects of $\delta_2$ and $\delta_3$. Figure 2 compares the spatial shift due to $\delta$ alone to that obtained with $\delta$ and $\delta_1$ acting together at different strengths of applied electric field, i.e., $E_0 = 10^5$ V/m, $E_0 = 2 \times 10^5$ V/m, and $E_0 = 5 \times 10^5$ V/m. The solid curves denote the dynamic evolutions...
of spatial shift on base of the first-order diffusion term for various bias fields and
the dashed curves denote the dynamic evolutions of spatial shift in various bias fields
when $\delta$ and $\delta_1$ act together. It is quite clear from the figure that at low bias fields
the process is dominated by first-order diffusion effects whereas at high bias one needs
to account for $\delta_1$ term, and the value of the spatial shift that is due to $\delta$ alone is always
smaller than that of $\delta$ and $\delta_1$ acting together. This behavior is similar to that of bright
screening solitons based on the single-photon photorefractive effects [14].

4. Conclusions

The effects of higher-order space charge field terms on the self-deflection of bright
screening solitons for two-photon photorefractive model have been investigated by
a numerical method. We have obtained an expression for the induced space charge
field in which higher-order space charge field terms are involved. Numerical results
indicate that the higher-order space charge field terms result in a considerable increase
in the self-deflection of bright screening solitons especially in the high bias field
strengths. That is, the value of the spatial shift that is due to both the first-order
diffusion term and the higher-order space charge field terms acting together is always
larger than that due to the first-order diffusion term alone.

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