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Model of spatial dependence of the transport coefficient of photons scattered in a tissue

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The paper presents the modeling of scattering of light photons in turbid medium, in which the relative change in light intensity by an elementary plane-parallel layer is proportional to the superposition of spatially homogeneous and spatially inhomogeneous summands. Taking into account the light intensity attenuation by a plane-parallel layer of the spatial nonlinearity transport coefficient of photons, the interaction with the scattering centers in the integral law does not significantly affect the anisotropy scattering regularity and statistical regularities, in which the transport coefficient is spatially inhomogeneous.

Keywords: Monte Carlo simulation, turbid medium, transport photon.

1. Introduction

It is known [1–3] that the attenuation regularities of photon flux (transport) in optically inhomogeneous medium with random fluctuations of interaction centers are usually modeled using statistical Monte Carlo. This is a flexible, but accurate approach to simulate the process in which local rules of photon migration are represented in the form of probability distribution of random variables, such as the displacement step size between two photons scattering acts, and azimuthal and polar deviation angles of the direction of this step.

Monte Carlo method is equivalent to the modeling of the photon migration by analyzing the differential equations of the radiation transport. However, its analytical solutions are often impossible to obtain, while Monte Carlo allows achieving the solution of the problem with assumed precision by increasing the number of photons. Through the use of the laser light sources, this approach has got a wide practical application, especially in biomedicine [4–11].

Monte Carlo method in scattering medium optics is the technique of obtaining and statistical analysis of the trajectories of a large number of scattered photons. In essence, it is a set of procedures for constructing ensembles of random numbers and functions, the statistical moments of which have been determined. As the statistical characteristics of the light field in optics of randomly inhomogeneous medium can be defined as the ensembles of photon trajectories and random waves, the approach based on the idea of the radiation propagation as a photons flux is called the "corpuscular" and the approach based on the wave conception – "wave".

Optically inhomogeneous medium with random fluctuations is mainly characterized by the following parameters [4–6]:

1. The factor of elastic scattering μ_s which is a numerically inverted optical path of the photon propagation in turbid medium in which the flux decreases in e times.

2. Absorption rate μ_a, which is numerically equal to the inverse optical distance at which the photon flux decreases in e times by absorption. Then the value

$$\mathrm{mfpl} = \frac{1}{\mu_s + \mu_a} = \frac{1}{\mu_t} \tag{1}$$

reflects the statistical average length of the photon free path between two scattered acts with dispersion $1/\mu_t^2$, with ratio $p_s = \mu_s/\mu_t$ being the probability of elastic scattering and $p_\alpha = \mu_\alpha/\mu_t$ – the probability of absorption or albedo of a lonely processes center of elastic scattering and absorption, where μ_t is a transport coefficient of the photons interaction with scattering centers.

3. Anisotropy parameter g. In the spherical coordinate system, the direction of the vector orientation of the photon in space is represented by the point on the unit sphere, that is why in the case of an isotropic distribution of spherical random variables $\rho = 1$, θ, φ (θ and φ denote polar and azimuthal angles, respectively) the probability density of joint distribution can be described by the dependence

$$p(\theta, \varphi)\mathrm{d}\theta\,\mathrm{d}\varphi = \frac{1}{4\pi}\sin(\theta)\mathrm{d}\theta\,\mathrm{d}\varphi = \left(\frac{\sin(\theta)}{2}\mathrm{d}\theta\right)\left(\frac{1}{2\pi}\mathrm{d}\varphi\right) \tag{2}$$

where θ and φ are independent random variables and $p(\cos(\theta))$ displays the indicatrix of photons scattering on a single particle, which is modeled by the Henyey–Greenstein function

$$p(\cos(\theta)) = \frac{1}{4\pi}\frac{1-g^2}{\left[1+g^2-2g\cos(\theta)\right]^{3/2}} \tag{3}$$

where $g = \langle\cos(\theta)\rangle$ is the anisotropy parameter.

4. A typical parameter of photon interactions with optically inhomogeneous medium is the mean free path l, which denotes the mean distance photon passes between

two scattered acts. It is a random characterization of the process and it is traditionally justified by postulating the linear dependence of relative intensity dI/I of light flux, transmitted through a plane parallel layer with thickness dl

$$\frac{\mathrm{d}I}{I} = -\mu_t \mathrm{d}l \tag{4}$$

Equation (4) is the base for the Bouguer–Lambert law justification. According to it, the decrease in the relative intensity of light is proportional to the differential of a spatial parameter in the first power l^1 ($\mathrm{d}l^1 = \mathrm{d}l$). Therefore, the probability that the photon will pass the distance ξ without collisions is equal to $\mathrm{d}p_\xi$, and at the next step after l the photon will be scattered with probability $\mathrm{d}p_\xi \exp(-l\mu_t)\, \mu_t\, \mathrm{d}\xi$, which is the product of probabilities, because the processes of interaction are statistically independent events. Then, the following equality comes true [11]:

$$s = \frac{-\log(r)}{\mu_t} \tag{5}$$

where $r \in [0;\, 1]$, and if the medium is statistically homogeneous μ_t = const, the processes of photon interactions with it will not depend on its spatial coordinates. Then

$$p(s)\mathrm{d}s = \frac{1}{l_{\mathrm{mean}}} \exp\left(-\frac{s}{l_{\mathrm{mean}}}\right) \tag{6}$$

$$\int p(s)\mathrm{d}s = 1 \tag{7}$$

$$\overline{s} = \int s p(s)\mathrm{d}s = l_{\mathrm{mean}} \tag{8}$$

But in this paper we investigate the statistical characteristics of the scattered light in turbid medium in which the transport coefficient μ_t is spatially inhomogeneous.

2. Theoretical model and analysis of the results

When modeling the scattering using Monte Carlo method, the transport length of photon

$$\ell_{\mathrm{tr}} = \frac{1}{(1-g)\mu_s + \mu_\alpha} \tag{9}$$

is an important characteristic and it is the distance needed to provide the stochasticity to the direction of photon propagation, when photon “forgets” its initial direction of motion as a result of scattering. Transport photon length is directly related to the mean free path l as a random variable that takes arbitrary positive values with probability

$p(\ell) = \mu\exp(-\mu\ell)$ and has the following statistics characteristics – mean value $\langle\ell\rangle$ and dispersion $\langle\ell^2\rangle$:

$$\langle\ell\rangle = \int_0^{\ell} \ell p(\ell)\mathrm{d}\ell = \frac{1}{\mu_t} \tag{10}$$

$$\langle\ell^2\rangle = \int_0^{\ell} \ell^2 p(\ell)\mathrm{d}\ell = \frac{2}{\mu_t^2} \tag{11}$$

Then the concrete realization of the mean free path is determined by the function

$$p(\ell) = \frac{1}{\langle\ell\rangle}\exp\left(-\frac{\ell}{\langle\ell\rangle}\right) \tag{12}$$

In this paper we considered the model of scattering photons of light in a tissue, in which the relative change in light intensity $\mathrm{d}I/I$ by an elementary parallel plane layer is proportional to the superposition of spatially homogeneous $\mu_t\mathrm{d}l$ and spatially inhomogeneous $\chi\mathrm{d}(l^2)$ summands. Then within the limits of the proposed approach we have the following equation:

$$\frac{\mathrm{d}I}{I} = -\mu_t\mathrm{d}l - \chi\mathrm{d}(l^2) \tag{13}$$

which is conveniently convertible to the form

$$\frac{\mathrm{d}I}{I} = -\mu_t\mathrm{d}l - 2\chi l\mathrm{d}l = -(\mu_t + 2\chi l)\mathrm{d}l \tag{14}$$

and the integral law of intensity reduction of the light plane-parallel medium will be:

$$I(l) = I_0\exp\left[-(\mu_t l + \chi l^2)\right] \tag{15}$$

or

$$I(l) = I_0\exp[-\mu_t'(l)l] \tag{16}$$

where $\mu_t'(l) = \mu_t(1 + l\chi/\mu_t)$.

It is worth to remind that the first summand Eq. (13) is the base for justification of the Bouguer–Lambert law, known in optics.

Then we calculate statistical parameters of the medium, which takes into account the spatial dependence of the transport coefficient of photons interaction with the scattering centers of the type $\mu_t'(l) = \mu_t(1 + l\chi/\mu_t)$.

The equation for simulating random values of the parameter *l* in this case has the form of a quadratic equations:

$$-\log(r) = \mu_t l + \chi l^2 \tag{17}$$

$$l^2 + \frac{\mu_t}{\chi} l + \frac{\log(r)}{\chi} = 0 \tag{18}$$

where we get two solutions

$$l_{NL} = -\frac{\mu_t}{2\chi} \pm \sqrt{\frac{\mu_t}{2\chi} - \frac{\log(r)}{\chi}} \tag{19}$$

where the notation l_{NL} has been introduced as the characteristics of the mean free path in the nonlinear model $\mu_t'(l) = \mu_t(1 + l\chi/\mu_t)$. Then the mean and the dispersion of the parameter l_{NL} equal to:

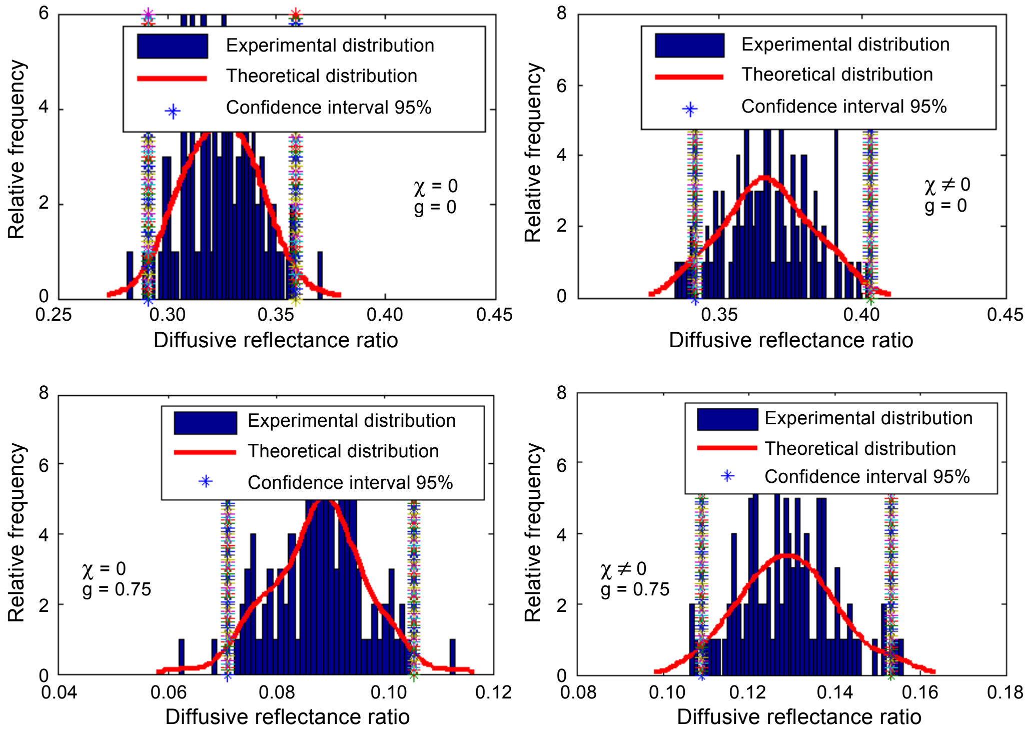
$$\overline{l_{NL}} = \bar{l}_L \left(1 - \frac{6\chi}{\mu_t^2}\right) \tag{20}$$

$$\overline{l_{NL}^2} = \bar{l}_L \left(1 - \frac{6\chi}{\mu_t^2}\right) \tag{21}$$

These formulas have been derived using the method of a small parameter [12] in the decomposition of the into the series, $\exp(-\chi l^2) \cong 1 - \chi l^2$, for integration.

The proposed model was tested in Matlab by calculation of reflection, transmission and absorption coefficients of diffusive photon scattered by turbid medium (tissue) with parameters: $\mu_s = 90\ \text{cm}^{-1}$, $\mu_a = 10\ \text{cm}^{-1}$ and thickness of the plane parallel layer max(Z) = 0.02 cm [13]. For diffusive scattering, the results are presented in the Figure. From the Figure, one can see that taking into account the appropriate nonlinearity leads to some increase in the average values of the diffuse reflection mean(R). And the difference $(\text{mean}(R)|_{g \neq 0} - \text{mean}(R)|_{g = 0})$ for diverse values of the anisotropy g does not significantly change. Thus, taking into account the law Eq. (15), the nonlinearity of the transport coefficient of photons interaction with the scattering centers does not significantly change the nature of dependence between the average value of the reflectance coefficient and the anisotropy factor mean($R(g)$).

The Figure also shows the curve of theoretical distributions as histogram envelopes. Based on the criterion of χ^2, we justified the probability distributions of relative frequency values R to be normal in both cases with 95% confidence. In these figures are also plotted the confidence limits with the confidence probability of 95% for the reflection coefficient R that determines the confidence interval. Taking into account



Distribution of the relative frequency values of the diffuse reflectance ratio for $\mu_t/2\chi \cong 10$ $(\chi \cong 0.05\mu_t)$, $\mu_s = 90$ cm^{-1}, $\mu_a = 10$ cm^{-1}, max$(Z) = 0.02$ cm, and 1000 photons.

the integral law, the intensity attenuation of light by a plane-parallel layer of the spatial nonlinearity transport coefficient of photons interaction with scattering centers does not significantly change the width of the confidence interval for the same confidence probability.

3. Conclusion

Results of N statistically independent measurements of spectral coefficients of reflection, transmission and absorption of photons in turbid medium are consistent with the general scheme of Monte Carlo method, which is based on the central limit theorem of the probability theory. Statistical parameters are changing if the integral law takes into account the light intensity attenuation by a plane-parallel layer of the spatial nonlinearity transport coefficient of the photons interaction with the scattering centers. But it does not significantly affect the previously installed statistical regularities – the central limit theorem and anisotropy scattering regularity, which follow from the analysis of the Henyey–Greenstein functions.

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