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Multi-objective optimization of dielectric layer photonic crystal filter

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The weighting factors method and the response surface methodology are used to achieve multi-objective optimization of a dielectric layer photonic crystal filter. The size of period and the transmission quantity are considered simultaneously and a multi-objective optimization model of filter is established, which takes the size of period and transmission quantity to be minimized in stop-band as objectives. Global approximate expressions of the objective and the constraint functions are found by response surface methodology. Then the weighting factors method is employed to convert the model into a quadratic programming model and the optimal parameters can be obtained using sequence quadratic programming. Examples provide the optimized results in three different weight coefficients. The effect of the weighting factors on the value of the objective function is also discussed. Results show that the present method is precise and efficient for multi-objective optimization of a dielectric layer photonic crystal filter.

Keywords: filter, photonic crystal, weighting factors method, response surface methodology (RSM).

1. Introduction

In recent years, a growing demand can be observed for dimension and characteristic of filter with the rapid development of microwave techniques. Photonic crystals are periodically layered structures that are filled with different dielectric materials and it is well-known that they have a special spectral structure, the so-called photonic band gap (PBG). This feature can be employed to design optical filters.

The design of a photonic crystal filter has already been undertaken by a number of research works. In [1], the particle swarm optimization method and the finite-difference time-domain method were used to improve the performance of a two-dimensional photonic crystal filter. A fabrication process of a tunable PBG filter that can be tuned in a very wide range of the central pass-band wavelength shifting is designed and simulated in [2]. In addition, the optimal design of the dielectric layer photonic crystal filter using the response surface methodology is described in [3]. For the works mentioned above, there are a single objective researches because only the property of the filters is considered. However, most real-world optimization problems that exist in practical engineering and scientific applications will be requested to optimize more than one objective. For the filter, the dimension and the characteristics should be considered to be equally important. In this paper, a multi-objective optimization model of the photonic crystal filter is proposed, and the weighting factors method and the response surface methodology (RSM) to solve this model are introduced.

In contrast to a single objective problem, a multi-objective problem is more difficult to solve because it has a set of solutions, called the Pareto-optimal set, but there is no limit to an optimal solution. Many methods for multi-objective optimization have been put forward and have shown great progress and success. The weighting factors method is the most commonly used technique and its basic idea is to transform the multi-objective problem into the single objective problem [4]. With the weighting factors method, we can issue a comprehensive quantitative analysis of aims and seek the best value to meet the system requirements.

RSM stemmed from experimental design and was later introduced into numerical simulation in reliability assessment of complex multivariable systems [5, 6]. The basic idea of RSM is to approximate the actual state function, which may be implicit or very time-consuming to evaluate, with the so-called response surface function that is easier to deal with complex problems. To construct approximate model with RSM, no sensitivity analysis is required, and thus it is more applicable to problems with sensitivity difficulty. Besides, response surface construction involves no information inside structural analysis procedure. For further reading about RSM, see [7].

In this paper, we use the weighting factors method and the quadratic RSM to achieve multi-objective optimization of the photonic crystal filter. A multi-objective optimization model of the filter is established first, which takes the size of period and transmission quantity to be minimized in stop-band as objectives. The weighting factors method is employed to merge two goals into a single target. Then global approximate expressions of the objective and the constraint functions are found by quadratic RSM. Finally, the model is converted into a quadratic programming model and the optimal parameters can be obtained using sequence quadratic programming. Examples show its precision and efficiency.

2. Bring forward the control model

Dielectric layer photonic crystal filter structures in waveguide are shown in Fig. 1. The periodic length a, the dielectric thickness d, the relative permittivity ε_r of the dielectric are the three major factors in determining the stop-band characteristic of the waveguide dielectric layer photonic crystal structures [8]. As is known to all, the less the transmission quantity in stop-band is and the more transmission coefficient beyond stop-band is, the better the property of the filter is. When the width of stop-band is

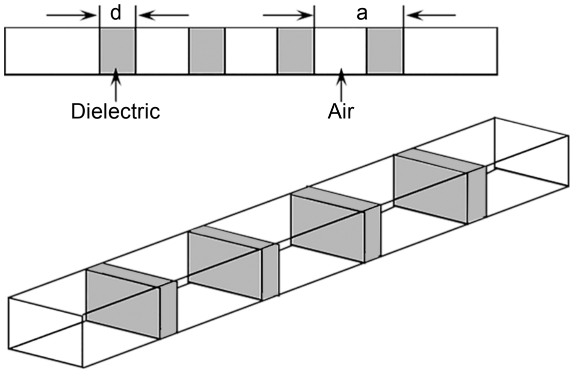

Fig. 1. Dielectric layer photonic crystal filter structures in waveguide.

fixed, we hope the area surrounded by the transmission coefficient curve and horizontal axis (frequency axis) should be maximum. Here we define (in the stop-band)

$$A_{\mathrm{sb}} = \int_f (-S_{21}) \mathrm{d}f \tag{1}$$

as the negative of transmission quantity in the stop-band, where S_{21} represents the transmission coefficient of filter during optimization process. Let $N_{\mathrm{sb}} = -A_{\mathrm{sb}}$, and thus the maximum value problem can be converted to a minimum value searching problem. So the less N_{sb} is, the less transmission quantity is. In this study, we take the size of the period and the transmission quantity to be minimized in the stop-band as objectives. Establishing the control model is as follows:

$$\left\{\begin{array}{ll} \text{Find:} & a, d, \varepsilon_{\mathrm{r}} \\ \text{Minimize:} & N_{\mathrm{sb}} = -A_{\mathrm{sb}} = \int_{f_2}^{f_3} S_{21} \mathrm{d}f \\ & a \\ \text{Subject to:} & \underline{a} \le a \le \overline{a} \\ & \underline{d} \le d \le \overline{d} \\ & d/a \le k \\ & \underline{\varepsilon_{\mathrm{r}}} \le \varepsilon_{\mathrm{r}} \le \overline{\varepsilon_{\mathrm{r}}} \\ & A_{\mathrm{L}} = \int_{f_1}^{f_2} (-S_{21}) \mathrm{d}f \le T_{\mathrm{L}} \\ & A_{\mathrm{R}} = \int_{f_3}^{f_4} (-S_{21}) \mathrm{d}f \le T_{\mathrm{R}} \end{array}\right. \tag{2}$$

where periodic length a, dielectric thickness d, relative permittivity ε_r of the dielectric are the design variables; f_2 and f_3 are the lower and upper bounds of the stop-band, and it is obvious that the bandwidth of the stop-band is between f_2 and f_3; f_1 is the lower bound of the concerned band on the left of the stop-band, f_4 is the upper bound of the concerned band on the right of the stop-band; A_L and A_R are the negative of transmission quantities at corresponding regions; T_L and T_R and are permitted maximum of transmission quantity's negative at corresponding regions. $\underline{a}$, $\overline{a}$, $\underline{d}$, $\overline{d}$, $\underline{\varepsilon_r}$, and $\overline{\varepsilon_r}$ are the lower and upper bound on the design variables a, d, and ε_r, respectively; k is a positive number less 1 because d is less than a.

Since N_{sb} is negative, we define $B_{sb} = -1/N_{sb} = 1/A_{sb}$ to transform the initial problem to the problem for searching a positive minimum. Nevertheless, as dimension and order of magnitude of the objective functions B_{sb} and a are incomparable to each other, we normalized them by $[B_{sb}]$ and $[a]$ which are the estimated average of B_{sb} and a, respectively. Namely we take two dimensionless values $B_{sb}/[B_{sb}]$ and $a/[a]$ as objectives simultaneously. After merging two goals into a single target by the weighting factors method, a new control model has been set up as:

$$\left\{\begin{array}{ll} \text{Find:} & a,\ d,\ \varepsilon_r \\ \text{Minimize:} & G = \alpha_1 a/[a] + \alpha_2 B_{sb}/[B_{sb}] \\ \text{Subject to:} & \underline{a} \le a \le \overline{a} \\ & \underline{d} \le d \le \overline{d} \\ & d/a \le k \\ & \underline{\varepsilon_r} \le \varepsilon_r \le \overline{\varepsilon_r} \\ & A_L = \int_{f_1}^{f_2} (-S_{21})\,\mathrm{d}f \le T_L \\ & A_R = \int_{f_3}^{f_4} (-S_{21})\,\mathrm{d}f \le T_R \end{array}\right. \tag{3}$$

where α_1 and α_2 are the weight coefficients of periodic length and transmission quantity in stop-band, respectively; α_1 and α_2 are positive numbers less 1, besides, $\alpha_1 + \alpha_2 = 1$.

It is very difficult to deduce an explicit expression of the objective function G with design variable a, d, and ε_r because of the strong nonlinear characteristics of the problem. Fortunately we can modify the original function (3) to an approximate one and make the optimization based on the approximate expression. In this study, such approximations can be carried out by RSM.

3. Response surface methodology

For objective function, the response surface generally takes a quadratic polynomial form. Higher order polynomials generally are not used for a conceptual reason (a computational one). In this paper, we use a quadratic form containing the crossing terms. Considering the full quadratic polynomial form, the response estimated equation for three designing variables is given by

$$\tilde{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1^2 + \beta_5 x_2^2 + \beta_6 x_3^2 + \beta_7 x_1 x_2 + \beta_8 x_1 x_3 + \beta_9 x_2 x_3 \tag{4}$$

where β_0, β_1, ..., β_9 are 10 coefficients to be determined, and x_1, x_2, x_3 represent a, d, and ε_r, respectively.

In order to determine all betas, we should select m ($m \geq 10$) experimental points. Putting the coordinates of m experimental points into Eq. (4), we can get m estimated response values

$$\tilde{y}_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i1}^2 + \beta_5 x_{i2}^2 + \beta_6 x_{i3}^2 + \beta_7 x_{i1} x_{i2} + \beta_8 x_{i1} x_{i3} + \beta_9 x_{i2} x_{i3} \tag{5}$$

where $i = 1, ..., m$, and x_{i1}, x_{i2}, x_{i3} represent a, d, and ε_r of the i-th experimental point, respectively.

In fact, we can also get actual values of m experimental points, represented by y_i ($i = 1, ..., m$).

Define error $\varepsilon = (\varepsilon_1, \varepsilon_2, ..., \varepsilon_m)^T$ between the actual and the estimated responses,

$$\varepsilon_i = \tilde{y}_i - y_i, \quad i = 1, ..., m \tag{6}$$

Using the least square technique, and minimizing the residual error measured by the sum of square deviations between the actual and the estimated responses, we have

$$S = \sum_{i=1}^{m} \varepsilon_i^2 = \sum_{i=1}^{m} (\tilde{y}_i - y_i)^2 \to \min \tag{7}$$

Let

$$\frac{\partial S}{\partial \beta_j} = 0, \quad j = 0, ..., 9 \tag{8}$$

Equation (8) is a system of 10 linear equations with 10 unknowns. Solving Eq. (8), we can find all betas and obtain the quadratic response function

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1^2 + \beta_5 x_2^2 + \beta_6 x_3^2 + \beta_7 x_1 x_2 + \beta_8 x_1 x_3 + \beta_9 x_2 x_3 \tag{9}$$

Equation (9) is the actual quadratic response function and β_0, β_1, ..., β_9 are determined.

The sequential quadratic programming is used to obtain the optimum. In the optimization process, suppose $x_l^{(v)}$ ($l = 1, 2, 3$) is the present designed point of l-th variable in v-th iteration, and specify artificially a step size $\Delta_l^{(v)}$ for l-th variable. The expressions of move limits are:

$$\underline{x_l^{(v)}} = x_l^{(v)} - \Delta_l^{(v)} \quad \text{and} \quad \overline{x_l^{(v)}} = x_l^{(v)} + \Delta_l^{(v)} \quad (l = 1, 2, 3) \tag{10}$$

where $\underline{x_l^{(v)}}$ and $\overline{x_l^{(v)}}$ represent the lower and upper bound respectively. The interval of l-th designed variable x_l is $\left[\underline{x_l^{(v)}}, \overline{x_l^{(v)}}\right]$ in v-th iteration.

Furthermore, to improve numerical stability, it is a good practice to scale all variables so that each variable changes in the range [–1, 1] [9]. Let ζ_l $(l = 1, 2, 3)$, represent the normalized variables. The transformation formula is as follows [10]:

$$\zeta_l = \frac{2x_l - (\overline{x_l} + x)}{\overline{x_l} - x_l} \quad (l = 1, 2, 3) \tag{11}$$

After the optimization, we can return to initial design variables and get their value by following transformation:

$$x_l = \frac{(\overline{x_l} - \underline{x_l})\zeta_l}{2} + \frac{\overline{x_l} + \underline{x_l}}{2} \quad (l = 1, 2, 3) \tag{12}$$

The choice of the experimental design can have a large influence on the accuracy of the approximation and the cost of constructing the response surface. For quadratic response models, the central composite design (CCD) is an attractive alternative [11]. There are 15 experimental points in CCD method for three designing variables, where 8 points are at vertices of a quadrilateral, 6 are along the three symmetry axis, and one is at the center. Figure 2**a** shows an example of CCD for objective response surface.

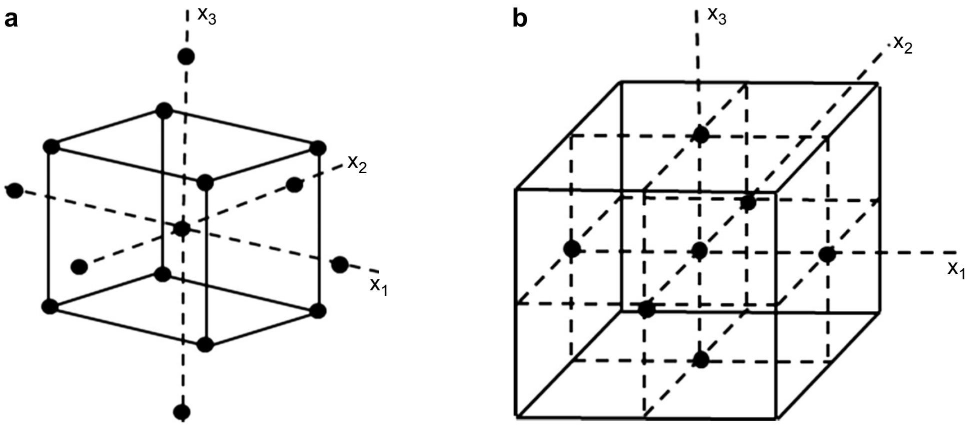

Fig. 2. Design of experiments for objective (**a**) and constraint (**b**) response surface.

In the paper, this method is used to choose the experimental design. This means that 15 experiment points ($m = 15$) are chosen to determine the value of betas. After three designed variables are normalized, in terms of the coordinates the corners of the cube are (–1, –1, –1), (1, –1, –1), (1, 1, –1), (–1, 1, –1), (–1, –1, 1), (1, –1, 1), (1, 1, 1), (–1, 1, 1); the center point is (0, 0, 0). According to [5], the distance between axial point and center point is 1.215, so the axial points are at (–1.215, 0, 0), (1.215, 0, 0), (0, –1.215, 0), (0, 1.215, 0), (0, 0, –1.215), (0, 0, 1.215).

For constraint functions, the response surfaces are constructed at the same value of the selected designing parameters. In this paper, the number of the selection of points for the constraint response is 7 for three variables. Of which, 6 are symmetrical distribution on the axis and one is at the center. Figure 2**b** shows an example of design of experiments for constraint response surface.

4. The control model used for solving

Based on the above discussion, the control model used for solving can be obtained as follows:

$$\begin{cases} \text{Find:} & a,\ d,\ \varepsilon_\mathrm{r} \\ \text{Minimize:} & G = x^\mathrm{T} H x/2 + f^\mathrm{T} x \\ \text{Subject to:} & \underline{x_1^{(v)}} \le a \le \overline{x_1^{(v)}} \\ & \underline{x_2^{(v)}} \le d \le \overline{x_2^{(v)}} \\ & d/a \le k \\ & \underline{x_3^{(v)}} \le \varepsilon_\mathrm{r} \le \overline{x_3^{(v)}} \\ & A_\mathrm{L}^\mathrm{T} x + B_\mathrm{L} \le T_\mathrm{L} \\ & A_\mathrm{R}^\mathrm{T} x + B_\mathrm{R} \le T_\mathrm{R} \end{cases} \tag{13}$$

where $x = (a, d, \varepsilon_\mathrm{r})^\mathrm{T}$, a series of coefficient matrices $H, f, A_\mathrm{L}, B_\mathrm{L}, A_\mathrm{R}, B_\mathrm{R}$ are obtained by RSM when objective and constraint functions are approximately explicated. This quadratic programming model is solved using quadratic programming and the optimal parameters can be obtained.

5. Numerical results

For dielectric layer photonic crystal filter structures in waveguide as shown in Fig. 1, the center frequency stop-band of this filter is designed at 6 GHz and the bandwidth is 2 GHz. The width and height of the waveguide are 57 and 23 mm, respectively. Let the number of the waveguide period be 9 in this paper. Choosing $f_1 = 3$ GHz, $f_2 = 5$ GHz, $f_3 = 7$ GHz, $f_4 = 9$ GHz, $T_L = T_R = 2.5$, $\underline{a} = 0.002$ mm, $\overline{a} = 100$ mm, $\underline{d} = 0.001$ mm, $\overline{d} = 100$ mm, $\underline{\varepsilon_r} = 1.1$, $\overline{\varepsilon_r} = 10$, $k = 0{,}9$, $[B_{sb}] = 1/40$, $[a] = 20$. Three selections of weight coefficients are discussed as follows: $\alpha_1 = 0$ and $\alpha_2 = 1$; $\alpha_1 = 0.5$ and $\alpha_2 = 0.5$; $\alpha_1 = 0.7$ and $\alpha_2 = 0.3$.

We set up the initial design variables according to the estimated equation [8]

$$\begin{cases} \varepsilon_e = \dfrac{d}{a}\varepsilon_r + \left(1 - \dfrac{d}{a}\right) \\ \lambda_g = \dfrac{\lambda\sqrt{\varepsilon_e}}{\sqrt{1 - (\lambda/\lambda_c)^2}} \\ a = \lambda_g/2 \end{cases} \tag{14}$$

where ε_e is the effective permittivity, λ and λ_g are the wavelength corresponding to the center frequency of the stop-band in the vacuum and waveguide, respectively, λ_c is the cutoff wavelength of TE_{10} mode in the rectangular waveguide. Here, according to Eq. (14), we choose the initial design variables as $a = 20$ mm, $d = 15$ mm and $\varepsilon_r = 2.25$.

5.1. Scenario 1

In the case of $\alpha_1 = 0$ and $\alpha_2 = 1$, the multi-objective optimization is transformed into the single objective optimization, which is a problem of searching the minimum value of the transmission quantity in stop-band. The optimization process is convergent and stable, which can be clearly seen in Fig. 3**a**. We see that the objective function value G decreases rapidly at the beginning, after 7 iterations the value starts to converge and after 16 iterations the value keeps constant at about 0.65. Here we obtain the minimum value of the transmission quantity, which is about 0.65. When the function value G converges, the periodic length a and the relative permittivity ε_r of the dielectric have a trend of slow increase and the dielectric thickness d is still at a rate of little decrease, as shown in Fig. 3**b**. After 16 iterations, we obtain the optimized function values, where a, d and ε_r are 22.21 mm, 6.71 mm, and 2.80, respectively. Stop-band characteristics before optimization and after optimization are given in Fig. 3**c**. It is obvious that, before optimization, the stop-band is not deep and wide, minimum value of the transmission coefficient and minimum periodic length are –15 dB and 20 mm, respectively. After optimization, the center frequency stop-band of this filter is 6 GHz and the bandwidth is 2 GHz, minimum value of the transmission coefficient is nearly –39 dB and minimum periodic length is 22.21 mm. The optimal design is carried out.

5.2. Scenario 2

All the data are the same as the scenario 1 except $\alpha_1 = 0.5$ and $\alpha_2 = 0.5$. The optimization process is given in Fig. 4 when $\alpha_1 = 0.5$ and $\alpha_2 = 0.5$. Clearly, Figs. 4**a** and 4**b**

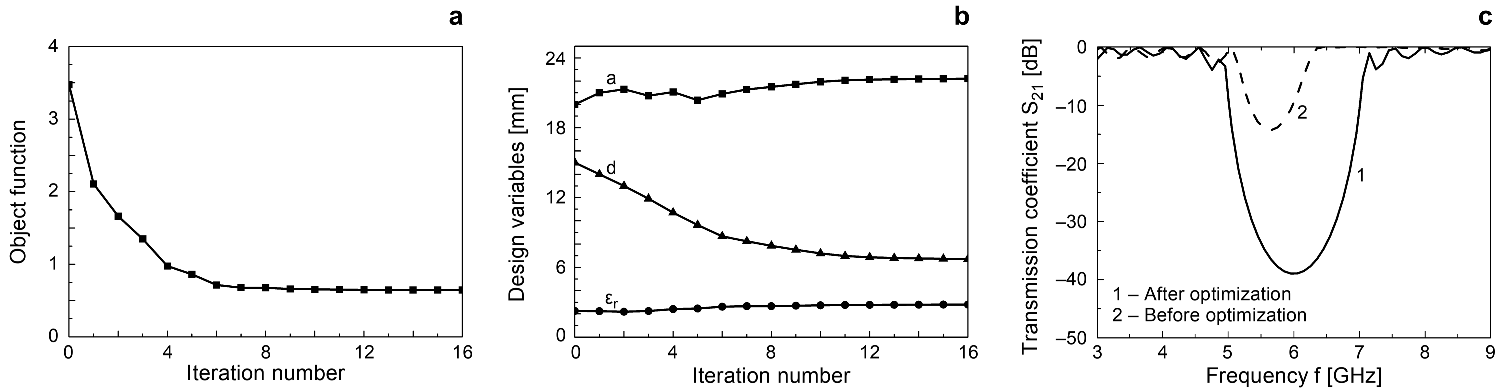

Fig. 3. The optimization process and results for scenario 1: object function *versus* iteration numbers when $\alpha_1 = 0$ (**a**), design variables *versus* iteration numbers when $\alpha_1 = 0$ (**b**), and stop-band characteristic before and after optimization when $\alpha_1 = 0$ (**c**).

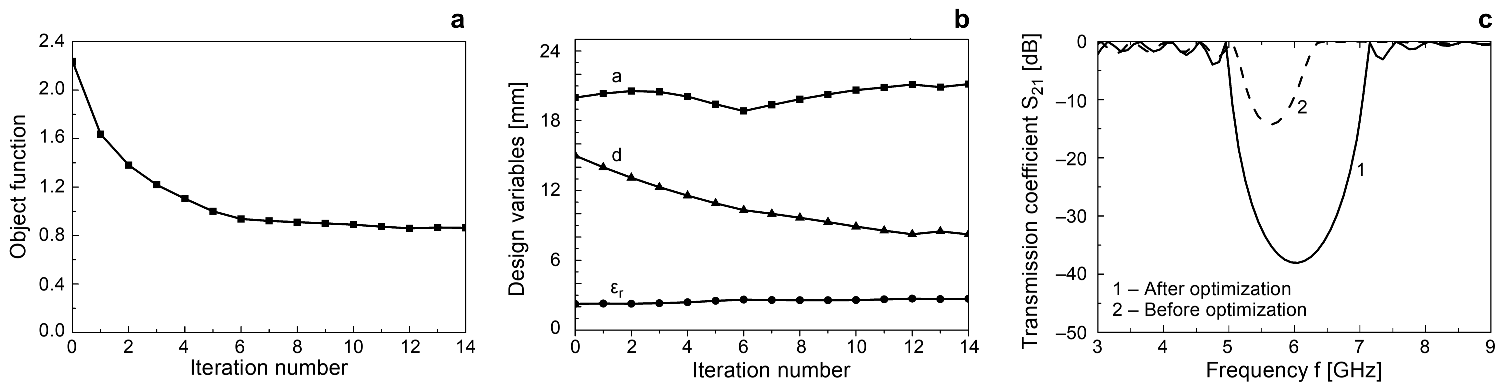

Fig. 4. The optimization process and results for scenario 2: object function *versus* iteration numbers when $\alpha_1 = 0.5$ (**a**), design variables *versus* iteration numbers when $\alpha_1 = 0.5$ (**b**), and stop-band characteristic before and after optimization when $\alpha_1 = 0.5$ (**c**).

show the objective function value G is convergent at about 0.86, and a, d and ε_r are about 21.15 mm, 8.24 mm, and 2.69, respectively, after 14 iterations. Figure 4**c** gives the stop-band characteristic before optimization and after optimization, and show after optimization that the stop-band of filter is deeper and wider than that before optimization, justifying the efficiency of our method. We can observe more from Fig. 4**c** that, after optimization, the minimum value of the transmission coefficient, which is –38.0 dB, is close to the minimum value of the transmission coefficient when $\alpha_1 = 0$, which is –39.0 dB (see Fig. 3**c**). Only the transmission quantity is taken as the objective when $\alpha_1 = 0$. This means that if we choose the values of the weight coefficients $\alpha_1 = 0.5$ and $\alpha_2 = 0.5$, the sub-objective, which is the transmission quantity, and the general objective G can achieve their optimal values simultaneously.

5.3. Scenario 3

Here we let $\alpha_1 = 0.7$ and $\alpha_2 = 0.3$. Figures 5**a** and 5**b** give the optimization process when $\alpha_1 = 0.7$ and $\alpha_2 = 0.3$. After 12 iterations, the objective function value G is convergent at about 0.93, where the optimized function value a, d and ε_r are about 18.31 mm, 10.39 mm, and 2.68, respectively. Figure 5**c** shows the minimum value of the trans-

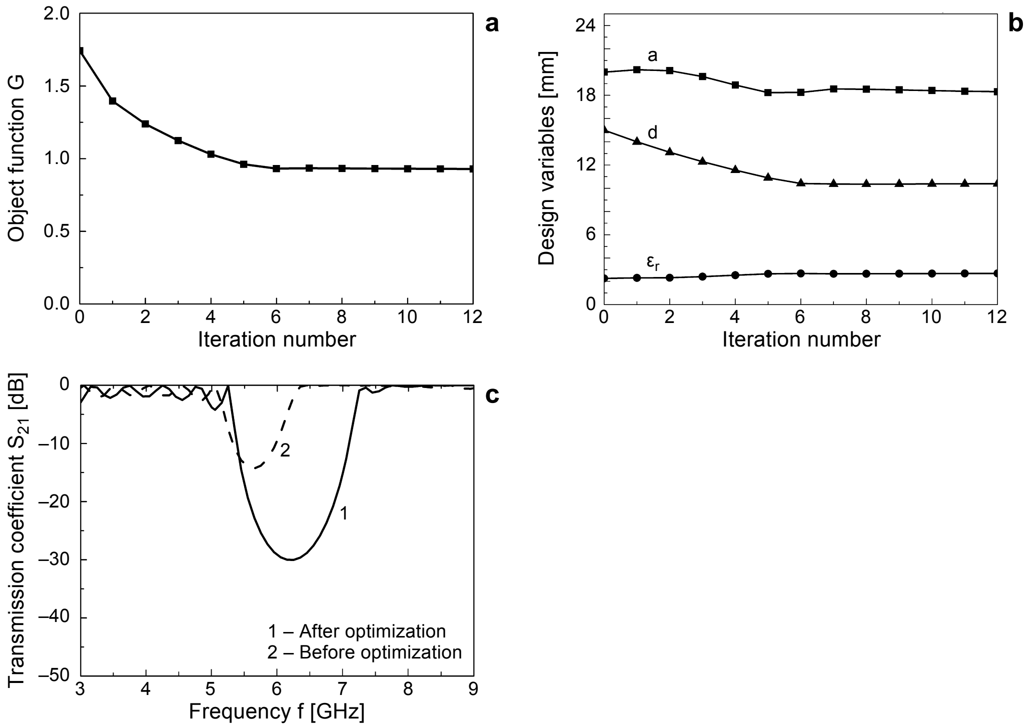

Fig. 5. The optimization process and results for scenario 3: object function *versus* iteration numbers when $\alpha_1 = 0.7$ (**a**), design variables *versus* iteration numbers when $\alpha_1 = 0.7$ (**b**), and stop-band characteristic before and after optimization when $\alpha_1 = 0.7$ (**c**).

mission coefficient is –30.0 dB after optimization. Furthermore, we can see that the optimized function value a is smaller than that of when $\alpha_1 = 0$, which is 22.21 mm (see Fig. 3**b**), whereas the minimum value of the transmission coefficient is larger than that of when $\alpha_1 = 0$, which is –39.0 dB (see Fig. 3**c**). It is worth noting that sometimes we merely desire smaller size of filter, and it is not necessary to minimize the transmission quantity in stop-band. In the case of this scenario, choosing $\alpha_1 = 0.7$ and $\alpha_2 = 0.3$, might just fit the bill.

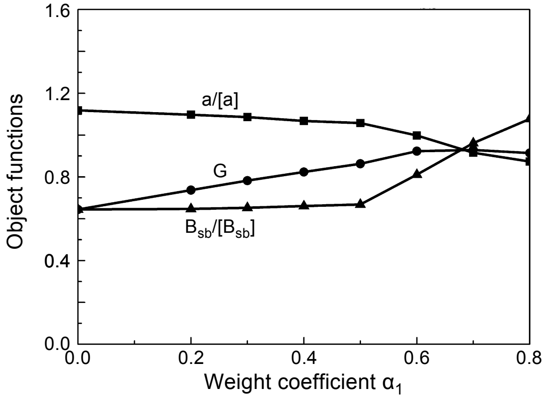

Fig. 6. Object functions *versus* α_1.

The above discussions imply the process of optimization depends strongly on the selection of weight coefficients. Figure 6 shows the curves of the objective functions varying with the weight coefficient α_1. It is observed that the value of the sub-objective $B_{sb}/[B_{sb}]$ does not change much with a small α_1, while the value of the sub-objective $a/[a]$ decreases gradually. In other words, the value of the design variable a keeps changing slowly when the transmission quantity becomes steady, which is similar to our previous discussion. What is more, when the weight coefficient α_1 is smaller than 0.5, the optimized value of the sub-objective $B_{sb}/[B_{sb}]$ is close to 0.65, which is the minimum value of the transmission quantity in the first scenario. Thus, it can be concluded that within this interval, the optimal value of the sub-objective $B_{sb}/[B_{sb}]$ is always obtained, *i.e.*, the sub-goal $B_{sb}/[B_{sb}]$ as well as the normalized general objective G is optimized simultaneously.

6. Conclusion

Multi-objective optimization model of the dielectric layer photonic crystal filter is proposed, and the objective functions are the size of period and the transmission quantity in stop-band. We use the weighting factors method in conjunction with the quadratic RSM to obtain a quadratic programming model and the optimal parameters can be obtained using sequence quadratic programming. The optimization results demonstrate that the present method is precise and efficient. According to the discussion on the effect of the weighting factors on the value of objective functions, the conclusions are drawn as follows:

1) When the objective value G converges, the periodic length a has a trend of slow increase;

2) In practice, we can choose the corresponding weight coefficients to achieve various requirements, including the size of the period and the transmission quantity;

3) When the weight coefficient α_1 is small, the optimized value of the sub-objective $B_{sb}/[B_{sb}]$ does not change dramatically and is close to the solution to the model with transmission quantity as the single objective. This implies that, within this interval, the optimal value of the sub-objective $B_{sb}/[B_{sb}]$ is always obtained, *i.e.*, the sub-goal $B_{sb}/[B_{sb}]$ as well as the normalized general objective G is optimized simultaneously.

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