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Dual reference beam digital off-axis holography for exact suppression of zero-order term and twin image without spectrum loss

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In this paper, we describe an effective approach to suppressing zero-order term and twin image by using pixel-by-pixel multiplication of double holograms in digital off-axis holography. This method records two holograms, respectively, by using the reference waves in different directions. It shows not only a simpler algorithm and easier implementation in experiment, but also exact and complete suppression of the zero-order term and twin image without any spectrum loss of the object in the image reconstruction, particularly when the image and the zero-order term are fairly close to each other. The experimental result approves of the theoretical prediction very well. This approach provides an effective solution to suppressing undesired noises in the digital off-axis holography.

Keywords: noise suppression, pixel-by-pixel multiplication, digital off-axis holography.

1. Introduction

The Fresnel digital holography will result in three components in the image plane: the reconstructed object image and its twin image as well as zero-order (dc) term. The last two terms are generally regarded as image noises to be removed in the hologram reconstruction. In the past two decades, multiple techniques for the suppression of zero -order term (ZOT) and twin image (TI) in digital holography were developed [1–9], and have already taken a certain effect particularly on the off-axis geometry. The existing approaches to the digital off-axis holography can be summarized and divided into three categories: 1) spectrum filtering techniques in frequency domain [1–3]; 2) numerical iterative computation methods [4–6]; 3) phase-shifting techniques [7–9]. However, these existing methods are still difficult in achieving a satisfying result with no image spectrum or resolution loss in digital off-axis holography due to some drawbacks. For instance, the current spectrum filtering techniques show that it is still difficult to find a proper mask which can exactly match the zero-order spectrum area in the frequency domain, although different linear and nonlinear filtering functions have been investi-

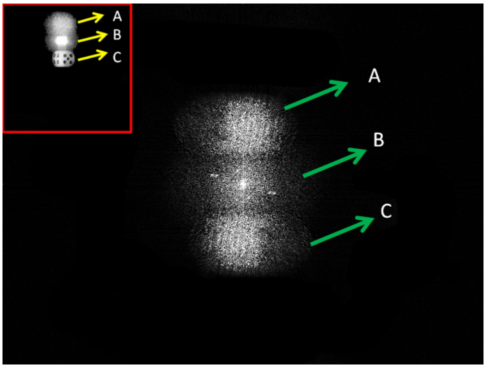

Fig. 1. Two-dimensional Fourier spectrum of the original hologram (insert), where A corresponds to the object image spectrum, B refers to ZOT spectrum, and C is the TI spectrum.

gated. This problem is particularly obvious when the object has an irregular profile and the image is extremely close to ZOT in the hologram reconstruction as shown in Fig. 1, in which both image and ZOT spectrum areas do not show cliffy boundaries. In this case, a small and regular mask is unable to exactly match the overall ZOT area. Conversely, the use of a large mask will partially cover the image area so that the object spectra are filtered as well. If the filtering process is achieved simply by increasing the spaces between the image and ZOT, the space bandwidth product (SBP) of the image available will be reduced [10].

In the numerical iterative computation, it is still difficult to achieve the absolute noise suppression without phase distortion in the hologram reconstruction for universal cases, as all the initial values of 2-dimensional signals (x_i, y_i) of a $M \times N$ array used to characterize the object wave-field are unknown. This means that at least $M \times N$ iterative equations are required, and some assumptions and approximations are introduced in the algorithm to obtain a convergence solution. These approximations will result in the image information loss.

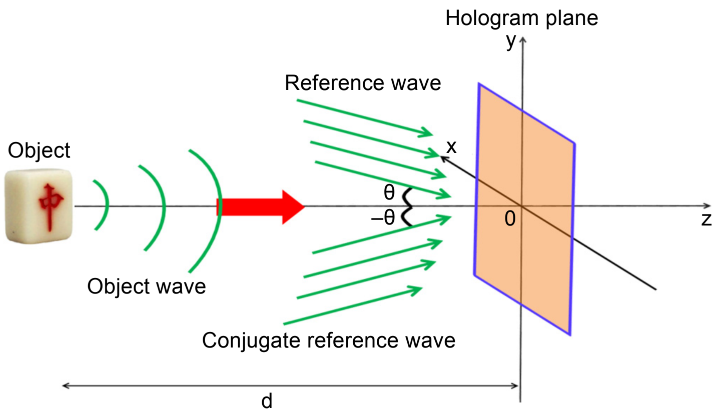

Fig. 2. Schematic diagram of double hologram recording from different reference waves.

The phase-shifting technique has been reported in many articles. It is a good approach to removing ZOT and TI in principle. Unfortunately, this method is quite sensitive to environmental disturbances, such as slight vibration, so that highly-accurate implementation and control are necessary. This will restrict its application as a practical instrumentation.

2. Theoretical model

The principle of pixel-by-pixel multiplication in digital off-axis holography is described as follows. A plane reference wave with an incident angle of θ and its conjugate wave with an incident angle of $-\theta$ in (y, z) plane are used to obtain two holograms based on the same object wave, respectively, as shown in Fig. 2. Both holograms can be expressed as

$$\begin{aligned} h_{\pm}(x, y) &= \left|\tilde{O}(x, y) + \tilde{R}_{\pm}(x, y)\right|^2 \\ &= |O(x, y)|^2 + \left|R_{\pm}(x, y)\right|^2 + \tilde{R}^*_{\pm}(x, y)\tilde{O}(x, y) + \tilde{R}_{\pm}(x, y)\tilde{O}^*(x, y) \end{aligned} \tag{1}$$

where the complex amplitudes of the object and reference waves are defined, respectively, as:

$$\tilde{O}(x, y) = O(x, y)\exp(i\varphi) \tag{2}$$

$$\tilde{R}_{\pm}(x, y) = R(x, y)\exp(\pm iky\sin(\theta)) \tag{3}$$

where k is the wave number and φ is the phase, the positive and negative signs denote the reference wave and its conjugate wave, respectively. With the holograms $h_{\pm}(x, y)$ the reconstruction wave-fields of the holograms can be described mathematically as [1]

$$\begin{aligned} \tilde{U}'_{\pm}(x', y') &= \tilde{C}\int_{\Sigma}\tilde{R}'_{\pm}(x, y)h_{\pm}(x, y)\frac{\exp(ikr)}{r}\mathrm{d}\Sigma \\ &= \tilde{U}_{\pm 0}(x', y') + \tilde{U}_{\pm}(x', y') + \tilde{U}^*_{\pm}(x', y') \end{aligned} \tag{4}$$

where $\tilde{U}_{\pm 0}(x', y')$ denotes the zero-order term, $\tilde{U}_{\pm}(x', y')$ and $\tilde{U}^*_{\pm}(x', y')$ are the reconstructed object wave-field and its conjugate wave-field, respectively; $(\tilde{U}_{+0}, \tilde{U}_{+}, \tilde{U}^*_{+})$ and $(\tilde{U}_{-0}, \tilde{U}_{-}, \tilde{U}^*_{-})$ form two groups of reconstruction wave-fields corresponding to the two reference waves, respectively; $\tilde{C}$ is a complex constant; Σ is the integral area of the object; r is the distance between an object point and a point in the hologram plane; $\tilde{R}'_{\pm}(x, y)$ denote the illustration waves for the hologram reconstruction, which are fully identical to the corresponding reference waves, respectively, *i.e.*, $\tilde{R}'_{\pm}(x, y) = \tilde{R}_{\pm}(x, y)$. These terms can be written in detail by substituting Eq. (1) into Eq. (4):

$$\tilde{U}_{\pm 0}(x', y') = \tilde{C}\int_{\Sigma}\left[O^2(x, y) + R^2_{\pm}(x, y)\right]R(x, y)\exp(\pm iky\sin(\theta))\frac{\exp(ikr)}{r}\mathrm{d}\Sigma \tag{5}$$

$$\tilde{U}_{\pm}(x', y') = \tilde{C}\int_{\Sigma}\tilde{O}(x, y)\frac{\exp(ikr)}{r}\mathrm{d}\Sigma \tag{6}$$

$$\tilde{U}^*_{\pm}(x', y') = \tilde{C}\int_{\Sigma}R^2_{\pm}(x, y)\tilde{O}^*(x, y)\exp(\pm i2ky\sin(\theta))\frac{\exp(ikr)}{r}\mathrm{d}\Sigma \tag{7}$$

It is clearly seen that additional linear phase factors of $ky\sin(\theta)$ and $2ky\sin(\theta)$ are involved in Eqs. (5) and (7), respectively. The additional linear phase factors will lead to different displacements of ZOT and TI in the hologram reconstruction. Furthermore, Eqs. (5)–(7) indicate that the ZOTs and TIs corresponding to different reference wave directions of θ and $-\theta$ shift just to opposite directions at different locations in the image plane. Meanwhile, the reconstructed object wave-field or image remains at the same position, *i.e.*, at the origin of the coordinates in the image plane, without any displacement in both cases. These characteristics are illustrated in Figs. 3**a** and 3**b**, in which the double holograms of a resolution plate are reconstructed from the simulation calculation with the reference wave directions of θ and $-\theta$, respectively. This offers a really perfect opportunity to effectively remove both ZOT and TI terms from the hologram reconstruction without spectrum filtering, as the filtering in frequency domain will lead to some spectrum loss of the object. This process can be easily realized by synthesizing Figs. 3**a** and 3**b** with pixel-by-pixel multiplication. As a result, ZOT and TI are fully removed without image spectrum loss and a noise-free object image with a square-enhanced contrast is obtained in the image plane, which is shown in Fig. 3**c**.

In fact, even if the incident angles of the two reference beams are slightly different from each other in experiment, the result of Eq. (6) is unchanged, *i.e.*, the theoretical model and result described by the Eqs. (1)–(7) are still correct. The only difference lies in that the spaces between the object image, ZOT and TI in the two groups of reconstruction wave-fields of ($\tilde{U}_{+0}$, $\tilde{U}_{+}$, $\tilde{U}^*_{+}$) and ($\tilde{U}_{-0}$, $\tilde{U}_{-}$, $\tilde{U}^*_{-}$) are different due to different incident angles of the two reference beams. This does not affect the final result

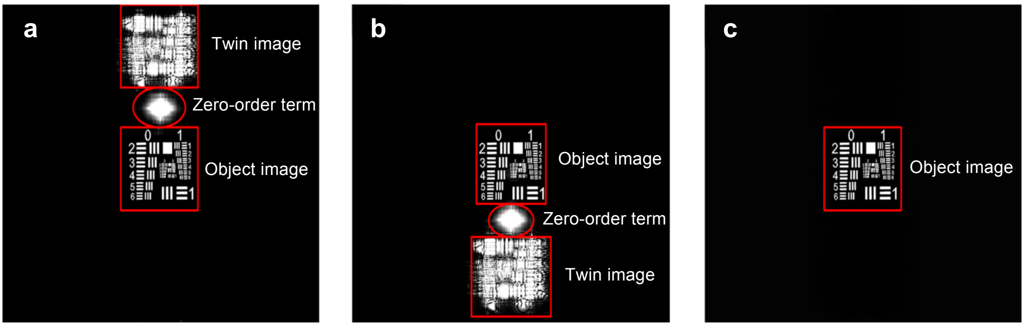

Fig. 3. Simulation of ZOT and TI suppression with pixel-by-pixel multiplication in digital off-axis holography. The hologram reconstructions of a resolution plate using the reference waves with the incident angles of θ (**a**) and $-\theta$ (**b**). Noise suppression from pixel-by-pixel multiplication of **a** and **b** parts (**c**).

from the image multiplication because the object image always locates at the center and ZOTs and TIs of $(\tilde{U}_{+0}, \tilde{U}_{+}, \tilde{U}_{+}^{*})$ and $(\tilde{U}_{-0}, \tilde{U}_{-}, \tilde{U}_{-}^{*})$ are still distributed at different sides of the object image, similar to the Figs. 3**a** and 3**b**.

In addition, in terms of the Nyquist sampling theorem, the incident angle θ of the reference wave must meet the following conditions to obtain a perfect hologram:

$$\theta \le \theta_{\max} = \operatorname{asin}\left(\frac{\lambda}{2\Delta x}\right) \tag{8}$$

$$\theta \le \theta_{\min} = \operatorname{asin}\left(\frac{3}{2}W\lambda\right) \tag{9}$$

where λ is the wavelength and Δx is the pixel pitch; W is the highest frequency in the Fourier frequency spectrum.

3. Experimental results

The experiment is carried out using a fundamental mode laser source at the wavelength of $\lambda = 532$ nm. The object is a dice. The holograms are recorded with a complimentary metal-oxide semiconductor (CMOS) sensor consisting of 1280 × 1024 pixels, each of which has an area of 5.2 × 5.2 μm. The recording distance of the hologram is 850 mm. The incident angles of the reference beam and its conjugate beam in (y, z) plane are 8.73 and –8.73 mrad, respectively, to obtain the maximum of SBP. The experimental optical path diagram is shown in Fig. 4. MS is the switch mirror. The first hologram is recorded by tilting the MS at 45° to choose the first reference beam. The second hologram is obtained when the MS is switched to 90° to choose the second reference beam. The results are shown in Fig. 5. Figures 5**a** and 5**b** show the Fresnel transformation reconstruction of the holograms of the dice from the use of reference beams at the incident angles of 8.73 and –8.73 mrad, respectively. Figure 5**c** shows a noise-free re-

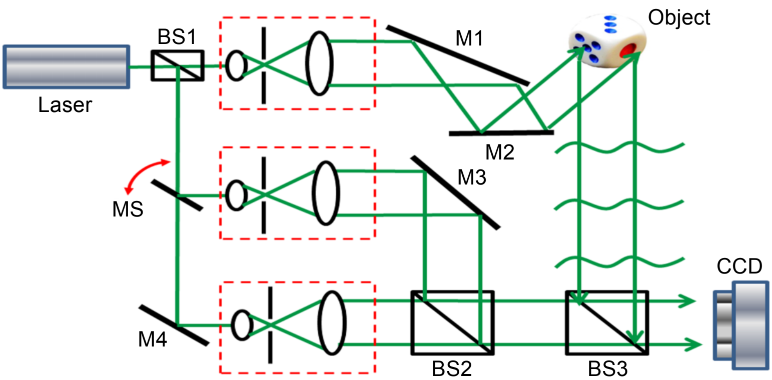

Fig. 4. Experimental setup of the holograms recording with the reference waves at different incident angles. MS – switch mirror, M – mirror, and BS – beam splitter.

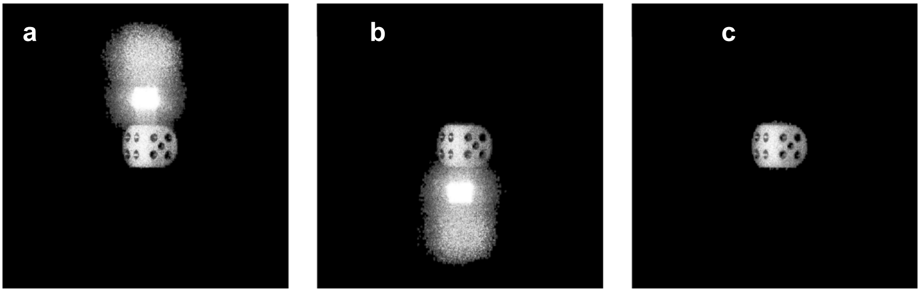

Fig. 5. Experimental results of pixel-by-pixel multiplication in digital off-axis holography. The Fresnel reconstruction of holograms using the reference waves with angles of 8.73 mrad (**a**) and of –8.73 mrad (**b**). Noises suppression from pixel-by-pixel multiplication of **a** and **b** parts (**c**).

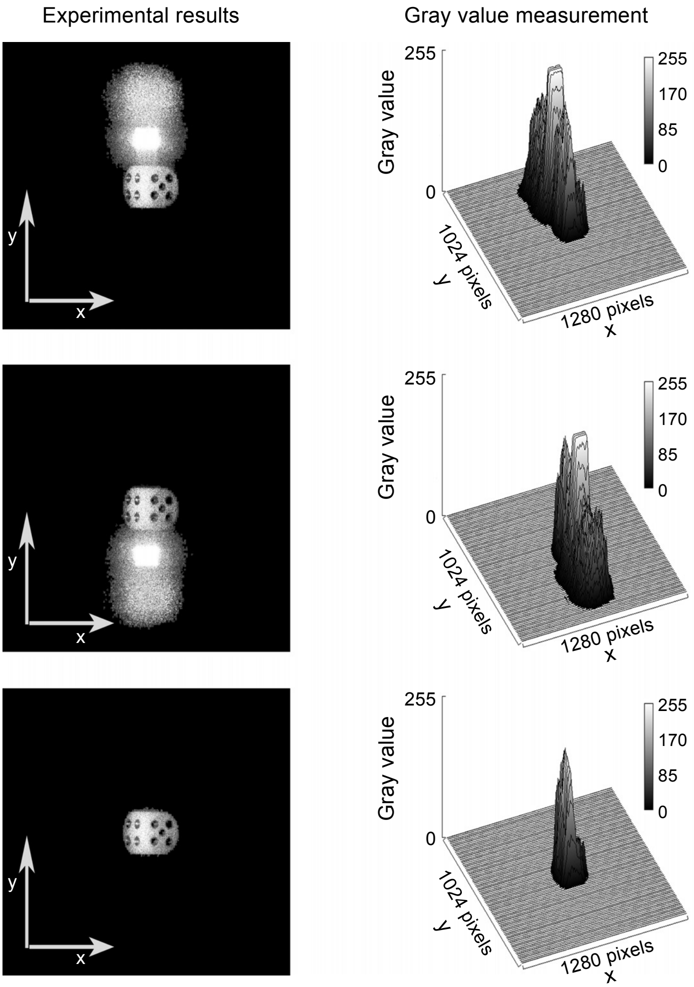

Fig. 6. Assessment of the ZOT and TI suppression level using grayscale measurement of the images for the pixel-by-pixel multiplication in digital off-axis holography.

constructed image with an intensity square-enhanced contrast from the pixel-by-pixel multiplication of the holograms. The experimental results are exactly consistent with the theoretical prediction.

A quantitative assessment of the noises suppression level is conducted by grayscale measurement of the images for the method described above. The result is shown in Fig. 6. It indicates that ZOT and TI term are removed very well. Meanwhile, the SBP is up to 1.03×10^7. It is close to the maximum value available for the given off-axis experimental setup.

In order to further analyze the spectrum loss or the phase variation with the proposed method, we design a phase object which is shown in Fig. 7**a**. According to Eq. (6), $U_+ = U_- = A\exp(i\varphi)$, the complex amplitude of the reconstruction object wave-field is expressed as

$$U = U_+U_- = A\exp(i\varphi)A\exp(i\varphi) = A^2\exp(i2\varphi) \tag{10}$$

where U is the final complex amplitude, A is the amplitude of a phase object, φ is the phase distribution. It is obvious from Eq. (10) that the reconstructed phase value is

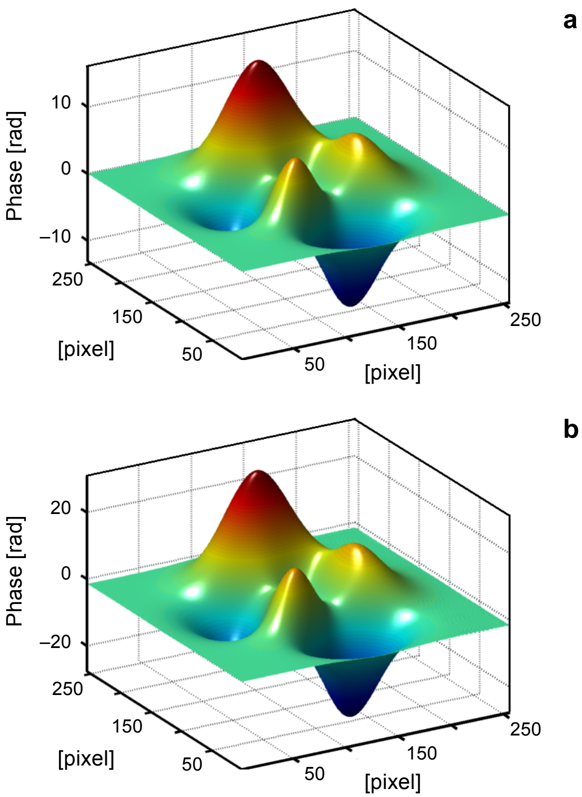

Fig. 7. Original phase object (**a**). Reconstructed phase object using the pixel-by-pixel multiplication of the holograms in digital off-axis holography (**b**).

twice the original object phase. Therefore, by simply dividing the phase values by 2, the real phase of the object can be obtained in the hologram reconstruction. This is proved in Fig. 7 by designing a peak function phase object. The original phase map is shown in Fig. 7**a** and the reconstructed phase of the object from the use of the proposed pixel-by-pixel multiplication of the holograms is shown in Fig. 7**b**. The phase object configuration is unchanged except its values are doubled after using the proposed pixel-by-pixel multiplication of the holograms

4. Conclusion

In conclusion, dual reference beam digital off-axis holography is proposed and described here, with which the zero-order term and twin image can be exactly removed without image spectrum loss, even if the image is extremely near to ZOT in comparison with the filtering technique. This is achieved by recording double holograms with two different directions of the reference waves, respectively, and then spatially synthesizing the reconstructed wave-filed of the holograms to remove all undesired noises. The experimental results are exactly consistent with the theoretical design and prediction. This approach provides a very practical and powerful solution to suppressing ZOT and TI noises in digital off-axis holography.

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