Optica Applicata, Vol. XLVII, No. 4, 2017
DOI: 10.5277/oa170413

Higher-order quantum dynamics of a generic quadratically-coupled optomechanical system: entanglement, antibunching and spin squeezing

Kousik MUKHERJEE[1, 2*], Paresh Chandra JANA[1]

[1]Department of Physics and Technophysics, Vidyasagar University, Midnapore, 721102, India

[2]Department of Physics, Government General Degree College, Gopiballavpur-II, 721517, India

[*]Corresponding author: kousikmukherjee89@gmail.com

To investigate the existence of higher-order intermodal entanglement, higher-order single-mode antibunching, higher-order intermodal antibunching and spin squeezing, a first order analytic operator solution of the Hamiltonian of quadratically-coupled optomechanical system is constructed using short time approximation for different field modes. Temporal variations of these nonclassical properties under different coupling strengths are studied, neglecting the effect of optical losses, environmental effects and also dissipation. With an increasing order number, the depth of these nonclassical properties is increased. Spin squeezed states are observed in any direction, *i.e.*, either in S_x or S_y direction.

Keywords: higher-order nonclassicality, entanglement, antibunching.

1. Introduction

Nonclassical states of electromagnetic field have been getting much attention for last few decades both in theory and experiments and now they are playing a major role in quantum computing and quantum information system. Nonclassical properties are basically quantum mechanical, not described by a classical stochastic process within the Glauber –Sudarshan P representation, *i.e.* a state is nonclassical if its Glauber–Sudarshan P representation is negative or more singular than δ-function, or in other words if it is only a nontrivial superposition of classical states according to a superposition principle. Such properties are characterized by Mandel Q-parameter, Fano factor, Wigner function, *etc*. These properties have potential applications in quantum teleportation [1], quantum cryptography [2], quantum dense coding [3], *etc*. There are several possibilities of different nonclassical features which are observed in the dynamics of optomechanical systems (OMSs), which can be exploited in different ways [4–8]. For long distance communication, OMSs can play the role of a transducer [4]. For implementation of a microwave sensor at the sub-photon level using OMSs, a scheme has been proposed [5].

In the study of optomechanically induced transparency [6], for detection of weak force such as a gravitational wave [8], an OMS plays an important role. An OMS represents a quantum hybrid system which will be useful in quantum information processing. Some systems are coherent which can be useful in long-term storage, some are strongly interacting which can be used in computation and others are easily transported over long distances to provide good communication. Again, the OMS provides a good platform to explore quantum effects of macroscopic or mesoscopic objects in the mechanical system [9]. Inspired by these facts, we decided to investigate the nonclassical properties in the quadratically coupled OMS. In our work, a group of nonclassical criteria are used to characterize a set of nonclassical properties such as intermodal entanglement, single -mode antibunching, compound mode antibunching, spin squeezing, *etc*. Recently, a theoretical study of higher-order nonclassicalities has been done by NGUYEN BA AN [10] and to observe higher-order nonclassical effects several experimental studies have also been carried out in different quantum optical systems [11, 12]. From these studies obviously it can be said that it is easy to characterize nonclassical properties by means of higher-order studies. However, no serious effort has been made to study the possibility of higher-order nonclassical properties in the quadratically coupled OMS. So, the possibility of higher-order intermodal entanglement, higher-order single-mode antibunching and higher-order compound mode antibunching in the quadratically coupled OMS, has been studied by us. These nonclassical properties are useful in quantum computation and quantum information processing, *e.g.* entangled states are necessary for quantum key distribution, teleportation of qubits, superdense coding, quantum metrology, *etc*. Antibunching is useful for a single photon source [13], a squeezed state is useful for teleportation of the wave function of a single-mode electromagnetic field [14] and reduction of noise in an optical signal [15], *etc*. Spin squeezed states [16] have been prepared experimentally [17] and are useful because of their connection with entanglement [18], squeezing of coupled radiation [19] and enhancement of the precision of atomic interferometers [20]. So, because of a large variety of possible applications of these nonclassical properties, we have investigated the possibility of finding nonclassicalities in quadratically coupled OMS. A current interest in different OMSs is realized both experimentally and theoretically: the entanglement between the cavity field mode and the mechanical mode for strong coupling has been analyzed for continuous-variable systems [21], a mechanical squeezing of the cavity with two beams [22], squeezing and entanglement of a generic quadratic coupled OMS [23]. All these studies have been carried out in a lower order, and the depth of nonclassical properties is weak. Motivated by these facts, we decided to investigate the depth of nonclassical properties in a higher-order. In our study it is assumed that optical losses inside the cavity are negligible, considering the fact that the quantum system is closed. Recent studies [24, 25] reveal that in OMSs the effect of the environment is considerably reduced. The Hamiltonian of a quadratically coupled OMS can be solved by employing different methods such as a numerical solution using a master equation approach, short-time approximation, the Langevin technique, *etc*. In our work, we have taken the first order analytical operator solution of the generalized Hamiltonian using short-time dynamics. Solutions of different field modes have been used for investigating higher-order nonclassical properties in quadratically coupled OMSs.

Our article is organized as follows: at first we introduce the model Hamiltonian which describes the quadratically coupled OMS. Using short-time dynamics, we find out perturbative solutions for different field modes by the corresponding Heisenberg equations of motion. Then, we introduce the Hillery–Zubairy criterion to investigate into the higher order intermodal entanglement, the Lee criterion for both single-mode and compound mode antibunching, the possibility of spin squeezing and also to study temporal variation of these nonclassical properties.

2. The model and its solution

The Hamiltonian for a quadratically coupled OMS may be written as

$$H = H_s + H_I + H_d \tag{1}$$

where $H_s = \omega_c a^\dagger a + \omega_m b^\dagger b$ describes the system with a mechanical resonator (dielectric membrane such as made by Si_3N_4) and an optical mode, the single-mode cavity field is characterized by the annihilation (creation) operators a ($a^\dagger$) and the field operators b ($b^\dagger$) correspond to the mechanical motion of the membrane, having resonant frequencies ω_c and frequency of the mechanical mode ω_m, respectively. These operators satisfy the bosonic commutation relation $[a, a^\dagger] = [b, b^\dagger] = 1$. $H_I = \lambda a^\dagger a(b^\dagger + b)^2$ represents the interaction Hamiltonian, where λ is the coupling strength between the cavity field and the membrane. Recent studies revealed that the value of λ may be of the order of several kilohertz [26, 27] and also it has been proposed, that for near-field optomechanical system, the coupling strength can be estimated as being of the order of few megahertz [28]. $H_d = iE[a^\dagger \exp(-i\omega_d t) - a\exp(i\omega_d t)]$ describes the external driving term where E is the external driving parameter which drives the system at the frequency ω_d. The Hamiltonian for such a system is given by ($\hbar = 1$) [29, 30]

$$H = \omega_c a^\dagger a + \omega_m b^\dagger b + \lambda a^\dagger a(b^\dagger + b)^2 + iE[a^\dagger \exp(-i\omega_d t) - a\exp(i\omega_d t)] \tag{2}$$

Here we are to study the system in the absence of driving as some previous studies [31–33] are done by zero drive Hamiltonian for different OMSs. In the absence of driving, *i.e.*, $E = 0$, the Hamiltonian of the system takes the form

$$H = \omega_c a^\dagger a + \omega_m b^\dagger b + \lambda a^\dagger a(b^\dagger + b)^2 \tag{3}$$

Using Heisenberg equations of motion, we obtain the coupled differential equations for two different modes. These are as follows:

$$\frac{\mathrm{d}a(t)}{\mathrm{d}t} = -i\left\{\omega_c a(t) + \lambda a(t)\left[b^{\dagger 2}(t) + b^2(t) + 2b^\dagger(t)b(t) + 1\right]\right\} \tag{4a}$$

$$\frac{\mathrm{d}b(t)}{\mathrm{d}t} = -i\left\{\omega_m b(t) + 2\lambda a^\dagger(t)a(t)\left[b^\dagger(t) + b(t)\right]\right\} \tag{4b}$$

To investigate the existence of higher-order entanglement, the higher-order anti-bunching and the spin squeezing for quadratically coupled OMSs, we use the Heisenberg equations of motion (4) for different field modes and short-time dynamics. This method is also employed to determine nonclassicality in couple-cavity OMSs [34]. The n-th order solutions of the mode a and b is assumed as

$$\begin{aligned} a^n(t) &= g_1^n a^n(0) + n g_1^{n-1}\Big[g_2 a^n(0) b^{\dagger 2}(0) + g_3 a^n(0) b^2(0) \\ &\quad + g_4 a^n(0) b^\dagger(0) b(0) + g_5 a^n(0)\Big] \end{aligned} \tag{5a}$$

$$\begin{aligned} b^n(t) &= l_1^n b^n(0) + n l_1^{n-1}\Big[l_2 a^\dagger(0) a(0) b^n(0) + l_3 a^\dagger(0) a(0) b^\dagger(0) b^{n-1}(0)\Big] \\ &\quad + \frac{n(n-1)}{2} l_1^{n-1} a^\dagger(0) a(0) b^n(0) \end{aligned} \tag{5b}$$

where all the parameters g_i and l_i are functions of time and evaluated from initial boundary conditions $g_1(0) = l_1(0) = 1$ and $g_i(0) = l_i(0) = 0$ ($i = 2, \ldots, 5$).

The coefficients, *i.e.*, $g_i(t)$ and $l_i(t)$, are given by:

$$\begin{aligned} g_1(t) &= \exp(-i\omega_c t) \\ g_2(t) &= \frac{\lambda}{2\omega_m} g_1(t) G(t) \\ g_3(t) &= -g_2^*(t) g_1^2(t) \\ g_4(t) &= -2i\lambda t g_1(t) \\ g_5(t) &= g_4(t)/2 \\ l_1(t) &= \exp(-i\omega_m t) \\ l_2(t) &= -2i\lambda t l_1(t) \\ l_3(t) &= -\frac{2i\lambda}{\omega_m} \sin(\omega_m t) \end{aligned}$$

where $G(t) = 1 - \exp(2i\omega_m t)$.

According to short-time dynamics, we may expand any operator $x(t)$ in Taylor's series as $x(t) = x(0) + t\dot{x}(t)|_{t=0} + \ldots$ where $\dot{x}(t) = i[H, x(0)]$. Here we take the solution up to the first order, *i.e.*, $\lambda t < 1$, for which perturbation calculations hold good, so we have neglected the term beyond $\dot{x}(t)$. To check the validity of the solutions, we used equal time commutation relation (ETCR) as $[a(t), a^\dagger(t)] = [b(t), b^\dagger(t)] = 1$.

In order to investigate the existence of various higher-order nonclassical properties and spin squeezing in quadratically coupled OMS, we assume that photon and phonon modes are initially coherent. So, the initial state is the product of two coherent states, *i.e.*, $|\varphi\rangle = |\alpha\rangle \otimes |\beta\rangle$, where $|\alpha\rangle$ and $|\beta\rangle$ are the eigenkets of field operators a and b, respectively. If the operator $a(t)$ operates on the state, it gives a complex eigenvalue α, *i.e.*, $a(t)|\varphi\rangle = \alpha|\varphi\rangle$, where $|\alpha|^2$ gives the number of photons for the cavity field mode a. In a similar manner, we obtain the phonon number for the phonon mode b. For stimulated process, these complex amplitudes are not necessarily zero and it seems that $\alpha > \beta$ should be considered.

3. Higher order entanglement

There is a number of inseparability criteria [35–37], out of them here we have used Hillery–Zubairy criteria to investigate the existence of the entangled state in a quadratically coupled optomechanical system. These conditions are sufficient for characterization of bipartite entanglement but not necessary.

Using quadratic operators $L_1 = \{a^m b^{\dagger n} + a^{\dagger m} b^n\}$ and $L_2 = -i\{a^m b^{\dagger n} - a^{\dagger m} b^n\}$ and adding up variances and using an uncertainty relation, HILLERY and ZUBAIRY [35, 36] established the states to be entangled if

$$|\langle a^m b^{\dagger n}\rangle| > [\langle a^{\dagger m} a^m b^{\dagger n} b^n\rangle]^{1/2}$$

Here, we define that condition as (HZ-I criterion)

$$E_{ab}^{m,\,n} = \langle a^{\dagger m} a^m b^{\dagger n} b^n\rangle - \left|\langle a^m b^{\dagger n}\rangle\right|^2 < 0 \tag{6}$$

Similarly, using quadratic operators $F_1 = \{a^m b^n + a^{\dagger m} b^{\dagger n}\}$ and $F_2 = -i\{a^m b^n - a^{\dagger m} b^{\dagger n}\}$ leads to conditions for the product state to be entangled if

$$|\langle a^m b^n\rangle| > [\langle a^{\dagger m} a^m\rangle\langle b^{\dagger n} b^n\rangle]^{1/2}$$

is satisfied. We may write this as (HZ-II criterion)

$$E'^{m,\,n}_{ab} = \langle a^{\dagger m} a^m\rangle\langle b^{\dagger n} b^n\rangle - \left|\langle a^m b^n\rangle\right|^2 < 0 \tag{7}$$

where m and n are non-zero positive integers. For the product state (bipartite) the choice of the integers m, n satisfy the conditions $m + n \geq 3$.

Using the above criteria and solutions, we obtain

$$\begin{aligned} E_{ab}^{m,\,n} = \Bigg\{ & mn\Big[2(g_2^* g_1 (\beta^*)^{n-1}\beta^{n-1} + \text{c.c.}) + (n-1)(g_2^* g_1 (\beta^*)^{n-2}\beta^n + \text{c.c.})\Big] \\ & + \frac{n(n-1)(m-1)}{2}(l_3^* l_1 (\beta^*)^{n-2}\beta^n + \text{c.c.})\Bigg\}|\alpha|^{2m} \end{aligned} \tag{8}$$

$$E'^{m,\,n}_{ab} = -mn\Big[(l_1^* l_3 (\beta^*)^2 + \text{c.c.})|\beta|^{2(n-1)} + \frac{n-1}{2}(l_1^* l_3 (\beta^*)^n \beta^{n-2} + \text{c.c.})\Big]|\alpha|^2 \tag{9}$$

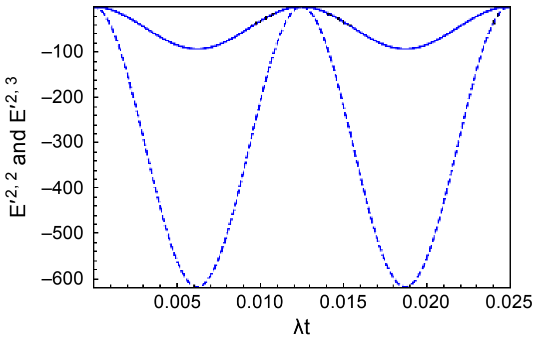

Fig. 1. Plot of $E'^{m,n}_{ab}$ for compound mode ab with rescaled time λt with $\alpha = 3$, $\beta = 2$, $\lambda = 2\pi \times 0.4$ kHz, $\lambda/\omega_m = 0.004$, $m = n = 2$ (solid line) and $m = 2$, $n = 3$ (dashed line).

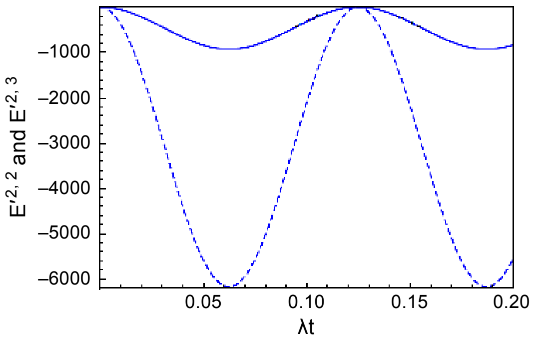

Fig. 2. Plot of $E'^{m,n}_{ab}$ for compound mode ab with rescaled time λt with $\alpha = 3$, $\beta = 2$, $\lambda = 2\pi \times 4$ kHz, $\lambda/\omega_m = 0.04$, $m = n = 2$ (solid line) and $m = 2$, $n = 3$ (dashed line).

Figures 1 and 2 show the graphical variation of Eq. (9) for different values of the order number and also the coupling strength λ. From the result, it is observed that according to HZ-I criterion, the result is positive but from HZ-II criterion the result is always negative, *i.e.* a bipartite entangled state exists for a higher-order. From figures it is clear that the degree of entanglement increases with the order number n and also the coupling strength λ. It is observed that the amount of entanglement increases considerably when the order number changes from $n = 2$ to $n = 3$. The amount of entanglement changes almost linearly with the coupling strength λ. The parameter $mn|\alpha|^{2m}$ in Eq. (9) plays the role of an amplification factor. The graphical variation shows that the entangled state is in a periodic repetition and its time period is determined by the relation $\omega_m t = s\pi$ where s is an integer. The periodicity of the entangled state is the same for all orders, *i.e.* the time period is independent of the value of n.

4. Higher order antibunching

Antibunching is a nonclassical phenomenon which is observed in different optical and optomechanical processes. There are different criteria for investigation of the existence of higher-order antibunching, but each of them can be explained by the Lee criteria [38]. According to LEE, the criteria for higher-order antibunching for a single-mode are expressed in terms of a higher-order factorial moment of the number operator and are defined by the inequality

$$\langle N_a^{n+1}\rangle\langle N_a^{m-1}\rangle < \langle N_a^{n}\rangle\langle N_a^{m}\rangle \tag{10}$$

where N_a is the usual number operator for a mode, n-th factorial moment of the number operator is $\langle N_a^n\rangle = \prod_{i=0}^{n-1}(N_a - i)$ and m, n are integers satisfying $m \ge n \ge 1$. The reduced criterion [10] for n-th order single-mode antibunching is $\langle N_a^{n+1}\rangle < \langle N_a\rangle^n\langle N_a\rangle$ in which $m = 1$ is taken. We may define the condition for n-th order single-mode antibunching as

$$A_a(n) = \langle N_a^{n+1}\rangle < \langle N_a\rangle^{n+1} < 0 \tag{11}$$

The smaller is the value of $A_a(n)$, the larger is the antibunching in degree. $\langle N_a^n\rangle = \langle a^{\dagger n}a^n\rangle$ is the measure of n photons of the same mode at the particular point in space time coordinate. The inequality implies that the probability of detection of a single photon pulse is greater than that for two photons in a bunch and so on. This feature is used for quantum cryptography. For $n = 1$, the condition is referred to lower order antibunching, while $n \ge 2$ is for higher-order antibunching. Again, for the compound mode a, b, the criterion for antibunching according to LEE [38], is expressed by the inequality

$$\langle N_a^{n+1}N_b^{m-1} + N_b^{n+1}N_a^{m-1}\rangle < \langle N_a^{n}N_b^{m} + N_b^{n}N_a^{m}\rangle \tag{12}$$

with $l \ge m \ge 1$. For $m = 1$ [10], the above condition reduces to

$$\langle N_a^{n+1} + N_b^{n+1}\rangle < \langle N_a^{n}N_b + N_b^{n}N_a\rangle$$

We may define the condition for n-th order compound mode ab antibunching

$$A_{ab}(n) = \langle N_a^{n+1} + N_b^{n+1}\rangle - \langle N_a^{n}N_b + N_b^{n}N_a\rangle < 0 \tag{13}$$

Here we discussed a higher-order sub-Poissonian photon statistics in terms of factorial moment, and as we are working with the quantum mechanical system satisfying commutation relation $[a, a^\dagger] = 1$ so, we use a single time correlation function.

Using solutions and conditions for n-th order single-mode antibunching, we find that

$$A_a(n) = 0 \tag{14}$$

$$A_b(n) = \frac{n(n+1)}{2}\left(l_3^* l_1 (\beta^*)^{n-1}\beta^{n+1} + \text{c.c.}\right) \tag{15}$$

From the result of Eq. (14) it is clear that the cavity field mode is coherent and also the mechanical mode shows super-Poissonian behaviour as the right side of Eq. (15)

shows a positive result. So, higher-order single-mode antibunching is not possible for quadratically-coupled OMS

$$A_{ab}(n) = \left[|g_1|^{2n}|\alpha|^{2n} - |l_1|^{2n}|\beta|^{2n}\right]\left[|\alpha|^2 - |\beta|^2\right]$$
$$+ l_3^* l_1 |\alpha|^2 (\beta^*)^{n-1} \beta^{n+1} \left[(n+1)\beta^* - n|\alpha|^2 + \frac{n(n-3)}{2}\right]$$
$$- l_3^* l_1 \left[|\alpha|^{2n} \beta^2 \left(n + |\alpha|^2\right) + \frac{n(n-1)}{2} |\alpha|^2 (\beta^*)^{n-2} \beta^n \left(2 + |\alpha|^2\right)\right] + \text{c.c.} \tag{16}$$

As the expression obtained in the right-hand side of Eq. (16) is not simple, it may be plotted to investigate the existence of compound mode antibunching. The variation of $A_{ab}(n)$ is plotted in Figs. 3–5 with the rescaled time λt. The figure shows that

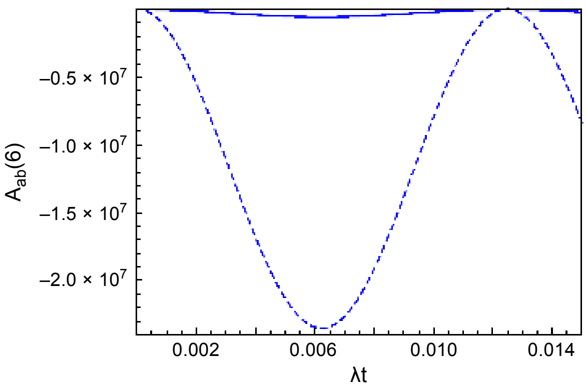

Fig. 3. Plot of $A_{ab}(n)$ for compound mode *ab* with rescaled time λt with $n = 6$, $\beta = 2$, $\lambda = 2\pi \times 0.4$ kHz, $\lambda/\omega_{\rm m} = 0.004$, $\alpha = 3$ (solid line) and $\alpha = 4$ (dashed line).

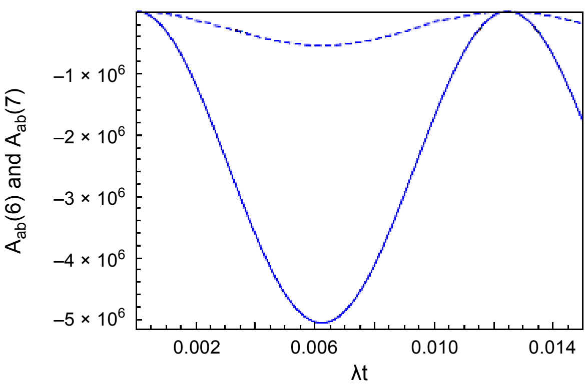

Fig. 4. Plot of $A_{ab}(n)$ for compound mode *ab* with rescaled time λt with $\alpha = 3$, $\beta = 2$, $\lambda = 2\pi \times 0.4$ kHz, $\lambda/\omega_{\rm m} = 0.004$, $n = 7$ (solid line) and $n = 6$ (dashed line).

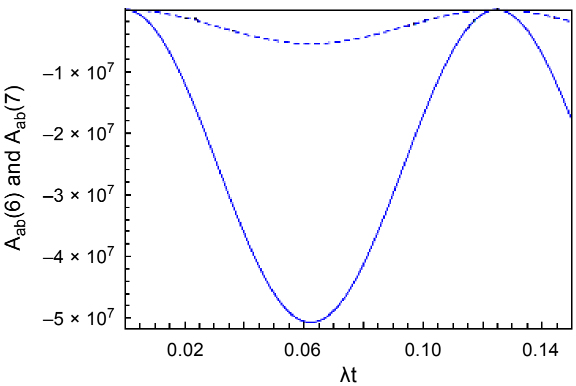

Fig. 5. Plot of $A_{ab}(n)$ for compound mode ab with rescaled time λt with $\alpha = 3$, $\beta = 2$, $\lambda = 2\pi \times 4$ kHz, $\lambda/\omega_{\rm m} = 0.04$, $n = 7$ (solid line) and $n = 6$ (dashed line).

$A_{ab}(n)$ is negative, *i.e.*, antibunching exists for the compound mode ab. Figure 3 shows the variation of depth of antibunching with the cavity photon number $|\alpha|^2$ with the order number $n = 6$. As the value of the cavity photon number increases from $|\alpha|^2 = 9$ to $|\alpha|^2 = 16$, the depth of antibunching increases very rapidly. The degree of antibunching increases rapidly with the order number as shown in Fig. 4 and also with the coupling strength.

5. Spin squeezing

There are several criteria to investigate the spin squeezing; we have used the relations [19, 39, 40] for spin squeezing existence identification in a quadratically-coupled OMS. Using concepts of second quantization formalism and the Schwinger bosonic representation [41], different spin components may be written in terms of creation and annihilation operators of two modes $a^\dagger$ ($b^\dagger$) and a (b) which are as follows:

$$S_x = \frac{1}{2}(S_+ + S_-)$$

$$S_x = \frac{1}{2i}(S_+ - S_-)$$

$$S_z = \frac{1}{2}(b^\dagger b - a^\dagger a)$$

where $S_+ = b^\dagger a$ and $S_- = a^\dagger b$. These three orthogonal spin components obey the commutation relation $[S_i, S_j] = i\varepsilon_{ijk} S_k$ (where ε_{ijk} is the Levi–Civita symbol). Therefore, any pair of spin operators obeys the uncertainty relation for which $(\Delta S_x)^2$ and $(\Delta S_y)^2$ are given as

$$\langle (\Delta S_x)^2 \rangle \langle (\Delta S_y)^2 \rangle \geq \frac{1}{4} \langle S_z \rangle^2$$

where $(\Delta S_x)^2 = \langle S_x^2\rangle - \langle S_x\rangle^2$ is the variance in S_x direction. From this relation it is observed that only one of the components, *i.e.* S_x or S_y, may be squeezed. So, the criterion for spin squeezing for S_x and S_y is written as

$$\langle(\Delta S_x)^2\rangle < \frac{1}{2}|\langle S_z\rangle| \quad \text{and} \quad \langle(\Delta S_y)^2\rangle < \frac{1}{2}|\langle S_z\rangle| \tag{17}$$

We may define squeezing factors as

$$S(x) = \frac{\langle(\Delta S_x)^2\rangle}{0.5|\langle S_z\rangle|} < 1 \quad \text{and} \quad S(y) = \frac{\langle(\Delta S_y)^2\rangle}{0.5|\langle S_z\rangle|} < 1 \tag{18}$$

For $n = 1$, the solutions of the Hamiltonian of Eqs. (5a) and (5b) give the values of $a(t)$ and $b(t)$. Using these solutions and criteria of spin squeezing, we obtain

$$\langle(\Delta S_x)^2\rangle = \frac{1}{4}\left\{|g_1|^2|l_1|^2\left(|\alpha|^2+|\beta|^2\right)+\left[g_1^2 l_3^* l_1^* \alpha^* \alpha^3 + 2(g_2^* g_1 + l_3^* l_1)|\alpha|^2\beta^2 + \text{c.c.}\right]\right\} \tag{19a}$$

$$\langle(\Delta S_y)^2\rangle = \frac{1}{4}\left\{|g_1|^2|l_1|^2\left(|\alpha|^2+|\beta|^2\right)+\left[2(g_2^* g_1 + l_3^* l_1)|\alpha|^2\beta^2 - g_1^2 l_3^* l_1^* \alpha^* \alpha^3 + \text{c.c.}\right]\right\} \tag{19b}$$

$$\langle S_z\rangle = \frac{1}{2}\left(|g_1|^2|\alpha|^2 - |l_1|^2|\beta|^2 - l_3^* l_1|\alpha|^2\beta^2 + \text{c.c.}\right) \tag{19c}$$

Figure 6 shows the variation of spin components S_x and S_y with $\omega_m t$. From the figure it is evident that any component of the spin, *i.e.* either S_x and S_y, is always squeezed due to the energy exchange between one another as one of the squeezing

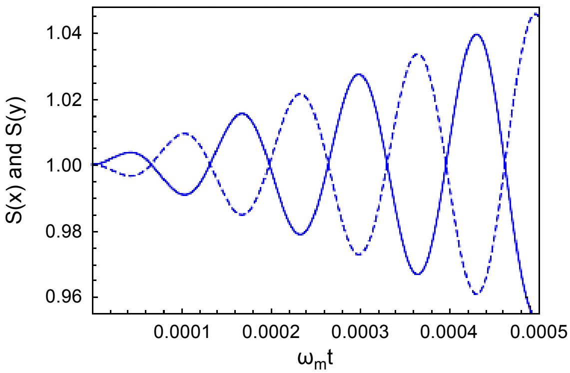

Fig. 6. Plot of $S(x)$ (solid line) and $S(y)$ (dashed line) with rescaled time $\omega_m t$ for $\lambda = 2\pi \times 0.4$ kHz, $\alpha = 3$, $\beta = 2$, $\lambda/\omega_m = 0.004$.

factors $S(x)$ or $S(y)$ is always less than 1. The depth of squeezing depends on the coupling strength λ and also on the photon number. Here, the monotonous increase of the envelope for both the cases is due to the truncation of the perturbation calculation for the coupling strength λ in the solutions.

6. Conclusions

In conclusion, we have discussed a higher-order nonclassical correlation such as higher -order entanglement, higher-order antibunching and spin squeezing in a quadratically coupled OMS. Higher order effects are more prominent for a higher value of the coupling strength λ and the cavity photon number. It is observed that the depth of nonclassical properties increases enormously with the order number. From our study it is clear that a higher-order entangled state exists for a quadratic OMS, according to HZ-II criterion. Higher-order single-mode antibunching is not observed as the photon mode shows statics which is of Poissonian in nature, *i.e*. coherent, and also the mechanical mode shows super-Poissonian statistics but a higher order compound mode, *i.e*. photon-phonon mode shows sub-Poissonian statistics, which shows the antibunched state. From the study it is also clear that a spin squeezed state is observed either in S_x or in S_y direction, which can be useful for the reduction of noise in the optical signal.

References

[1] Furusawa A., Sørensen J.L., Braunstein S.L., Fuchs C.A., Kimble H.J., Polzik E.S., *Unconditional quantum teleportation*, Science **282**(5389), 1998, pp. 706–709.

[2] Hillery M., *Quantum cryptography with squeeze state*, Physical Review A **61**(2), 2000, article ID 022309.

[3] Braunstein S.L., Kimble H.J., *Dense coding for continuous variables*, Physical Review A **61**(4), 2000, article ID 042302.

[4] Stannigel K., Rabl P., Sørensen A.S., Zoller P., Lukin M.D., *Optomechanical transducers for long-distance quantum communication*, Physical Review Letters **105**(22), 2010, article ID 220501.

[5] Keye Zhang, Bariani F., Ying Dong, Weiping Zhang, Meystre P., *Proposal for an optomechanical microwave sensor at the subphoton level*, Physical Review Letters **114**(11), 2015, article ID 113601.

[6] Weis S., Rivière R., Deléglise S., Gavartin E., Arcizet O., Schliesser A., Kippenberg T.J., *Optomechanically induced transparency*, Science **330**(6010), 2010, pp. 1520–1523.

[7] Teufel J.D., Dale Li, Allman M.S., Cicak K., Sirois A.J., Whittaker J.D., Simmonds R.W., *Circuit cavity electromechanics in the strong-coupling regime*, Nature **471**(7337), 2011, pp. 204–208.

[8] Caves C.M., Thorne K.S., Drever R.W.P., Sandberg V.D., Zimmermann M., *On the measurement of a weak classical force coupled to a quantum-mechanical oscillator. I. Issues of principle*, Reviews of Modern Physics **52**(2), 1980, p. 341.

[9] Aspelmeyer M., Kippenberg T.J., Marquardt F., *Cavity optomechanics*, Reviews of Modern Physics **86**(4), 2014, p. 1391.

[10] Nguyen Ba An, *Multimode higher-order antibunching and squeezing in trio coherent state*, Journal of Optics B: Quantum and Semiclassical Optics **4**(3), 2002, pp. 222–227.

[11] Avenhaus M., Laiho K., Chekhova M.V., Silberhorn C., *Accessing higher order correlations in quantum optical states by time multiplexing*, Physical Review Letters **104**(6), 2010, article ID 063602.

[12] Allevi A., Olivares S., Bondani M., *Measuring higher-order photon-number correlation in experiments with multimode pulsed quantum states*, Physical Review A **85**(6), 2012, article ID 063835.
[13] Zhiliang Yuan, Kardynal B.E., Stevenson R.M., Shields A.J., Lobo C.J., Cooper K., Beattie N.S., Ritchie D.A., Pepper M., *Electrically driven single-photon source*, Science **295**(5552), 2002, pp. 102–105.
[14] Braunstein S.L., Kimble H.J., *Teleportation of continuous quantum variables*, Physical Review Letters **80**(4), 1998, p. 869.
[15] Caves C.M., *Quantum-mechanical noise in an interferometer*, Physical Review D **23**(8), 1981, pp. 1693–1708.
[16] Jian Ma, Xiaoguang Wang, Sun C.P., Nori F., *Quantum spin squeezing*, Physics Reports **509**(2–3), 2011, pp. 89–165.
[17] Hald J., Sørensen J.L., Schori C., Plozik E.S., *Spin squeezed atoms: a macroscopic entangled ensemble created by light*, Physical Review Letters **83**(7), 1999, p. 1319.
[18] Sørensen A., Duan L.-M., Cirac J.I., Zoller P., *Many-particle entanglement with Bose–Einstein condensates*, Nature **409**(6816), 2001, pp. 63–66.
[19] Walls D.F., Zoller P., *Reduced quantum fluctuations in resonance fluorescence*, Physical Review Letters **47**(10), 1981, p. 709.
[20] Wineland D.J., Bolinger J.J., Itano W.M., Heinzen D.J., *Squeezed atomic states and projection noise in spectroscopy*, Physical Review A **50**(1), 1994, pp. 67–88.
[21] Paternostro M., Vitali D., Gigan S., Kim M.S., Brukner C., Eisert J., Aspelmeyer M., *Creating and probing multipartite macroscopic entanglement with light*, Physical Review Letters **99**(25), 2007, article ID 250401.
[22] Nunnenkamp A., Børkje K., Harris J.G.E., Girvin S.M., *Colling and squeezing via quadratic optomechanical coupling*, Physical Review A **82**(2), 2010, article ID 021806(R).
[23] Shi H., Bhattacharya M., *Quantum mechanical study of a generic quadratically coupled optomechanical system*, Physical Review A **87**(4), 2013, article ID 043829.
[24] Singh S., Phelps G.A., Goldbaum D.S., Wright E.M., Meystre P., *All-optical optomechanics: an optical spring mirror*, Physical Review Letters **105**(21), 2010, article ID 213602.
[25] Gieseler J., Deutsch B., Quidant R., Novotny L., *Subkelvin parametric feedback cooling of a laser -trapped nanoparticle*, Physical Review Letters **109**(10), 2012, article ID 103603.
[26] Sankey J.C., Yang C., Zwickl B.M., Jayich A.M., Harris J.G.E., *Strong and tunable nonlinear optomechanical coupling in a low-loss system*, Nature Physics **6**(9), 2010, pp. 707–712.
[27] Flowers-Jacobs N.E., Hoch S.W., Sankey J.C., Kashkanova A., Jayich A.M., Deutsch C., Reichel J., Harris J.G.E., *Fiber-cavity-based optomechanical device*, Applied Physics Letters **101**(22), 2012, article ID 221109.
[28] Hao-Kun Li, Yong-Chun Liu, Xu Yi, Chang-Ling Zou, Xue-Xin Ren, Yun-Feng Xiao, *Proposal for a near-field optomechanical system with enhanced linear and quadratic coupling*, Physical Review A **85**(5), 2012, article ID 053832.
[29] Thompson J.D., Zwickl B.M., Jayich A.M., Marquardt F., Girvin S.M., Harris J.G.E., *Strong dispersive coupling of a high-finesse cavity to a micromechanical membrane*, Nature **452**(7183), 2008, pp. 72–75.
[30] Cheung H.K., Law C.K., *Nonadiabatic optomechanical Hamiltonian of a moving dielectric membrane in a cavity*, Physical Review A **84**(2), 2011, article ID 023812.
[31] Mukherjee K., Jana P.C., *Squeezing and entanglement in quadratically-coupled optomechanical system*, Journal of Physical Sciences **19**, 2014, pp. 143–155.
[32] Bose S., Jacobs K., Knight P.L., *Preparation of nonclassical states in cavities with a moving mirror*, Physical Review A **56**(5), 1997, p. 4175.
[33] Rai A., Agarwal G.S., *Quantum optical spring*, Physical Review A **78**(1), 2008, article ID 013831.
[34] Mukherjee K., Jana P.C., *Nonclassical properties (squeezing, antibunching, entanglement) for couple-cavity optomechanical system*, Journal of Optics **016**, 2016, article ID 0339.

[35] HILLERY M., ZUBAIRY M.S., *Entanglement conditions for two-mode states*, Physical Review Letters **96**(5), 2006, article ID 050503.

[36] HILLERY M., ZUBAIRY M.S., *Entanglement conditions for two-mode states: applications*, Physical Review A **74**(3), 2006, article ID 032333.

[37] MIRANOWICZ A., BARTKOWIAK M., XIAOGUANG WANG, YU-XI LIU, NORI F., *Testing nonclassicality in multimode fields: a unified derivation of classical inequalities*, Physical Review A **82**(1), 2010, article ID 013824.

[38] CHING TSUNG LEE, *Higher-order criteria for nonclassical effects in photon statistics*, Physical Review A **41**(3), 1990, p. 1721.

[39] KITAGAWA M., UEDA M., *Squeezed spin states*, Physical Review A **47**(6), 1993, p. 5138.

[40] WÓDKIEWICZ K., *Reduced quantum fluctuations in the Josephson junction*, Physical Review B **32**(7), 1985, p. 4750.

[41] SAKURAI J.J., *Modern Quantun Mechanics*, Pearson, 2013.

Received February 18, 2017
in revised form April 11, 2017