Optica Applicata, Vol. XLVIII, No. 1, 2018
DOI: 10.5277/oa180106

Slowing of optical signals in generalized Cantor structure

NAIM BEN ALI[1, 2*], WAY YUSOFF[1, 3], YOUSSEF TRABELSI[2], MOUNIR KANZARI[2]

[1]College of Engineering, Industrial Engineering Department, Haïl University, Haïl City 2440, Saudi Arabia

[2]Photovoltaic and Semiconductor Materials Laboratory, El-Manar University-ENIT, PO Box 37, Le belvedere 1002-Tunis, Tunisia

[3]College of Engineering, Manufacturing and Materials Department, International Islamic University, Malaysia

[*]Corresponding author: naimgi2@yahoo.fr

In this paper, a simulation study of the slowing optical signals in generalized Cantor structure is proposed. The materials constituting the structure's layers are SiO_2 and the TiO_2. The slowing down factor is determined using the transfer matrix method whose purpose is to study the slowing of light. We show that the slowing down factor value depends on the generalized Cantor parameters (a, b and c) and the reference wavelength λ_0. These parameters are optimized to better slowing the optical signals, minimize the number and the thickness of structure's layers. At the end of the paper we compare our results with previous research work.

Keywords: slow of light, optic communication, photonic, generalized Cantor, slowing down factor.

1. Introduction

Until this day, the old communication networks use electrons to transport information and are characterized by high-power consumption and by the heating problems associated with the Joule effect. Recently the researchers have concentrated on the development of these circuits. They propose the photon to replace the electron and to transport the information in the new generation of communication networks [1, 2]. Indeed, the photon is faster than the electron; it travels in vacuum at the speed 3×10^8 m/s and the optical communication circuit consumes less power and it is not accompanied with the problem of heating. However new generations of optical communication network confront some problems with information distribution nodes [3]. Here, the signal processing is not all optical and requires an optical-electronic conversion that introduces a lot of inefficiency. Here the solution is to use the optical buffer: a device that temporarily stores and adjusts the timing of optical packets. At present, solutions are based on mechanical variable delay lines and a combination of different delay lines with an optical switch, however these approaches are not ideal owing to their slow response [4]. If the velocity of slow light can be controlled with a response speed much faster than the mechanical method, it could be a solution for buffering and various types of time-domain processing, such as retiming, multiplexing and performing convolution integrals [4]. So, the search for devices to slow the light opens a window towards the feasibility of an optical router that processes all-optical signals. One of the structures proposed for this function is the photonic crystal and it has become part of the modern research axes to improve the optical properties of communication networks and other scientific fields [4, 5].

Photonic crystals (PCs) also known as photonic band-gap materials are periodic optical nanostructures that affect the motion of photons [6]. This photonic band gap (PBG) forbids the propagation of a certain frequency range of light. PC can be one, two or three dimensional, and the simplest form of this crystal is the one-dimensional (1D); it can be made of layers of different dielectric constant deposited or stuck together [7]. Recently a new family of PCs has attracted the attention of several researchers by their characteristics. These are the quasiperiodic photonic crystals (QPC) like those of Fibonacci, Thue–Morse and Cantor [8–11]. The most important property and the advantage of these crystals compared with conventional periodic crystals is the existence of strong resonances in the forbidden band (PBG), which allows to have a strong localization and to slow the light.

Using the transfer matrix method (TMM) [12], all the results of this article focus on the simulation study of the slow light in a non-periodic photonic crystal constructed according to the generalized Cantor distribution. Firstly, we show the effect of the parameters of the generalized Cantor on the velocity of the optical signals. Then, and by comparing other hybrid photonic structures with the generalized Cantor structure, we show the advantage of the latter one to slow down the light while optimizing the geometric thickness and the number of layers of the whole photonic system.

2. Problem formulation

2.1. Calculation's method

The method that we introduce for calculating the group velocity in one-dimensional photonic structures is the transfer matrix method (TMM) [12]. This method showed that the relation between the amplitudes of the incident E_0^+, reflected E_0^- and transmitted E_{m+1}^+ wave is expressed as the following expression:

$$\begin{pmatrix} E^+(z_0) \\ E^-(z_0) \end{pmatrix} = C_1 C_2 C_3 \dots C_{m+1} \begin{pmatrix} E^+(z) \\ E^-(z) \end{pmatrix} \tag{1}$$

Here C_j is the product of the propagation matrix C_{pr} and the interface matrix C_{int} with the matrix elements:

$$C_{\mathrm{pr}} = \begin{pmatrix} \exp(i\varphi_{j-1}) & 0 \\ 0 & \exp(-i\varphi_{j-1}) \end{pmatrix} \tag{2}$$

$$C_{\text{int}} = \frac{1}{t_j}\begin{pmatrix} 1 & r_j \\ r_j & 1 \end{pmatrix} \tag{3}$$

where t_j and r_j are respectively the Fresnel transmission and reflection coefficients between the $(j-1)$-th and j-th layer.

The values φ_{j-1} in Equation (2) indicate the change in the wave's phase between $(j-1)$-th and j-th interfaces and are expressed by the following equations:

$$\varphi_0 = 0 \tag{4}$$

$$\varphi_{j-1} = \frac{2\pi}{\lambda}\hat{n}_{j-1}d_{j-1}\cos(\theta_{j-1}) \tag{5}$$

except for $j \geq 1$; λ is the wavelength of the incident wave in vacuum and d_{j-1} is the thickness of the $(j-1)$-th layer.

The group velocity can be calculated from the dispersion curve of an optical mode for a long lattice using the following relation [13]:

$$V_{\text{g}}(\omega) = \frac{\mathrm{d}\omega}{\mathrm{d}k} = L\frac{\partial\omega}{\partial\varphi} \tag{6}$$

where L represents the physical length of the multilayer system, φ is the total change in the phase of the wave and ω is the angular frequency (rad/s).

The relative group velocity of a propagating mode is

$$\text{RV}_{\text{g}} = \frac{V_{\text{g}}(\omega)}{C} \tag{7}$$

the slowing down factor is defined as

$$\text{SDF} = \frac{V_{\text{m}}}{V_{\text{g}}} \tag{8}$$

with V_{g} is the group velocity, and V_{m} is the average group velocity defined as

$$V_{\text{m}} = \frac{1}{2}\left(\frac{c}{n_{\text{H}}} + \frac{c}{n_{\text{L}}}\right) \tag{9}$$

where n_{H} and n_{L} are the high and low refractive indices, respectively [14].

2.2. Generalized Cantor structure

The generalized Cantor structure GC(a, b, c) is characterized by the parameters a, b and c [15]. To construct multilayer photonic structures according to this sequence, we start from an initiator of length l and of high refractive index n_{H}, we subdivide it into m unities of equal length (l/m), the first layer of length $a(l/m)$ is a layer of high refractive index n_H, the second layer of length (b/m) is a layer of low refractive index n_L and the last layer of length $[l - (a + b)l/m]$ is of high refractive index n_H, where a and b are integer and m is the homothetic ratio. If $m = 2$, the interferential mirror is a periodic multilayer. When $m > 2$, we have to consider different cases according to the algebraic properties of the ratio. This is the first step ($N = 1$) of the model.

In the second iteration ($N = 2$), we subdivide the first layer into m units of equal length $a(l/m^2)$ and take a segment of length $a(l/m^2)$ as a layer of high refractive index n_H, the second layer of length $ba(l/m^2)$ is a layer of low refractive index n_L and the last layer of length $[al/m - (a + b)(a^2l/m^2)]$ is of high refractive index n_H. We repeat this procedure for the $[l - (a + b)l/m]$ high refractive index layer. We subdivide it into m unities of equal length $[l - (a + b)l/m]/m$, the first layer of length $a\{[l - (a + b)l/m]/m\}$ is a layer of high refractive index n_H, the second layer of length $b\{[l - (a + b)l/m]/m\}$ is a layer of low refractive index n_L and the last layer of length: $l - (a + b)(l/m) - (a + b)\{[l - (a + b) \times (l/m)]\}/m$ is of high refractive index n_H, continuing this procedure *ad infinitum*. We note that a, b and m obey to $a + b < m$. In the case where $m = a + b + c = 3$ and $(a, b) = (1, 1)$, we obtain the classical triadic structure of Cantor [16, 17].

Table 1 shows some examples of generalized Cantor structures for the third iteration and for different values of a, b and c. Here we can conclude that by changing the parameters a, b and c we can have structures with the same number of layers but with different distributions of H and L layers. Also Fig. 1 shows an example of the GC(a, b, c) for

Table 1. Generalized Cantor multilayer sequences for the third iteration and for different values of a, b and c.

Parameters			Generalized Cantor structure	Number
a	b	c	GC(a, b, c)	of layers
1	1	1	HLHLLLHLH	9
2	1	1	HHL(3H)LH(4L)HHLH	16
1	2	1	HLLH(8L)HLLH	16
1	1	2	HLHH(4L)HL(3H)LHH	16

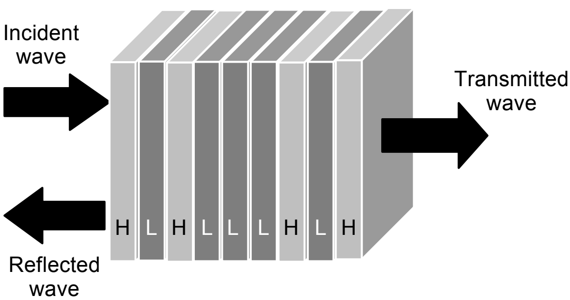

Fig. 1. Schematic representation showing the generalized Cantor photonic structure (third generation and $a = b = c = 1$).

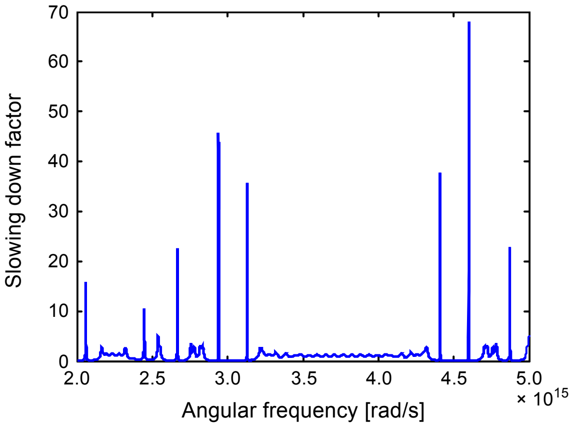

Fig. 2. Slowing down factor according to the angular frequency for GC($a = 2$, $b = 1$, $c = 2$) structure with iteration number $m = 4$ and $\lambda_0 = 0.5$ μm.

the third iteration and for $a = b = c = 1$. Figure 2 illustrates the SDF for the generalized Cantor structure with the parameters $a = 2$, $b = 1$ and $c = 2$, the iteration number is fixed at $m = 4$ and the reference wavelength is fixed at $\lambda_0 = 0.5$ μm. Here in Fig. 2 the SDF takes the maximum value 68.02.

3. Results and discussion

In this numerical investigation, we study the slowing down factor (SDF) of one-dimensional, quasiperiodic generalized Cantor photonic crystals. The SiO_2 (L) and TiO_2 (H) were chosen as two elementary layers, with refractive indices $n_L = 1.45$ and $n_H = 2.3$, respectively. The geometric thickness of the layers H and L are $d_L = \lambda_0/(4n_L)$ and $d_H = \lambda_0/(4n_H)$, respectively, where λ_0 is the reference wavelength. Here, all materials are assumed to be linear, homogenous, non-absorbing, and with no optical activity in the considered spectral region from 318 to 796 THz.

3.1. Effect of the generalized Cantor parameters

The effect of the generalized Cantor parameters (a, b, c) on SDF is proposed in this part. In Table 2 we show the proposed values of a, b, c, the number of layers of the structure, the geometric thickness, the relative group velocity and the slowing down factor. These parameters are chosen to be between 1 and 3 and the iteration number is fixed at $m = 4$, because our goal is to limit the structures' number of layers. Also Fig. 3 shows the variation of the slowing down factor according to the parameters a, b and c. From Table 2 and Fig. 3, and if $a = b = c = 1$, the photonic structure is a classical triadic structure of Cantor. Here the number of layer is 27 and the slow down factor is very weak (SDF = 3.54).

T a b l e 2. Variation of the SDF and the RV_g according to the parameters *a*, *b* and *c* of the 4th iteration of GC structure.

a	*b*	*c*	Number of layers *P*	Geometric thickness *L* [μm]	RV_g	SDF
1	1	1	27	2.07	0.158	3.54
2	1	1	64	4.65	0.069	8.072
1	2	1	64	5.26	0.147	3.806
1	1	2	64	4.65	0.165	3.396
2	2	1	125	9.91	0.073	7.605
2	1	2	125	8.73	0.008	68.02
1	2	2	125	9.915	0.09	6.15
3	1	1	125	8.73	0.033	16.67
1	3	1	125	10.52	0.114	4.93
1	1	3	125	8.73	0.063	8.821

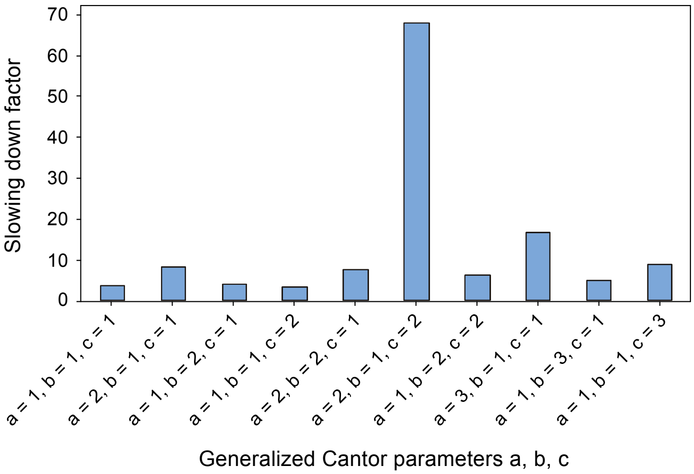

Fig. 3. Variation of the slowing down factor according to the parameters *a*, *b* and *c* of the 4th iteration ($m = 4$) of GC structure.

To change from the conventional Cantor structure to the generalized Cantor one, we modify the values of *a*, *b* and *c* to verify this condition: $a + b + c > 3$. In Table 2, we can see that if one of the parameters (*a*, *b* and *c*) is fixed at 2 and the others at 1, the total number of layers becomes $P = 64$. For example if $a = 2$, $b = 1$ and $c = 1$, the SDF = 8.072 and if $a = 1$, $b = 2$ and $c = 1$, the SDF = 3.806. Also if $a = 1$, $b = 1$ and $c = 2$, the SDF = 3.396, so here it is clear that if $a > b$ and $a \geq c$, the slowing down factor takes its maximum value. In the case where the total number of layers becomes $P = 125$, the slowing down factor takes its maximum value (SDF = 68.02) for $a = 2$, $b = 1$ and $c = 2$. Therefore for $P = 125$, we can conclude that if $a > b$ and $a \geq c$, the SDF takes its maximum value.

Consequently from Table 2 and Fig. 3, it is clear that the parameter *a* of the GC(*a*, *b*, *c*) structure has the greater effect on the slowing down factor. Also we conclude that by changing the values of the parameters (a or b or c), we can keep the same number of layers, change the position of high and low layers, change the values of the SDF and even more change the geometric thickness of the structure. For example if $a = 2$, $b = 1$ and $c = 2$, the number of layers is 125, the SDF is 68.02 and the geometric thickness is 8.73 μm and if $a = 1$, $b = 2$ and $c = 2$, the number of layers is also 125, however the SDF is 6.15 and the geometric thickness is changed to be 9.915 μm. So finally, we conclude that by manipulating the parameters (a, b and c) of the GC structure we can improve the value of SDF while optimizing the geometric thickness of the structure and keeping the same number of layers.

3.2. Effect of the reference wavelength

In this part, we study the effect of the reference wavelength λ_0 on the slowing down factor (SDF). Here, the iteration number is chosen to be $m = 4$, and the parameters a, b and c are fixed at 2, 1 and 2, respectively. In Table 2, these parameters' values give us the best slowing down factor (SDF = 68.02, λ_0 =0.5 μm, P = 125 and L = 8.73 μm).

Figure 4 shows the variation of SDF according to λ_0 (varied from 0.32 to 0.68 μm with a steps of 0.02 μm). So here the number of layers is kept at 125 and only the geometric thickness of the structure is changed. From this figure, it is clear that the maximum slowing down factor (SDF = 157.1) is found for λ_0 =0.46 μm; here the thickness is L = 8.037 μm. If we compare this result with the best result found in Table 1, we conclude that by keeping the same values for the GC parameters (a = 2, b = 1 and c = 2) and changing only the reference wavelength λ_0 from 0.5 to 0.46 μm, we can keep the same number of layers (P = 125), decrease the geometric thickness from 8.73 to 8.03 μm and increase the SDF from 68.02 to 157.1.

The reduction of the reference wavelength λ_0 makes it possible to reduce the optical thickness ($\lambda_0/4$) of different layers constituting the photonic structure. This reduction of the optical thickness near the scale of the incident wavelength makes it possible to have a strong light-localization and SDF.

The previous research work of Ben Ali *et al*. [8] focused on the study of light slowing in one-dimensional hybrid Cantor and periodic photonic crystals. In [8], the best slow

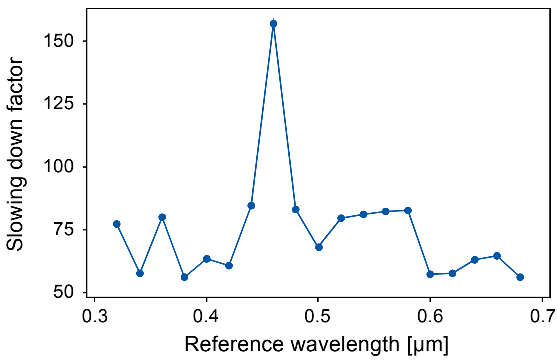

Fig. 4. Variation of the slowing down factor according to the reference wavelength λ_0.

down factor (SDF = 90.49) is found with the hybrid structure constructed by periodic photonic structure sandwiched between two classical triadic Cantor structures (Cantor1/periodic/Cantor2). The materials constituting the hybrid structure are the SiO_2 and the TiO_2. The first Cantor structure is fixed at the 5th iteration, the second one at the 4th iteration and the periodic structure has 24 layers. Also the λ_0 =0.6 μm, so the total number of layers and the geometric thickness of the hybrid structure are 132 and 11.22 μm, respectively [8]. By comparing our best result (SDF = 157.1) found in Fig. 4, with the result of the hybrid structure [8], we can see clearly that our photonic structure of the GC (a = 2, b = 1 and c = 2), makes it possible to have the maximum slowing down factor, while optimizing the number of layers and the geometric thickness of the structure.

4. Conclusion

The study of generalized Cantor structure GC(a, b, c) makes it possible to clearly improve the slowing of light. In fact, by optimizing the parameters of this structure such as a, b and c, an improvement of the slowing down factor value is observed. The best result (SDF = 68.02) is found for a = 2, b = 1 and c = 2, and here the number of layers and the geometric thickness of the structure are 125 and 8.73 μm, respectively. Then, by optimizing the value of the reference wavelength to become λ_0 =0.46 μm, another improvement is observed for the slowing down factor (SDF = 157.09). Also the geometric thickness of the structure is decreased to be 8.03 μm, however the number of layers remains at 125. This improvement is due to the reduction in the optical thickness ($\lambda_0/4$) of layers constituting the photonic structure.

References

[1] Gisin N., Thew R., *Quantum communication*, Nature Photonics **1**(3), 2007, pp. 165–171.

[2] Tao Li, Guo-Jian Yang, Fu-Guo Deng, *Heralded quantum repeater for a quantum communication network based on quantum dots embedded in optical microcavities*, Physical Review A **93**(1), 2016, article ID 012302.

[3] Saif Islam M., Logeeswaran Vj, *Nanoscale materials and devices for future communication networks*, IEEE Communications Magazine **48**(6), 2010, pp. 112–120.

[4] Toshihiko B., *Slow light in photonic crystals*, Nature Photonics **2**(8), 2008, pp. 465–473.

[5] Ben Ali N., Kanzari M., *A broad slow frequencies band and high slowing down factor by using one -dimensional hybrid periodic/Fibonacci photonic crystals*, IOP Conference Series: Materials Science and Engineering **13**(1), 2010, article ID 012016.

[6] Wen-Kai Kuo, Hsueh-Ping Weng, Jyun-Jheng Hsu, Hsin Her Yu, *Photonic crystal-based sensors for detecting alcohol concentration*, Applied Sciences **6**(3), 2016, p. 67.

[7] Yi Yu, Heuck M., Hao Hu, Weiqi Xue, Peucheret C., Yaohui Chen, Oxenløwe L.K., Yvind K., Mørk J., *Fano resonance control in a photonic crystal structure and its application to ultrafast switching*, Applied Physics Letters **105**(6), 2014, article ID 061117.

[8] Ben Ali N., Jihene Zaghdoudi, Kanzari M., Kuszelewicz R., *The slowing of light in one-dimensional hybrid periodic and non-periodic photonic crystals*, Journal of Optics **12**(4), 2010, article ID 045402.

[9] Vasconcelos M.S., Mauriz P.W., Albuquerque E.L., *Optical filters based in quasiperiodic photonic crystal*, Microelectronics Journal **40**(4–5), 2009, pp. 851–853.

[10] BIANCALANA F., *All-optical diode action with quasiperiodic photonic crystals*, Journal of Applied Physics **104**(9), 2008, article ID 093113.
[11] ARAÚJO C.A.A., VASCONCELOS M.S., MAURIZ P.W., ALBUQUERQUE E.L., *Omnidirectional band gaps in quasiperiodic photonic crystals in the THz region*, Optical Materials **35**(1), 2012, pp. 18–24.
[12] YEH P., YARIV A., *Optical Waves in Crystals*, Wiley Series in Pure and Applied Optics, Wiley – Interscience, New York, 1984, p. 589.
[13] TRABELSI Y., BENALI N., BOUAZZI Y., KANZARI M., *Microwave transmission through one-dimensional hybrid quasi-regular (Fibonacci and Thue–Morse)/periodic structures*, Photonic Sensors **3**(3), 2013, pp. 246–255.
[14] CHUNFANG OUYANG, ZHIQIANG XIONG, FANGYUAN ZHAO, BIQIN DONG, XINHUA HU, XIAOHAN LIU, JIAN ZI, *Slow light with low group-velocity dispersion at the edge of photonic graphene*, Physical Review A **84**(1), 2011, 015801.
[15] KANZARI M., BOUZIDI A., REZIG B., *Interferential polychromatic filters*, The European Physical Journal B **36**(4), 2003, pp. 431–443.
[16] OKAMOTO T., FUKUYAMA A., *Light amplification from Cantor and asymmetric multilayer resonators*, Optics Express **13**(20), 2005, pp. 8122–8127.
[17] SAHEL S., AMRI R., BOUAZIZ L., GAMRA D., LEJEUNE M., BENLAHSEN M., ZELLAMA K., BOUCHRIHA H., *Optical filters using Cantor quasi-periodic one dimensional photonic crystal based on Si/SiO$_2$*, Superlattices and Microstructures **97**, 2016, pp. 429–438.

Received April 8, 2017
in revised form April 24, 2017