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Propagation of solitary wave in non-uniform fiber system with high-order nonlinear effects

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The ultra-short pulse propagation in a non-uniform fiber system is investigated based on the variable coefficient coupled higher-order nonlinear Schrödinger equation with the dispersion gain and nonlinear gain terms. By using the ansatz method and the split-step Fourier method, we get the exact solitary wave solution, with which the transmission process of the solitary wave is studied. Furthermore we obtain the stability of the solitary wave under finite initial perturbations. The interaction between two neighboring solitary waves is also studied.

Keywords: solitary wave, coupled higher-order nonlinear Schrödinger equation, high-order nonlinear effect.

1. Introduction

Optical soliton, one of the best information carriers for large capacity and long distance optical transmission systems, has attracted much attention and has became the important resources in optical communications [1, 2]. Propagation of optical pulse in fiber was described by the nonlinear Schrödinger (NLS) equation, in which the group velocity dispersion (GVD) and the self-phase modulation (SPM) were considered. Optical soliton can propagate over a long distance without the shape change in optical fiber depended on the balance between GVD and SPM [3]:

$$iU_z + \frac{1}{2}\beta_2 U_{tt} + \gamma_1 |U|^2 U = 0 \tag{1}$$

where U describes the envelope amplitude of the electric field, the subscripts represent the partial derivatives, z and t are the normalized distance along the direction of the propagation and retarded time, β_2 denotes the GVD, and γ_1 represents the SPM parameter.

The solutions of NLS equations have been extensively studied. Two families of analytical light bullet solutions with two types of PT-symmetric potentials are obtained based on the (3 + 1)-dimensional NLS equation with variable-coefficient dispersion/diffraction and cubic-quintic-septimal nonlinearities [4]. The (2 + 1)-dimensional variable-coefficient NLS equation with partial nonlocality is studied by CHAO-QING DAI *et al.* and they have found the hierarchies of Peregrine solution and breather solution [5]. Furthermore, the (3 + 1)-dimensional partially nonlocal NLS equation is also considered, from which they obtained the approximate spatiotemporal Hermite–Gaussian soliton solutions [6].

Solitary wave solutions under the special relationship of system parameters of NLS equation were obtained [7–11] already for ultra-short optical pulse, such as picosecond and femtosecond optical pulses. In order to consider the higher-order nonlinear effects such as third-order dispersion, self-steepening and self-frequency-shift on the transmission media, the nonlinear Schrödinger equation is extended to higher-order nonlinear Schrödinger (HNLS) equation [12–16],

$$iU_z + \frac{1}{2}\beta_2 U_{tt} + \gamma_1 |U|^2 U + \frac{1}{6} i\beta_3 U_{ttt} + i\gamma_3 (|U|^2 U)_t + i\gamma_4 (|U|^2)_t U = 0 \tag{2}$$

where β_3 describes the third-order dispersion, γ_3 describes the self-steepening, γ_4 represents the delayed nonlinear process, and the imaginary part of γ_4 represents the low frequency component of self-frequency-shift. It was shown that the stably solitary wave can be used to describe the propagation of femtosecond pulses in an optical fiber under certain parametric conditions [17, 18].

The influence of the inter-mode coupling on nonlinear dynamics in optical fibers was discussed [19, 20]. Taking into account the case where two polarized components of one optical pulse or two optical pulses propagate at the same time with higher-order effects, the higher-order nonlinear Schrödinger equation was then extended to the coupled higher-order nonlinear Schrödinger (CHNLS) equation:

$$\begin{aligned} iU_{1,z} &+ \frac{1}{2}\beta_2 U_{1,tt} + \gamma_1 (|U_1|^2 + |U_2|^2) U_1 + \frac{1}{6} i\beta_3 U_{1,ttt} \\ &+ i\gamma_3 \left[(|U_1|^2 + |U_2|^2) U_1 \right]_t + i\gamma_4 (|U_1|^2 + |U_2|^2)_t U_1 = 0 \end{aligned} \tag{3a}$$

$$\begin{aligned} iU_{2,z} &+ \frac{1}{2}\beta_2 U_{2,tt} + \gamma_1 (|U_2|^2 + |U_1|^2) U_2 + \frac{1}{6} i\beta_3 U_{2,ttt} \\ &+ i\gamma_3 \left[\left(|U_2|^2 + |U_1|^2 \right) U_2 \right]_t + i\gamma_4 (|U_2|^2 + |U_1|^2)_t U_2 = 0 \end{aligned} \tag{3b}$$

which are proposed to model the ultra-short pulse propagation in optic fiber, z and t are the normalized distance along the direction of the propagation and retarded time, respectively, U_1 and U_2 represent the two complex envelope amplitudes of the electric field, respectively. The exact solitary wave solutions of CHNLS equation were also extensively studied [14, 21].

In this paper, based on the CHNLS equation, we also take the dispersion gain, the nonlinear gain, self-steepening and self-frequency-shift into account. In order to be more in line with the actual situation, we consider the parameters of fiber as variable coefficients. In Section 2 of this paper, by using the ansataz method we obtained the exact solitary wave solution of our theoretical model. In Section 3, by using the split-step Fourier method, we present the numerical simulations of the propagation characteristics. The stability analysis and the interaction behaviors is also discussed. Section 4 contains our conclusions.

2. Theoretical analysis and soliton solutions

The coupled higher-order nonlinear Schrödinger (CHNLS) equation with the dispersion gain and the nonlinear gain is used to describe the propagation of ultra-short optical pluses. It is given by:

$$iU_{1,z} + \left(\frac{1}{2}\beta_2 + i\alpha_1\right)U_{1,tt} + (\gamma_1 + i\gamma_2)(|U_1|^2 + |U_2|^2)U_1 + \frac{1}{6}i\beta_3 U_{1,ttt} + i\gamma_3\Big[(|U_1|^2 + |U_2|^2)U_1\Big]_t + i\gamma_4(|U_1|^2 + |U_2|^2)_t U_1 + gU_1 = 0 \tag{4a}$$

$$iU_{2,z} + \left(\frac{1}{2}\beta_2 + i\alpha_1\right)U_{2,tt} + (\gamma_1 + i\gamma_2)(|U_2|^2 + |U_1|^2)U_2 + \frac{1}{6}i\beta_3 U_{2,ttt} + i\gamma_3\Big[(|U_2|^2 + |U_1|^2)U_2\Big]_t + i\gamma_4(|U_2|^2 + |U_1|^2)_t U_2 + gU_2 = 0 \tag{4b}$$

where U_1 and U_2 represent the two complex envelope amplitudes of the electric field, z and t are the normalized distance along the direction of the propagation and retarded time, respectively; β_2 denotes the GVD, α_1 is the dispersion gain, γ_1 is the SPM parameter and γ_2 is the nonlinear gain, β_3 describes the third-order dispersion, γ_3 describes the self-steepening, and γ_4 represents the self-frequency-shift.

Solitary wave solution has been achieved under certain parametric choice by the ansatz method of the HNLS equation. In this paper, we will concentrate on Eqs. (4a) and (4b) to find its solitary wave solution by assuming a solution of the following form [22–24]:

$$U_i(z, t) = A(z)\,\mathrm{sech}\Big[\eta(z)(t - T(z))\Big]\exp\Big[i\varphi(z, t)\Big], \quad i = 1, 2 \tag{5}$$

$$\varphi(z, t) = \rho(z)\ln\left\{\mathrm{sech}\Big[\eta(z)(t - T(z))\Big]\right\} + a(z) + b(z)t + c(z)t^2 \tag{6}$$

where $\rho(z)$ denotes the nonlinear chirp, $A(z)$, $\eta(z)$, $T(z)$ and $\varphi(z, t)$ are real functions of amplitude, inverse pulse width, time position and phase of pulse, respectively. The parameters $a(z)$, $b(z)$ and $c(z)$ describe the initial phase, frequency and linear chirp effects, respectively. Substituting Eqs. (5) with (6) into Eqs. (4a) and (4b), removing the exponential terms, then separating the real and imaginary parts, and equating the coefficients of independent terms, we can obtain the following expressions:

$$\rho(z) = \beta_3(z) = 0 \tag{7}$$

$$b(z) = c(z) = 0 \tag{8}$$

$$\eta(z)T'(z) = 0 \tag{9}$$

$$A(z)\left[2A^2(z)\gamma_1 - \beta_2\eta^2(z)\right] = 0 \tag{10}$$

$$\frac{1}{2}\beta_2\eta^2(z) + g - a'(z) = 0 \tag{11}$$

$$A(z)\left[2A^2(z)\gamma_2 - 2\alpha_1\eta^2(z)\right] = 0 \tag{12}$$

$$A^3(z)\eta(z)(-6\gamma_3 - 4\gamma_4) = 0 \tag{13}$$

$$A'(z) = 0 \tag{14}$$

$$\eta'(z) = 0 \tag{15}$$

We obtain the relation of the model parameters and the soliton solution parameters of Eqs. (4a) and (4b) after performing some algebra. From the calculation process, a noteworthy feature of the result is that $\rho(z) = \beta_3(z) = 0$ and $b(z) = c(z) = 0$. It means that Eqs. (4a) and (4b) has no linear chirp nor nonlinear chirp, thus third-order dispersion needs to be compensated. Since the amplitude of the pulse is real in practice, we can infer from Eq. (14) that $A(z)$ is constant, indicating that the amplitude is unchanged under the transmission process. Besides, the inverse pulse width $\eta(z)$ is a constant too, namely, pulse width will not change during propagation along the fiber. It means that energy is conserved. Further, we can know that $T(z)$ is also a constant. That is to say, the center position of the pulse is unchanged. From Eqs. (10)–(13) we can know that β_2 and γ_1, γ_2 and α_1, γ_3 and γ_4 have a certain constraint relationship. To further simplify the above formula, we can get:

$$\gamma_1 = \frac{\beta_2\eta^2(z)}{2A^2(z)} \tag{16}$$

$$a'(z) = \frac{1}{2}\beta_2\eta^2(z) + g \tag{17}$$

$$\alpha_1 = \frac{\eta^2(z)}{A^2(z)}\gamma_2 \tag{18}$$

$$\frac{\gamma_3}{\gamma_4} = -\frac{2}{3} \tag{19}$$

So we find the exact solitary wave solution of the Eqs. (4a) and (4b). However, during the academic calculating, it is difficult to determine each model parameter and each solitary wave solution parameter. We can only determine the mutual relation of the parameters by Eqs. (16)–(19). So, if giving a part of the model parameters, such as β_2, γ_2, γ_3 and g, we can determine the other model parameters and the solitary wave solution parameters from Eqs. (16)–(19).

3. Numerical simulation

In what follows, we analyze the stability of the exact solitary wave solution by employing the numerical split-step Fourier method. As in the practical non-uniform fiber, the parameters of fiber could fluctuate nearby the ideal value. If the amplitude of fluctuation is small, we distribute the parameters of fiber in variable coefficients forms:

$$\beta_2(z) = \beta_{20}\Big[1 + a_1\sin(\sigma z)\Big]\exp(\mu z) \tag{20}$$

$$\gamma_2(z) = \gamma_{20}\Big[1 + a_1\sin(\sigma z)\Big]\exp(\mu z) \tag{21}$$

$$\gamma_3(z) = \gamma_{30}\Big[1 + a_1\sin(\sigma z)\Big]\exp(\mu z) \tag{22}$$

where β_{20}, γ_{20} and γ_{30} are ideal fiber parameters, a_1 – small quantities that characterize the amplitudes of fluctuations, μ – small real constants, and σ is related to the variation period of the fiber parameters. In this paper, we take the system parameters as: $a_1 = 0.01$, $\mu = -0.04$, $\sigma = 0.01$, and $g = 0.0005$.

For such a set of parameters, we demonstrate a typical example, in which the parameters of the solitary wave solution and the parameters of the fiber system we adopted are: $\eta_0 = 0.15$, $A(z) = 0.8$, $\beta_{20} = 0.5$, $\gamma_{20} = 0.002$ and $\gamma_{30} = 0.05$. Then the parameters γ_1, α_1 and γ_4 are determined by Eqs. (16)–(19). In our numerical simulation, we take the parameter $T(0) = 0$, which said the initial time position of the pulse is zero.

First, we get as the input pulse:

$$U_i(0, t) = A(0)\,\mathrm{sech}\Big[\eta_0(t - T(0))\Big]\exp\Big[ia(0)\Big], \qquad i = 1, 2 \tag{23}$$

Through the check of simulation, the evolution of the transmission diagram is shown in Fig. 1**a**. This clearly indicates that solitary wave keeps its shape in propagating along

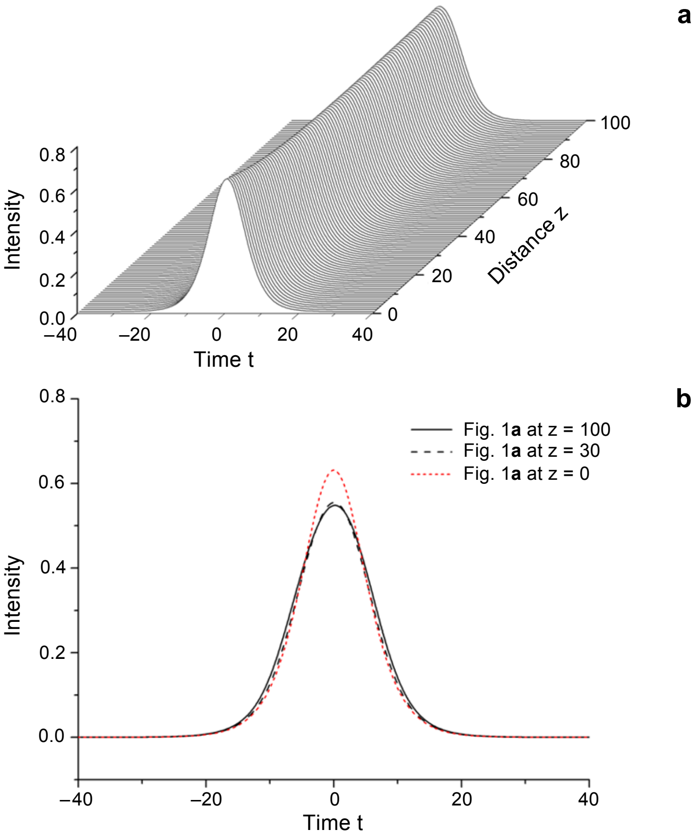

Fig. 1. The evolution plot of the solitary wave (**a**). The compared plots of the initial pulse at different transmission positions (**b**).

the fiber after self-adjustment at the beginning, and transmission stably. In Fig. 1**b**, we compare the initial pulse at $z = 0$ and $z = 100$, which shows that after self-adjustment the soliton pulse is widened and the amplitude is reduced slightly. We also compare the initial pulse at $z = 30$ and $z = 100$. It turns out that the solitary wave is almost the same, keeps amplitude and pulse width unchanged. This simulation result is also coinciding with the theoretical analysis.

In order to investigate the stability of the solitary wave with the effect of the dispersion gain and the nonlinear gain and high order nonlinearity, we consider finite initial perturbations. We performed three types of numerical simulation experiments:

1. We perturbed the amplitude in the initial distribution. The second condition was

$$U_i(0, t) = 0.95A(0)\,\mathrm{sech}\left[\eta_0(t - T(0))\right]\exp\left[ia(0)\right], \qquad i = 1, 2 \tag{24}$$

The parameters of the solitary wave solution and the parameters of the fiber system are the same as first condition. Then we get the transmission diagram as shown in

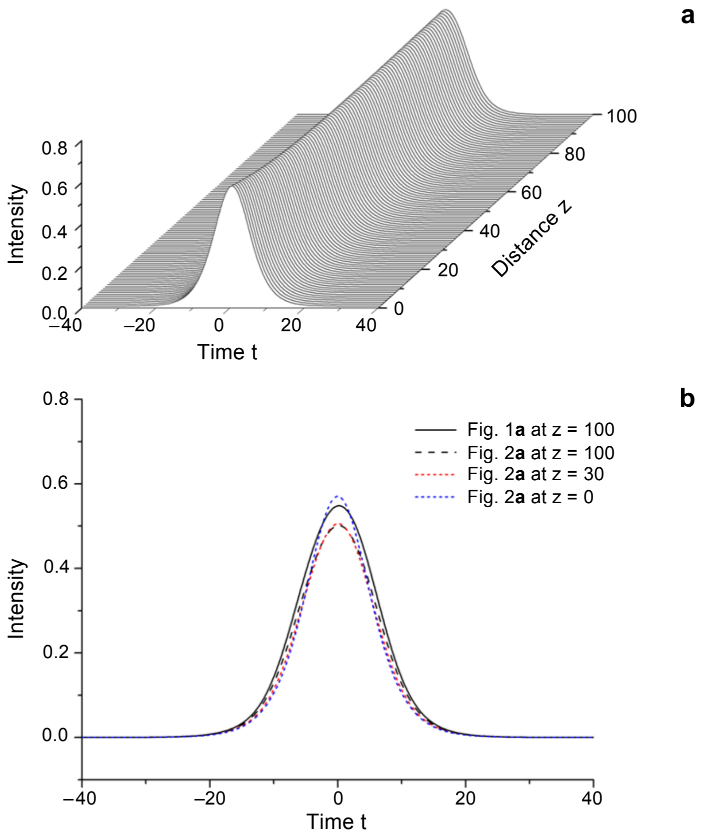

Fig. 2. The evolution plot of the solitary wave with amplitude perturbation (**a**). The compared plots at different transmission positions (**b**).

Fig. 2**a**. From it we can find that, after a short period of self-adjustment, the pulse is rather stable and can propagate 100 dispersion lengths along fiber. In Fig. 2**b**, we compare the initial pulse and the amplitude perturbation pulse, which shows that the solitary wave shape is basically similar, except that the amplitude is reduced. After the self-adjustment, the pulse transmission is stable. From Fig. 2 we demonstrate that the small perturbations of amplitude will not affect the pulse stability if we take the appropriate parameter values.

2. We added white noise in the initial pulse, and the third condition was

$$U_i(0, t) = \left\{ A(0)\operatorname{sech}\left[\eta_0(t - T(0))\right] + 0.1\operatorname{rand}(t) \right\} \exp\left[ia(0)\right], \quad i = 1, 2 \tag{25}$$

The parameters of the solitary wave solution and the parameters of the fiber system are the same as first condition. Then we get the transmission diagram as shown in Fig. 3**a**. From it we can find that, after a short period of self-adjustment, the pulse sta-

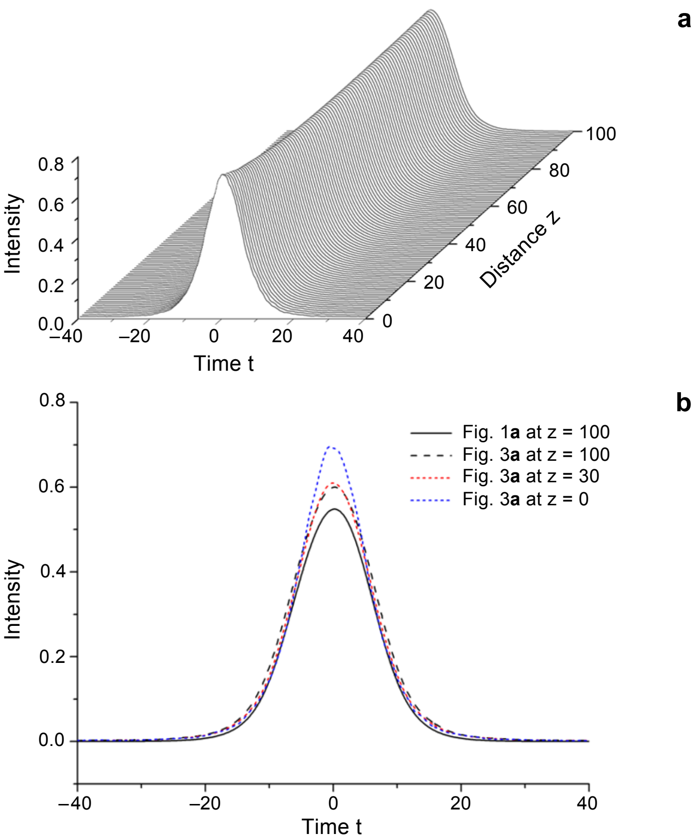

Fig. 3. The evolution plot of the solitary wave with white noise (**a**). The compared plots at different transmission positions (**b**).

bilizes and can propagate 100 dispersion length along fiber. In Fig. 3**b**, we compare the initial pulse and the pulse with white noise, which shows that the solitary wave shape is basically similar. We can also find that the pulse is widened and the amplitude increases a little compared to the initial pulse. After the self-adjustment, the pulse transmission is stable. From Fig. 3 we demonstrate that the white noise will not affect the pulse stability if we take the appropriate parameter values.

3. We added phase perturbation in the initial pulse, and the fourth condition was

$$U_i(0, t) = A(0)\,\mathrm{sech}\Big[\eta_0(t - T(0))\Big]\exp\Big[ia(0) + 0.5i\Big], \qquad i = 1, 2 \tag{26}$$

The parameters of the solitary wave solution and the parameters of the fiber system are the same as first condition. Then we get the transmission diagram as shown in Fig. 4**a**. From it we can find that, after a short period of self-adjustment, the pulse stabilizes and can propagate 100 dispersion length along fiber. In Fig. 4**b**, we compare the initial pulse and the phase perturbation pulse, which shows that the solitary wave

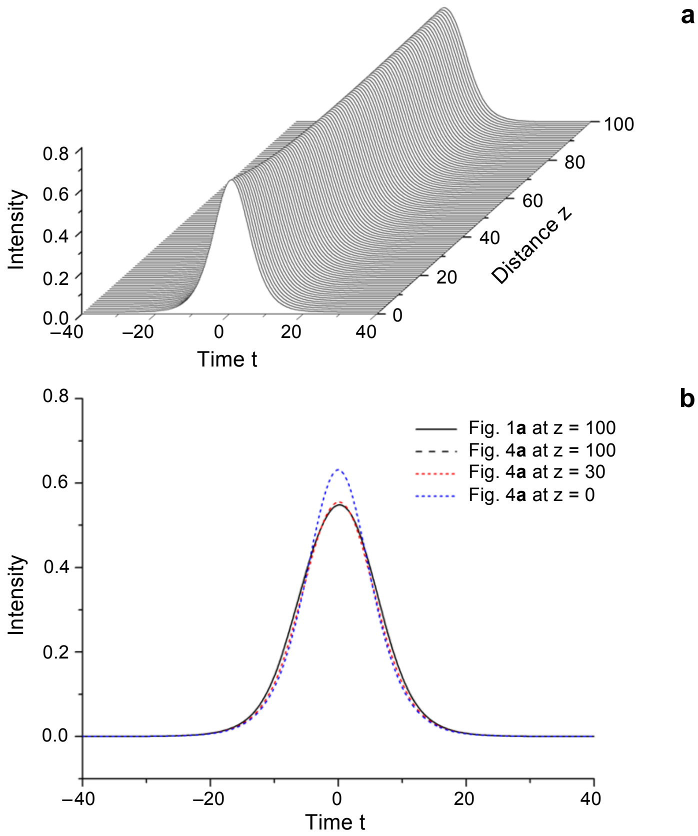

Fig. 4. The evolution plot of the solitary wave with phase perturbation (**a**). The compared plots at different transmission positions (**b**).

shape is almost identical. After self-adjustment, the pulse transmission is stable. In Figs. 4**a** and 4**b**, we demonstrate that the phase perturbation will not affect the pulse stability if we take the appropriate parameter values.

In addition, we investigate the evolution features of the interaction between two neighboring pulses in the fiber system with the effect of the dispersion gain and the nonlinear gain. The input pulse forms are as follows:

$$U_i(0, t) = A(0)\,\mathrm{sech}\left[\eta_0\left(t - T(0) - \frac{q_0}{2}\right)\right]\exp\left[ia(0)\right] + A(0)\,\mathrm{sech}\left[\eta_0\left(t - T(0) + \frac{q_0}{2}\right)\right]\exp\left[ia(0)\right], \quad i = 1, 2 \tag{27}$$

Here q_0 is the initial separation between two adjacent pulses. The parameters of the solitary wave solution and the parameters of the fiber system are the same as first con-

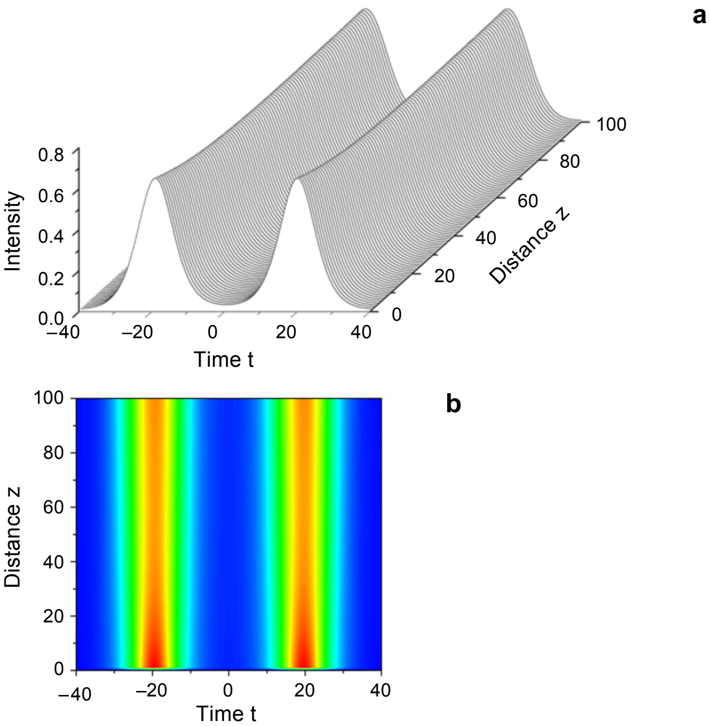

Fig. 5. The evolution (**a**) and the contour (**b**) plots of two neighboring solitary waves.

dition. Using the same method we get the interaction diagram of two neighboring solitary waves as in Fig. 5.

From Fig. 5**a** we can find that after a short period of self-adjustment, for the two neighboring solitary waves the elastic collision did not happen, and they can propagate 100 dispersion lengths steady in the fiber system. From the contour plot of Fig. 5**b**, we can clearly see that the soliton pulses are independent of each other. Through a series of numerical simulations, we find that as the initial separation reaches up a certain value, the interaction of the solitary wave exhibits neither the elastic interaction nor the mutually exclusive effect. Therefore, we may infer that the solitary wave can restrain the interaction between the neighboring pulses. This is also an advantage in improving the information bit rate in optical communication.

4. Conclusion

In this paper, we have investigated the coupled higher-order nonlinear Schrödinger equation with variable coefficients, which describe the ultra-short pulse propagation in the non-uniform fiber system. The exact solitary wave solution is presented by using the ansataz method. In the numerical simulation experiment, we find that the solitary wave keeps its shape in propagating along the fiber system, and the small perturbations of amplitude, phase and white noise will not affect the stability of the solitary wave.

Meanwhile, we investigate the interaction between the two neighboring solitary waves. It turns out that the two neighboring solitary waves can propagate steady along the fiber system. Our analytic results can be used to improve the information bit rate in optical communication.

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