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Phase recovery from intensity distributions generated by differential operators in one-dimensional coherent imaging

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In the paper some examples of differential operators L are analysed which modify the amplitude of the band-limited signal $f(x)$ is such a way that the measurement of intensities $|f(x)|^2$ and $|L[f(x)]|^2$ assures a unique recovery of the signal. Particular attention is drawn to an operator which may be realized by locating transmittance $T(\omega) = 2A\omega+B$ in the exit pupil of the optical system. A simulation of the reconstruction process has been performed by applying na algorithm proposed in [10].

Introduction

As it is well known the classical coherent optical field is defined by two quantities: amplitude $A(x)$ and phase $\varphi(x)$, the combination of which gives the complex amplitude

$$f(x) = A(x)\exp[i\cdot\varphi\cdot(x)]. \tag{1}$$

So far is optics there exist no method of direct detection of the phase in (1), as all the detectors are subject to the square-law detection (i.e. they measure the signal intensity proportional to $|f(x)|^2$). However, if an additional information about the type of the signal is available the phase information is not entirely lost in detection. Knowing, for instance, that $f(x)$ is a band-limited signal it is possible to determine the class of all the band-limited functions giving the same measurement result [1–3]. This follows from the fact that the real and imaginary parts of the band-limited signals are mutually connected by the Hilbert transform. However, this condition does not assure a unique dependence between the phase and the amplitude.

Therefore, for a unique phase determination not only the $A(x)$ distribution should be known but an additional information about the measured signal is needed. This information can be obtained most conveniently by additional square-law measurements made on some other distribution of the complex amplitude obtained from the primary complex amplitude by a known optical transformation. Thus, the problem of phase reconstruction may be reduced to finding such an optical operator L, for which the following system of functional equations

$$|f(x)|^2 = |g(x)|^2, \tag{2a}$$

$$|L[f(x)]|^2 = |L[g(x)]|^2 \tag{2b}$$

will have a unique solution with respect to the unknown function $g(x)$ belonging to the class of optically realizable functions. If L is a linear operator then the unique solution of the system (2a,b) means, in reality, a solution determined with the accuracy to the constant phase factor $\exp(i\cdot c)$. In the further part of this work it is assumed that the class of admissible solutions of the system (2a,b) is the class of band-limited functions.

In the papers [4–7] the case of

$$L[f(x)] = \int_{-\infty}^{\infty} f(x)e^{-2\pi i\omega x}dx \tag{3}$$

has been examined. Then the system of equations (2a,b) may be interpreted as then respective results of the intensity measurements performed in the image (2a) and frequency (2b) planes (the latter may be identical with the exit pupil plane). In the paper [4] it has been shown that if $L[f(x)]$ is an analytic function (this condition being fulfilled in an aberration-free system) then the system (2a,b) has, in general, a unique solution. However, in few cases, e.g. when the object $f(x)$ is symmetric, a two-fold ambiguity arises.

The unique solution of equation system (2a,b) has been obtained for the case, when $f(x)$ is a complex amplitude in the image plane of the microscopic system and $L[f(x)]$ is a complex amplitude in the image given by defocussed version of this system [8–11]. Then $L[f(x)]$ may take the form of an integral operator

$$L[f(x)] = \int_{-\infty}^{\infty} f(x')h(x-x')dx', \tag{4}$$

where the kernel of this operator is of the form

$$h(x) = \int_{-\omega_0}^{\omega_0} e^{-i\Delta\omega^2} e^{2\pi i\omega x}d\omega. \tag{5}$$

Here, ω_0 denotes the cut-off frequency transmitted by the microscopic system, and Δ is a measure of the system defocussing. In the paper [12] the case is discussed when L is a differential operator d/dx. Then the phase may be recovered up to the sign accuracy, i.e. the system (2a,b) has two solutions: $f(x)\cdot\exp(i\cdot c)$, and $f^*(x)\exp(i\cdot c)$.

The goal of this paper is to determine such differential operators L, which assure a unique phase recovery. Special attention will be paid to the optically realizable operators.

Unique phase recovery by using a linear differential operator of the first order

Let us assume that an intensity comming from the signal $L[f(x)]$ is determined in the second measurement, where

$$L[f(x)] = \alpha(x)\frac{df(x)}{dx} + \beta(x)f(x), \tag{6}$$

and $\alpha(x)$, and $\beta(x)$ are complex functions. By using the eq. (1) the operator (6) may be represented in the following form

$$L[f(x)] = [\alpha(x)(A'(x)+iA(x)\cdot\varphi'(x))+\beta(x)A(x)]e^{i\varphi(x)}, \tag{7}$$

where prime denotes derivative d/dx[1]. After simple rearrangements we may show that the intensity in the second measurement is expressed by the formula

$$|L[f(x)]|^2 = |\alpha(x)A(x)|^2|\varphi'(x)|^2+2Im(\alpha^*(x)\cdot\beta(x))A^2(x)\cdot\varphi'(x) + |\alpha(x)A'(x)|^2+|\beta(x)A(x)|^2+2Re(\alpha^*(x)\cdot\beta(x))A(x)A'(x). \tag{8}$$

The functions: real part, imaginary part and the complex conjugate are denoted by *Re*, *Im*, and asterisk, respectively. First measurement gives the amplitude $A(x)$, thus each signal $g(x)$ which gives the same result as that of the first measurement of $f(x)$ must be of the form

$$g(x) = A(x)e^{i\psi(x)}. \tag{9}$$

Let us determine the phase $\psi(x)$ necessary for the signal (9) to give the same second measurement result as that produced by the signal defined in (1). For this purpose it is necessary to know when the eq. (2b) is satisfied. By using (8) and (9) the eq. (2b) takes the form

$$|L[f(x)]|^2-|L[g(x)]|^2 = A^2(x)(\varphi'(x)-\psi'(x))\left[|\alpha(x)|^2(\varphi'(x)+\psi'(x)) + 2Im(\alpha^*(x)\cdot\beta(x))\right]. \tag{10}$$

Obviously, the left hand side of the above equation must be equal to zero for each x. Consequently, there must exist two subsets A, B of the set of real numbers R that $A \cup B = R$, and that

for $x \in A$ $$w_1(x) = A^2(x)(\varphi'(x)-\psi'(x)) = 0, \tag{11}$$

for $x \in B$ $$w_2(x) = \varphi'(x)+\psi'(x)+2\frac{Im(\alpha^*(x)\cdot\beta(x))}{|\alpha(x)|^2} = 0. \tag{12}$$

It turns out that if $f(x)$ and $g(x)$ are band-limited then only one of the cases below may occur:

1. $R \setminus A$ is at most a discrete set with no condensation point.
2. A is at most a discrete set with no condensation point.

In order to show this property of the set A, it must be first proved that $w_1(x)$ is a meromorphic function. For this purposes the phase difference $\varphi-\psi$ is represented in the following form

$$\varphi(x)-\psi'(x) = \mathrm{arctg}\frac{v(x)}{u(x)}+c(x), \tag{13}$$

where $v(x) = Im(f(x)g^*(x))$, and $u(x) = Re(f(x)g^*(x))$, $c(x)$ is a piece-wise constant

[1] The formula (7) is valid outside the points for which $A(x) = 0$, because then $A'(x)$ and $\varphi(x)$ are undertermined.

function defined as follows

$$c(x) = \begin{cases} 0 & u(x) > 0 \text{ and } v(x) \geq 0, \\ \pi & u(x) < 0, \\ 2\pi & u(x) > 0, \text{ and } v(x) < 0. \end{cases}$$

The eq. (13) may be differentiated everywhere except for the points, in which $u(x)$ or $v(x)$ changes its sign (these points create a discrete set). Then, after multiplying by $A^2(x)$, the following expression is obtained for $w_1(x)$[1]:

$$w_1(x) = \frac{v'(x)u(x) - v(x)u'(x)}{A^2(x)}. \tag{14}$$

From the formula (14) it follows that $w_1(x)$ is a meromorphic function, because $u(x)$, $v(x)$ and $A^2(x)$ are analytical functions[2]. Hence, if the set A had a condensation point then $w_1(x)$ should be equal to zero everywhere outside the points, in which $A(x) = 0$ [14].

From the properties 1. and 2. of the set A it follows that one of the sets $R \setminus A$ or $R \setminus B$ is a discrete set having no condensation points.

If $R \setminus A$ is of this form then after dividing $w_1(x)$ by $A^2(x)$ the eq. (11) may be integrated. At the points which do not belong to A the undetermined constant of integrate may have a jump. Then

$$\psi(x) = \varphi(x) + c_1(x), \tag{15}$$

where $c_1(x)$ is a piece-wise constant function having jumps at the points belonging to the set $R \setminus A$. Since $g(x)/f(x)$ is a meromorphic function, it may be shown that the $\exp[i \cdot c_1(x)]$ function must be continuous. This means that the difference

$$\lim_{x \to x_0^-} c_1(x) - \lim_{x \to x_0^+} c_1(x),$$

where x_0 is a point of discontinuity of the function $c_1(x)$, is a integer multiple of 2π. Therefore, it may be assumed that

$$c_1(x) = \text{constant}, \tag{15a}$$

thus, the system (2ab) has a unique solution.

If $R \setminus B$ is a discrete set then after integrating the eq. (12) the following relation can be obtained

$$\psi(x) = -\varphi(x) + p(x) + c_2(x), \tag{16}$$

where

$$p(x) = -2 \int_0^x \frac{Im\left(\alpha^*(x) \cdot \beta(x)\right)}{|\alpha(x)|^2} dx, \tag{17}$$

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[1] In the case when $u(x) \equiv 0$, the phase difference may be expressed by the function $\operatorname{arc\,ctg} \frac{u(x)}{v(x)}$, but it will not influence the final results.

[2] The complex variable functions $f(z) \cdot f^*(z^*)$ and $f(z) \cdot g^*(z^*)$ are integral functions of the exponential type equal to $A^2(x)$ and $f(x) \cdot g^*(x)$, respectively, on the real axis.

and where $c_2(x)$ is a piece L-wise constant function having jumps at the points belonging to the set $R \setminus B$. Hereafter, it is assumed that the functions $\exp[ip(x)]$ is a meromorphic function. Then, taking advantage of the fact that the function $\exp[-ip(x)] \cdot g(x)/f^*(s)$ is also meromorphic, it may be shown that

$$c_2(x) = \text{constant}. \tag{16a}$$

Finally, it may be stated that the system of eqs. (2a,b) will have a unique solution if the function $p(x)$ has such a property that the equality (16), (16a) is not fulfilled for any phases φ, ψ of band-limited signals of the same amplitude. Then the eqs. (15), (15a) will be satisfied automatically, thus

$$g(x) = f(x)\exp(i \cdot c). \tag{18}$$

Below a general condition for $p(x)$ has been formulated, the fulfillment of which assures a unique recovery.

Condition 1

If

$$e^{ip(x)} \neq e^{2\pi iax} \prod_{n_k} \frac{x - z_n^*}{x - z_n} \tag{19}$$

for each real number a and each subsequence n_k of the complex number sequence $\{z_n\}$ such that the series

$$\sum_{n_k} \frac{1}{|z_n|^{1+\varepsilon}} \tag{20}$$

is convergent for every $\varepsilon > 0$, then the measurement of $|f|^2$ and $|L[f]|^2$ assures that unique determination of the signal $f(x)$ within the class of band-limited functions.

The proof of the above condition will be obvious if the fact that the argument of the right hand side of (19) represents all the possible phases by which two signals with the same amplitude may differ from each another, is taken into account [1], as well as that the convergence of the series (20) determines the position of the zero places of the integral functions of order one [14]. However, the Condition 1 is too general to construct the function $p(x)$, and next the functions $\alpha(x)$ and $\beta(x)$ fulfilling (19). Therefore, we will formulate below such conditions for $p(x)$ which allow to find the functions $\alpha(x)$ and $\beta(x)$, for which the system (2a,b) has a unique solution.

Condition 2

If $\exp[ip(x)]$ is a periodic function expandable into Fourier series with an infinite number of Fourier-components, then the uniqueness is obtained. Evidently, if

$$e^{ip(x)} = \sum_n a_n e^{2\pi icnx}, \tag{21}$$

then from the eq. (16) it may be shown that the spectrum of the signal $g(x)$ is given by the following formula

$$G(\omega) = \sum_n a_n F^*(-\omega + c \cdot n), \tag{22}$$

where $F(\omega)$ is the spectrum of the signal $f(x)$. If for infinite number of n Fourier-components $a_n \neq 0$, then $G(\omega)$ has a unrestricted support, in other words, $g(x)$ is not a band-limited signal.

An example: It is easy to verify that if

$$a(x) = e^{iax}, \quad \beta(x) = 1, \tag{23}$$

then $p(x) = \frac{2}{a}\cos(a\cdot x)$. Thus $p(x)$ fulfills the condition mentioned above.

However, the optical realization of an operator

$$L[f(x)] = e^{iax}\frac{df(x)}{dx} + f(x), \tag{24}$$

though possible, is very difficult. Therefore, this case will not be discussed further,

Condition 3

If $p(x) = a\cdot x^2$, then the uniqueness is assumed. From eq. (16) it may be shown that the spectrum of the signal $g(x)$ may be expressed by the formula

$$G(\omega) = \left(\frac{\pi}{a}\right)^{1/2} e^{(\pi/4-\pi^2/a\omega^2)} \int_{-\omega_0}^{\omega_0} F^*(-\mu)e^{-i\frac{\pi^2}{a}\mu^2} e^{\frac{2\pi^2 i}{a}\omega\mu} d\mu. \tag{25}$$

The integral in eq. (25) is the Fourier transform of the function of limited support. thus $G(\omega)$ cannot have a limited support.

An example: The operator

$$L[f(x)] = i\frac{df(x)}{dx} + axf(x) \tag{26}$$

satisfies of the Condition 3. However, the optical realization of such an operator seems to be impossible. Therefore, this case will not be discussed either.

Condition 4

We assume that the cut-off frequency ω_0 of the signal $f(x)$ is known, i.e. it is known that

$$|F(\omega)| = 0 \text{ for } |\omega| > \omega_0, \tag{27}$$

then if $p(x) = 2\pi ax$, and if

$$|a| > 2\omega_0 \tag{28}$$

the system (2a,b) has a unique solution in the class of functions satisfying the condition (27) for the fixed value of ω_0.

From the eq. (16) it may be shown that

$$G(\omega) = F^*(-\omega+a). \tag{29}$$

Then taking account of (27) and (28) it may be seen that except for the case $F(\omega) \equiv 0$, the function $G(\omega)$ cannot fulfill the condition (27).

An example: Let us assume that $\alpha(x) = -iA/\pi$, $\beta(x) = B$, where A, B are real

Then the system (36) may be represented in the form

$$I_k(\omega_j+\omega_0)-h\sum_{i=1}^{j-1} F(\omega_i)F^*(\omega_i-\omega_j-\omega_0)T_k(\omega_i)T_k^*(\omega_i-\omega_j-\omega_0)$$
$$=\frac{1}{2}h[F(\omega_j)F^*(-\omega_0)T_k(\omega_j)T_k^*(-\omega_0)+F^*(-\omega_j)F(\omega_0)T_k(\omega_0)T_k^*(-\omega_j)], \quad (40)$$

for $k = 1, 2$, and $j = 1, \ldots, N$.

Let us assume that $F(\omega_0) = F_0$, and $F^*(-\omega_0) = -D/F_0$, where F_0 is an arbitrarily chosen complex number different from zero. In the first step, for $j = 1$, the sum on the left-hand side of (40) disappears. Then $F(\omega_1)$ and $F^*(-\omega_1)$ could be calculated from the system of eq. (40) if the values $F(\omega_0)$ and $F^*(-\omega_0)$ were known. Instead of this the following quantities may be calculated:

$$X(\omega_1) = c_1 F(\omega_1), \text{ where } c_1 = \frac{F_0}{F(\omega_0)},$$
$$Y(-\omega_1) = c_2 F^*(-\omega_1), \text{ where } c_2 = -\frac{D}{F_0 F^*(-\omega_0)}, \quad (41)$$

by inserting F_0, and $-D/F_0$ into (40) in the place exact values $F(\omega_0)$ and $F^*(-\omega_0)$. Next the values $X(\omega_1)$, and $Y(-\omega_1)$ are substituted to the sume on the left hand-side of (40), for $h = 2$, instead of the values $F(\omega_1)$ and $F^*(-\omega_1)$. Considering that $c_1 c_2 = 1$ the sum calculated in this way will have an exact value. Then, the following values may be calculated

$$X(\omega_2) = c_1 F(\omega_2),$$
$$Y(-\omega_2) = c_2 F^*(-\omega_2). \quad (42)$$

This procedure is repeated until $j = N$, which results in two sequences of data

$$X(\omega_j) = c_1 F(\omega_j),$$
$$Y(\omega_j) = c_2 F^*(\omega_j), \quad (43)$$

from which $F(\omega_j)$ may be calculated. It suffices to assume that at some point, say at ω_n, the phase is equal to zero, in other words, $F(\omega_n) = |F(\omega_n)|$. Since

$$F(\omega_n) = \sqrt{X(\omega_n)Y(\omega_n)}, \quad (44)$$

the value of $F(\omega_n)$ is known, and we may calculate c_1. Next dividing $X(\omega_j)$ by c_1 we get $F(\omega_j)$. The algorithm discussed can be used for selected transmittances $T_1(\omega)$, and $T_2(\omega)$, if the determinant of the system (40) differs from zero in each step. This determinant is expressed by the formula

$$\det_j = -\frac{1}{4}h^2 D\cdot[T_1(\omega)T_1^*(-\omega_0)T_2(\omega_0)T_2^*(-\omega_j)$$
$$-T_1(\omega_0)T_1^*(-\omega_j)T_2(\omega_j)T_2^*(-\omega_0)]. \quad (45)$$

It may be verified that for case discussed this determinant has the following form

$$\det_j = -D\cdot A\cdot B\cdot h^3\cdot j. \tag{46}$$

Thus, the algorithm may find an application when using the method described in the previous section.

Simulation of the phase recovery procedure with the help of the direct method

The above algorithm was tested for the following distributions of the complex amplitude in the frequency plane:

$$\begin{aligned} F_1(\omega) &= 1+2i,\\ F_2(\omega) &= 1+2\cos 2\pi\omega,\\ F_3(\omega) &= \exp(i\omega^2),\\ F_4(\omega) &= \sin\omega^3+i\cos[\omega\sin\omega],\\ F_5(\omega) &= \cos\pi\omega+i\sin\pi\omega^3. \end{aligned}$$

It has been assumed that the cut-off frequency $\omega_0 = 1$, and the filter parameters are: $A = 0.03$, $B = 0.8$. Such a choice assured the fulfillament of both the uniquenes condition: $|B/A| > 2\omega_0$, and normalized transmittance condition: $|T_2(\omega)| \leqslant 1$, for $|\omega| \leqslant \omega_0$. The quantities $I_1(\omega_j+\omega_0)$, $I_2(\omega_j+\omega_0)$ were calculated directly from the formulae (40), for $N = 50$[1]. Instead of calculating the derivative (38) the exact value of $F(\omega_0)$, and $F^*(-\omega_0)$ were acceptable as the initial values F_0 and $-D/F_0$. In this way the normalizing process of $X(\omega_j)$, and $Y(\omega_j)$ was eliminated. Therefore, the results of reconstruction were "exact", i.e. the undetermined phase factor $\exp(i\cdot c)$ was equal to unity. Such an approach facillitated the interpretation of the results and calculation of errors.

For the cases F_1-F_5 r.m.s. errors were not greater than $10^{-4}\%$. The errors were calculated from the formula

$$\varepsilon = \Big[\sum_{j=1}^{N} |F(\omega_j)-F^r(\omega_j)|^2 / \sum_{j=1}^{N} |F(\omega_j)|^2\Big]^{1/2}\, 100\%, \tag{47}$$

where $F^r(\omega_j)$ is the recovered value of the function $F(\omega)$ at the point ω_j.

Also the recovery error of the real part was estimated at the point ω_j by using the formula

$$Re(\varepsilon_j) = \frac{|ReF(\omega_j)-ReF^r(\omega_j)|}{|ReF(\omega_j)|}\cdot 100\%. \tag{48}$$

The error of the imaginary part $Im(\varepsilon_j)$ was calculated similarly. In this case the real

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[1] All the calculations were made on a Odra 1305 computer in Fortran 1900, by using the single-precision procedure.

parts in the formula (48) ought to be replaced by the imaginary parts. In fig. 1 the errors $Re(\varepsilon_j)$ at the consecutive points $\omega_j(\omega_j > \omega_{j+1})$ are presented in a logarithmic scale, the graph of $Re\ F(\omega)$ being shown in a linear scale, for the case of F_5. For the other cases as well as for the errors $Im(\varepsilon)$ the dependences were similar. However,

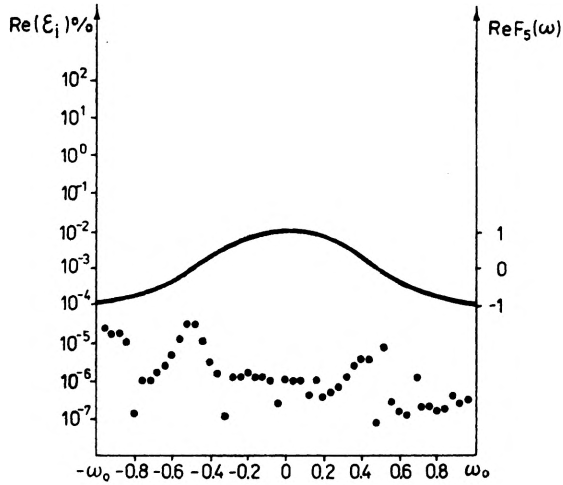

Fig. 1

it has been noted that ε increases with the increase of N. It happens that this algorithm cumulates the errors due to rounding the data by the computer which results in increasing of $Re(\varepsilon_j)$, and $Im(\varepsilon_j)$ in subsequent steps. There are some complex amplitude distributions, in which the errors cumulate very quickly. For instance for

$$F_6(\omega) = 1+2\sin 2\pi\omega, \quad \varepsilon = 30\%,$$

and for

$$F_7(\omega) = \cos 10\pi\omega + i\sin(\omega^2 + 1), \quad \varepsilon = 2\%.$$

This cumulation is distinctly seen in figs. 2 and 3, where the relations $Re(\varepsilon_j)$ and $Re\ F(\omega)$ are shown for F_6 and F_7, respectively. From the graphs it follows that at each 6 steps on average the errors increase by one order of magnitude. It has been also stated that the errors of reconstruction depend weakly on the values A, and B and also that these errors do not depend on the form of the function $T_2(\omega)$. For instance, for $T_2(\omega) = \exp(i\omega^2)$ or $T_2(\omega) = \exp(i\cos 2\omega)$ the errors for the functions 1–5 were also small, while for the functions 6–7 they were great but by one or one half order of magnitude smaller than for the discussed differential operator.

Summarizing remarks

A unique recovery of the phase is possible if a measurement of the respective two intensity distributions is made. If the signal measured in the second measurement is obtained by the differentiation and addition operators the signal measured in the

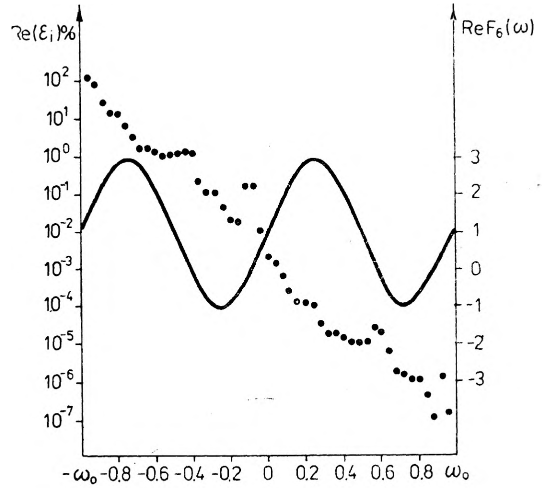

Fig. 2

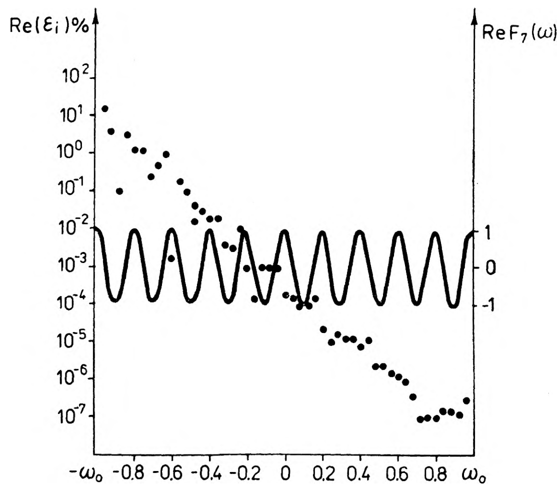

Fig. 3

first measurement according to (26), then uniqueness of the phase reconstruction may be assured in several ways (Conditions 2, 3, 4). The operator (31) seems to be of the greatest importance due to its easy realization. It sufficed to insert a transparency of transmittance $T(\omega) = 2A\omega+B$ (provided that $|B/A| > 2\omega_0$) in the frequency plane before the second measurement. It seems that in the microscopy of coherent light the suggested method may be used as the first step to the complete reconstruction of the microscopic objects. This method is not restricted to signals of weak phase, but may be applied to arbitrary microscopic objects. However, the application of the algorithm described in the section *The phase reconstruction by solving the system of integral equation* (direct method) gives sometimes results suffering from great error. Therefore, it seems that some further efforts should be used to find an algorithm, enabling the solution of the system (35) and having no shortcomings of the direct method.

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Реконструкция фазы из распределений интенсивностей, происходящих от двух комплексных амплитудь полученных с помощью дифференциальных операторов в однородном случае

В работе даны примеры дифференциальных операторов L, которые модифицируют амплитуду полосно-ограниченного сигнала $f(x)$ таким образом, что измерение интенсивностей $|f(x)^2$ и $|L[f(x)]|^2$ обеспечивает однозначную реконструкцию сигнала $f(x)$. Подробно исследован оператор, который может осуществляться посредством помещения в выходном зрачке оптической системы трансмитации $T(\omega) = 2A\omega+B$. Приведеба имитация процесса реконструкции при применении предлагаемого алгоритма в работе [10].