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Effect of a quadratic phase factor on the partially coherent far-field diffraction in the presence of primary astigmatism

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The influence of primary astigmatism has been investigated on the intensity distributions in the Fraunhofer diffraction patterns formed by an optical system with a circular aperture under partially coherent illumination when the mutual coherence function contains a spatially non-stationary quadratic phase term. Results are presented to illustrate the degrading effects of various amounts of astigmatism on the image quality for several values of the coherence interval and the phase parameter. Besinc form of correlation has been assumed for the mutual coherence function. The extent of improvement in the intensity distribution has been illustrated for non-uniformly illuminated aperture in case of typical apodising functions.

Introduction

It is now well known that the cases of complete coherence or incoherence normally do not occur in practice. Hence considerable significance is attached to the studies dealing with the diffraction in optical system under partially coherent illumination. Numerous investigations were carried out in the past [1-3] concerning the subject and we refer to a comprehensive bibliography [4] for references.

Fraunhofer and Fresnel diffraction patterns have been investigated for partially space coherent illumination for aperture with different shapes, apodizing filters and aberrations. Defocussing and spherical aberration have received considerable attention. Off-axis aberrations, such as coma and astigmatism become of considerable importance while designing the optical systems for large field of view. Considerable work has been done in [5-15] concerning the evaluation of images of point, line, edge and infinitely periodic objects in the presence of primary astigmatism. Visual optical systems [16] have been treated recently. Images of bar objects have also been reported [17]. This analysis is tantamount to a study of effect on Fraunhofer diffraction patterns of partially coherent light with sinc correlation. Besinc correlation is assumed for a circular source. We have, therefore, provided the results of the intensity distribution in the diffraction pattern under partially coherent illumination in the presence of primary astigmatism. The results for coma have already been reported [18].

In most of the studies, except a few [19-22], the illumination across the aperture is usually assumed to be real and spatially stationary. However, it is evident from the van Cittert-Zernike theorem [23] that a spatially non-stationary quadratic phase term appears in the complex degree of coherence for the field illuminated by an extended incoherent source. The effect of this quadratic phase term has been studied on the Fraunhofer [19], and Fraunhofer and Fresnel diffraction patterns [21, 22] for slit and circular apertures. It has been shown that the effect of phase on the diffraction patterns becomes quite significant in certain cases, especially when the source size becomes small. Recent results of investigations of ZAJĄC [24] come out to be the same as those of SHORE [19] and ASAKURA [21]. We have included the effect of phase term in the complex degree of coherence to calculate the irradiance in the Fraunhofer diffraction patterns of astigmatic circular aperture.

It is also well known that there are techniques for modifying the imaging properties of an optical system by manipulating its pupil function [25-28]. Optimum apodisers are used in optical instruments as filters that maximize the concentration of energy within a circle of arbitrary fixed radius concentric with the diffraction pattern. The design of optimum apodisers was initially based on the assumption that the diffracting aperture receives coherent illumination and is free from all aberrations. In practical situations, however, these ideal conditions are hardly achieved. SOM and BISWAS [27] have shown that the performance of optimum apodisers in partially coherent illumination may be quite unexpected and contrary to their predicted performance. The presence of aberrations in optical systems also markedly influences the performance of apodisers. Results have been reported [29-31] on the improvement of the optical transfer function in the presence of defocussing and astigmatism. Recently, BISWAS [32] has investigated the influence of spherical aberration on the performance of apodisers. BISWAS and BOIVIN [14] have studied the performance of optimum apodisers using Zernike-circle polynomials in the presence of primary astigmatism for coherent case. Our studies generalize some of these results in partially coherent light for the case of a system with astigmatism.

Theory

The calculation of the far field diffraction pattern of a plane aperture illuminated with quasi-monochromatic partially space-coherent radiation is facilitated by using the Fourier-transform relation [1, 19, 22]:

$$I(P) = \frac{A\cos^2\theta}{\lambda^2 R^2}\int_\sigma \gamma(\mathbf{S})\,C_0(\mathbf{S})\exp(\mathrm{i}k\hat{p}\cdot\mathbf{S}\sin\theta)\,d\sigma, \tag{1}$$

where A is the area of the aperture, λ is the mean wavelength and $k = 2\pi/\lambda$; R, θ, $\hat{p}$ (a unit vector), and $\mathbf{S}$ are defined in fig. 1; $\gamma(\mathbf{S})$ is the

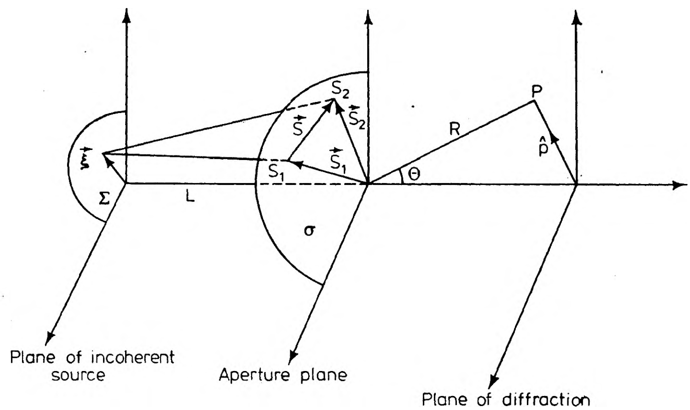

Fig. 1. Coordinate system and illustration of symbols used in various formulae

complex degree of coherence across the aperture, $C_0(\boldsymbol{S})$ is the autocorrelation function of the aperture amplitude and σ is the integration range of S. While deriving the eq. (1), the mutual coherence function across the aperture was assumed to be spatially stationary and to take the following form:

$$\Gamma(\boldsymbol{S}_1, \boldsymbol{S}_2) = I(\boldsymbol{S}_1)^{1/2} I(\boldsymbol{S}_2)^{1/2} \gamma(\boldsymbol{S}_2 - \boldsymbol{S}_1) = I(\boldsymbol{S}_1)^{1/2} I(\boldsymbol{S}_2)^{1/2} \gamma(\boldsymbol{S}). \quad (2)$$

However, the van Cittert-Zernike theorem [23] states that the complex degree of coherence $\gamma(\boldsymbol{S}_1, \boldsymbol{S}_2)$ for the field illuminated by an extended incoherent quasi-monochromatic light source is usually spatially non stationary and takes the form

$$\gamma(\boldsymbol{S}_1, \boldsymbol{S}_2) = \frac{e^{i\Psi'} \int\limits_{\Sigma} I(\xi) \exp[-ik\xi(\boldsymbol{S}_1 - \boldsymbol{S}_2)] d\Sigma}{\int\limits_{\Sigma} I(\xi) d\Sigma}, \quad (3)$$

where

$$\Psi' = k(|\boldsymbol{S}_1|^2 - |\boldsymbol{S}_2|^2)/2L, \quad (4)$$

$\boldsymbol{S}_1$, $\boldsymbol{S}_2$, ξ and L are defined in fig. 1. Because of the spatially non-stationary quadratic phase term $e^{i\Psi'}$, eq. (1) cannot be directly evaluated. However, by some straight forward mathematical manipulation it is found [19, 22] that the quadratic phase factor is simply incorporated into the auto-correlation function of the source amplitude.

The complex degree of coherence can be written in the form

$$\gamma(\boldsymbol{S}_1, \boldsymbol{S}_2) = J(\boldsymbol{S}_1) J^*(\boldsymbol{S}_2) U(\boldsymbol{S}_2 - \boldsymbol{S}_1), \quad (5)$$

where

$$J(\boldsymbol{S}) = \exp(ik|\boldsymbol{S}|^2/2L), \quad (5b)$$

and

$$U(S_2 - S_1) = U(S) = \frac{\int_{\Sigma} I(\xi) \exp\left[-\frac{k}{L}\,\xi(S_1 - S_2)\right] d\Sigma}{\int_{\Sigma} I(\xi)\, d\Sigma}. \tag{5c}$$

Substitution of eqs. (5a) and (5b) in (3) and insertion of resultant equation into the eq. (1), yield

$$I(P) = \frac{A \cos^2\theta}{\lambda^2 R^2} \int_{\sigma} C'(S) U(S) \exp[ik(\hat{p}\cdot S \sin\theta] d\sigma, \tag{6}$$

where

$$C'(S) = \frac{1}{A} \int_{\sigma'} I(S_1) J(S_1) I^*(S+S_1) J^*(S+S_1) d\sigma'. \tag{7}$$

In eq. (7) σ' is the portion of the aperture to which S_1 is restricted in order that (S_1+S) lie on the aperture. For aperture radius $a' \gg \lambda$ and small diffraction angle, i.e. $\cos\theta \approx 1$, and $\sin\theta \approx \theta$, diffraction of circular aperture, the eq. (6) becomes

$$I(v,\,\psi) = \left[\frac{\pi a'^2}{\lambda^2}\right] \int_0^{2a'} \varrho' d\varrho' \int_0^{2\pi} d\Phi\, U(\varrho') C'(\varrho',\,\Phi) \exp[iv\varrho' \cos(\Phi-\psi)] \tag{8}$$

under far field approximation. Here $\varrho' = |S_2 - S_1|$, and $v = ka'\theta$. Putting the normalized distance $\varrho'/a' = \varrho$, eq. (8) for normalized intensity is reduced to

$$I(v,\,\psi) = \frac{1}{\pi} \int_0^2 \int_0^{2\pi} \varrho\, d\varrho\, d\Phi \left[\frac{2J_1(a\varrho)}{a\varrho}\right] C(\varrho,\,\Phi) \exp[iv\varrho\cos(\Phi-\psi)]. \tag{9}$$

Here $\dfrac{2J_1(a\varrho)}{a\varrho} = U(\varrho)$ is the besinc form of correlation given by a quasi-monochromatic incoherent circular source, $a = ka'r'/L$ the number of correlation intervals contained in the aperture, r' is the radius of the source.

The equation (9) will now be applied to calculate the far-field intensity distribution for the following cases:

i) In the presence of primary astigmatism when the quadratic phase term is neglected the auto-correlation function of aperture amplitude distribution is the incoherent transfer function for a system suffering from primary astigmatism. It has been calculated by DE [6], and is given by

$$C(\varrho,\,\Phi) = [C(0,\,0)^{-1} \int_{-a}^{a} \int_{-b}^{b} \exp\left[ik\left\{W\left(x + \frac{1}{2}\,\varrho,\,y\right) - W\left(x - \frac{1}{2}\,\varrho,\,y\right)\right\}\right] dx\,dy, \tag{10}$$

W is the aberration polynomial and in the present case is given by

$$W = W_{20}(x_0^2+y_0^2)+W_{22}y_0^2, \tag{11}$$

which — on the rotation of axes [6] — reduces to

$$W = W_{20}(x^2+y^2)+W_{22}(x^2\sin^2\Phi+xy\sin 2\Phi+y^2\cos^2\Phi). \tag{12}$$

Substituting eq. (12) in eq. (10) we get

$$C(\varrho,\Phi) = [C(0,0)]^{-1}\int_{-a}^{a}\int_{-b}^{b}\exp[i(px+qy)]dxdy \tag{13}$$

with

$$p = 2k\varrho[W_{20}+W_{22}\sin\Phi], \tag{14}$$

$$q = k\varrho[W_{22}\sin\Phi], \tag{15}$$

$$a = \sqrt{1-\varrho^2/4}, \quad b = \sqrt{1-y^2}-\varrho/2 \tag{16}$$

W_{20} and W_{22} here represent the amounts of defocussing and astigmatism, respectively for the mid focal plane $W_{20} = -1/2\, W_{22}$.

ii) In the presence of astigmatism , ut when the quadratic phase term is preserved, the eq. (7) becomes

$$C(\varrho,\Phi) = [C'(0,0)]^{-1}\int_{-a}^{a}\int_{-b}^{b}\exp[i(Y\varrho x+px+qy)]dxdy, \tag{17}$$

where $Y = ka'^2/2L$ is the phase parameter.

iii) To evaluate the modified intensity distribution, due to nonuniformly illuminated astigmatic circular aperture, the amplitude transmittances for the non-uniform pupil are assumed to be circularly symmetric and mathematically represented as:

$$\text{(I)}\ f(r) = (1-\beta r^2), \tag{18}$$

$$\text{(II)}\ f(r) = 0.181+0.426(1-r^2)+0.257(1-r^2)^2+0.136(1-r^2)^3. \tag{19}$$

For the case (I) the auto-correlation of the modified aperture amplitude distribution (quadratic phase factor in eq. (7) is neglected) is written [31] as

$$C(\varrho,\Phi) = [C(0,0)]^{-1}\int_{-a}^{a}\int_{-b}^{b}\left[1-\beta\left(x-\frac{1}{2}\varrho\right)^2-\beta y^2\right]\left[1-\beta\left(x+\frac{1}{2}\varrho\right)^2-\beta y^2\right]\exp[i(px+qy)]dxdy \tag{20}$$

here β determines the extent of non-uniformity of illumination.

Case (II) represents the optimum Straubel apodiser for the apodized interval $3.8317 \leqslant v < \alpha$. The auto-correlation of the aperture amplitude distribution can be derived in a straight forward manner like eq. (20).

Results for coherent case have also been verified by evaluating the point spread function directly

$$t(v, \psi) = D\left|\int_0^1\int_0^{2\pi} \exp[iv\varrho_1\cos(\theta_1-\psi) - ik(W_{22}\varrho_1^2\cos^2\theta_1 + W_{20}\varrho_1^2)]\varrho_1 d\varrho_1 d\theta_1\right|^2,$$

where D is a constant, (ϱ_1, θ_1) and (v, ψ) are polar co-ordinates in aperture and image plane, respectively. W_{20} represents the plane of defocussing; and for $W_{20} = 0, -\frac{1}{2} W_{22}, -W_{22}$ we obtain the intensity distribution in the sagittal (Gaussian), mid focal and tangential focal planes, respectively.

Results and discussions

The equation (10) was first evaluated for the mid focal plane by using 20 point Gauss quadrature method, the quadratic phase term being neglected. Results obtained agree with those of DE [6]. This eq. (10) was then evaluated after the quadratic phase term was included. The results were obtained by a suitably manipulation of the aberration polynomial for the following two cases (i): when the tangential plane is towards the aperture plane (i.e. positive astigmatism), (ii) when the sagittal focal plane is towards the aperture (i.e. negative astigmatism). It is found that in the above two cases for certain combination of Y and N (e.g. $N = 1.0$, $Y = 1.0$) we get results for transfer function in tangential and sagittal planes. These agree with the corresponding results of KAPANY and BURKE [10]. The results for transfer function so obtained were fed into eq. (9) to evaluate the intensity distribution in mid focal plane for the above two cases by using 40-point Gauss quadrature method.

The figures 2-9 show the computed results along $\psi = 0$ for the Fraunhofer diffraction patterns for various combinations of the phase parameter Y and the coherence parameter a. Two values of the aberration coefficient $W_{22} = N\lambda/\pi$ are taken with $N = 1.0$ and 2.0. The curves for $a = 0.0$ represent the case of point source illumination of circle aperture, so that the magnitude of the complex degree of coherence is unity and only the effect of quadratic phase factor is observed. The curve for $a = 0$ with $Y = 0{,}0$, and $N = 1.0$ (shown by dotted line in fig. 2) is in agreement with the results of NIENHUIS and NIJBOER [5]. Results were also obtained for aberration free case for different values of Y and a and these agree with the results of SHORE [19].

It is clear from figs. 2-9 that, like in aberration free case, the main effect of the phase term is to reduce the central intensity and decrease the fringe contrast for small values of Y. As Y increases beyond 3, the

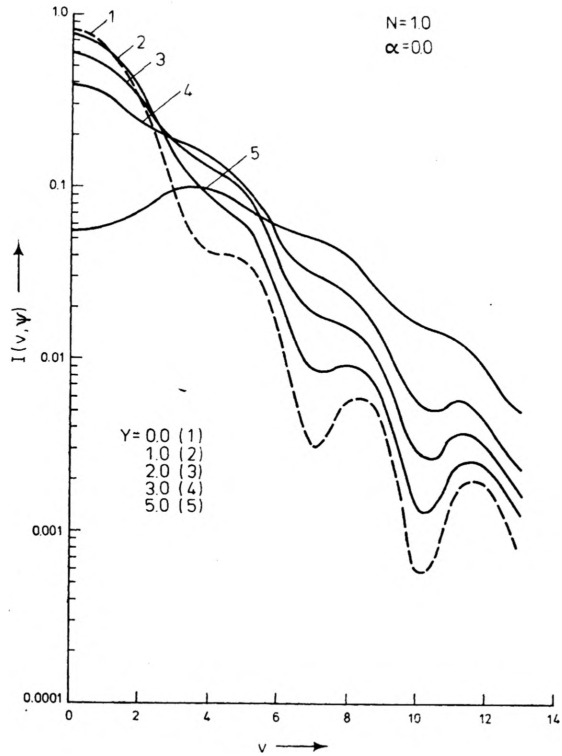

Fig. 2. Effect of different amounts of phase parameters on the intensity distribution in the far-field diffraction patterns. For case (i), i.e. when the tangentia focus is towards the aperture plane, for N = 1.0, ψ = 0.0, a = 0.0

main beam splits and a minimum occurs at $v = 0$. The central minimum is due to the fact that contributions from the centre of the aperture and its edge are 180° out of phase. It is also seen that with the increase in a the effect of the phase term becomes less pronounced. It is also interesting to note that the effect of including the quadratic phase term is tantamount to obtaining the results in the plane drifted away from the mid focal plane by an amount corresponding to different values of N and Y. For example the curves for $N = 1.0$ and $Y = 1.0$ represent the intensity distribution in the tangential plane for various values of a (figs. 2-5). Other combinations of Y and N represent the result for the plane between mid focal plane and the tangential plane (e.g. for $N = 2.0$, $Y = 1.0$) or for planes beyond the tangential plane (e.g. for $N = 1.0$, $Y = 3.0$). These results were also verified for the coherent case (i.e. $a = 0.0$.) by evaluating directly the point spread function in different planes.

The figures 10-17 show the results for the intensity distribution for case (ii), i.e. when the sagittal plane is towards the pupil plane. It is observed that the effect of the phase term on the trend of the curves is not

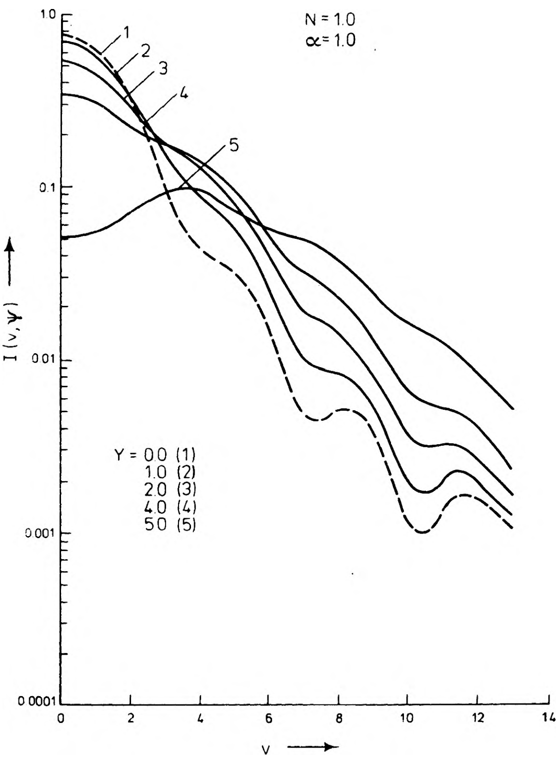

Fig. 3. Same as fig. 2, but for $\alpha = 1.0$

the same as discussed previously for case (i). Here, though the central intensity is decreased by the same amount, curves become sharper for different values of Y. This can be explained on the basis of the fact that in the present case we are moving away from the mid focal plane towards the sagittal plane and the curve for $\alpha = 0.0$ with $N = 1.0$, and $Y = 1.0$ represents the intensity distribution in the point image in the sagittal plane. Other combinations of Y and N represent the results for the planes between the mid focal plane and sagittal plane (e.g. $N = 2.0$, and $Y = 1.0$) or beyond the sagittal plane (e.g. $N = 1.0$ and $Y = 3.0$). For coherent case the results were again verified for different planes by evaluating directly the point spread function.

Therefore, it is concluded that the effect of including the phase term is to drift away from the mid plane and consequently decrease in the central intensity especially for large values of Y. The effect of the quadratic phase term becomes less important as the coherence decreases, i.e. α increases.

The figure 18 shows a comparative study of the intensity distribution in the presence of astigmatism without apodiser, with $(1 - r^2)$ (β is taken to be unity) type apodiser and with optimum Straubel apodiser. Perform-

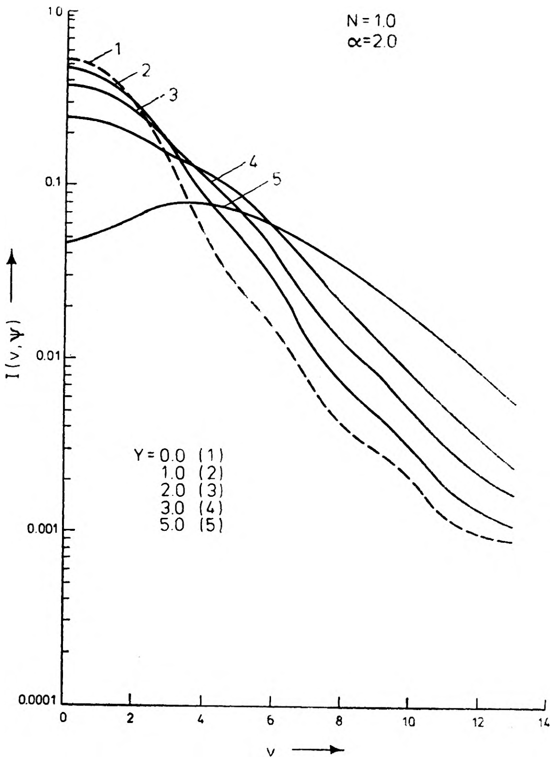

Fig. 4. Same as fig. 2, but for $\alpha = 2.0$.

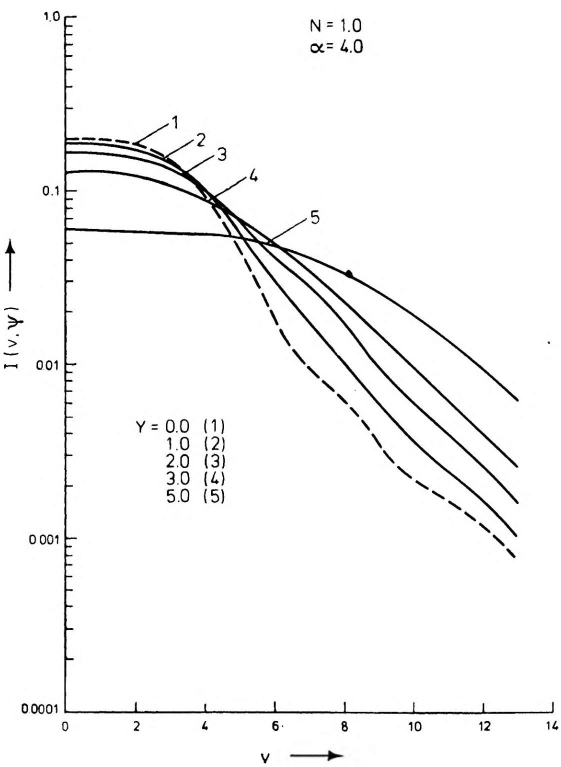

Fig. 5. Same as fig. 2, but for $\alpha = 4.0$

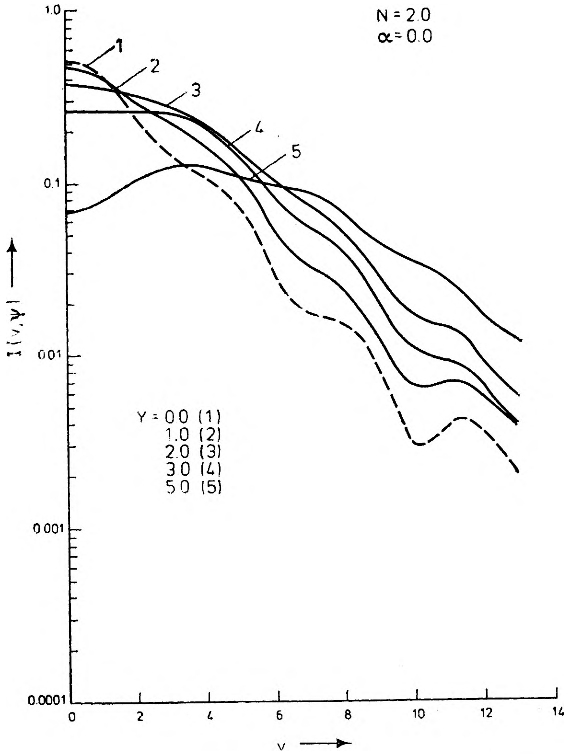

Fig. 6. Same as fig. 2, but for N = 2.0, α = 0.0

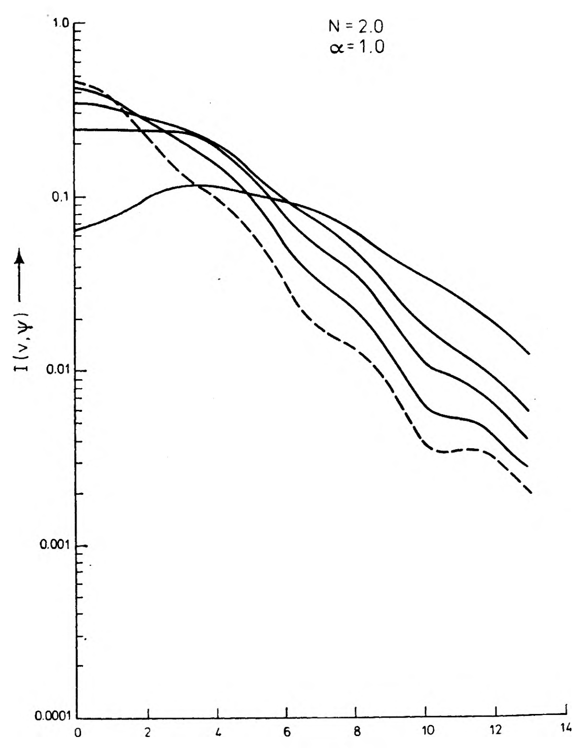

Fig. 7. Same as fig. 6, but for α = 1.0

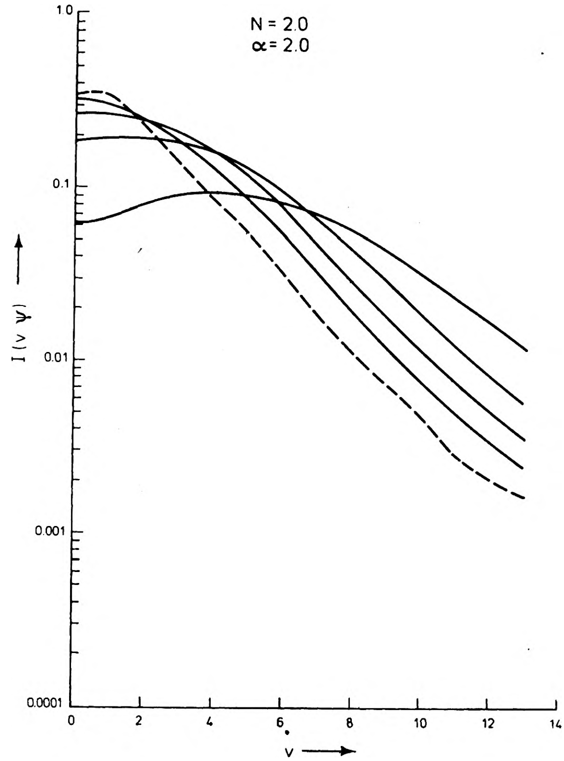

Fig. 8. Same as fig. 6, but for $\alpha = 2.0$

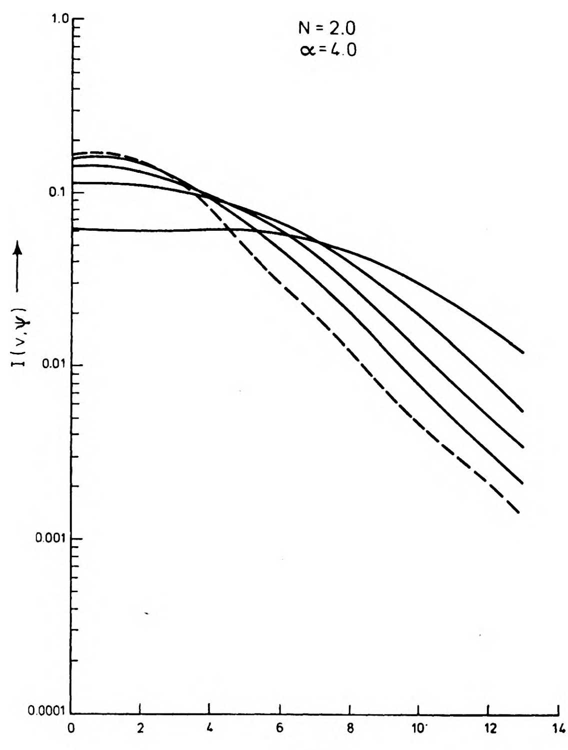

Fig. 9. Same as fig. 6, but for $\alpha = 4.0$

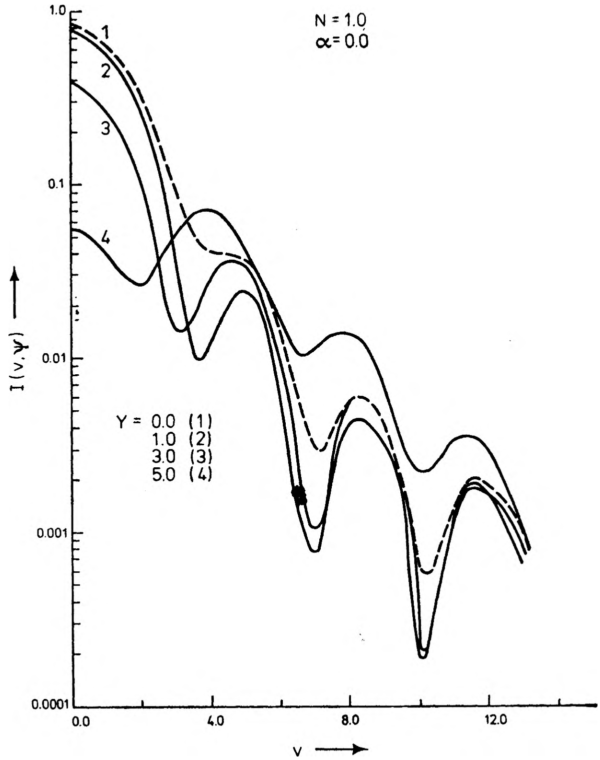

Fig. 10. Same as fig. 2, but for case (ii), when the sagittal focus is towards the aperture plane

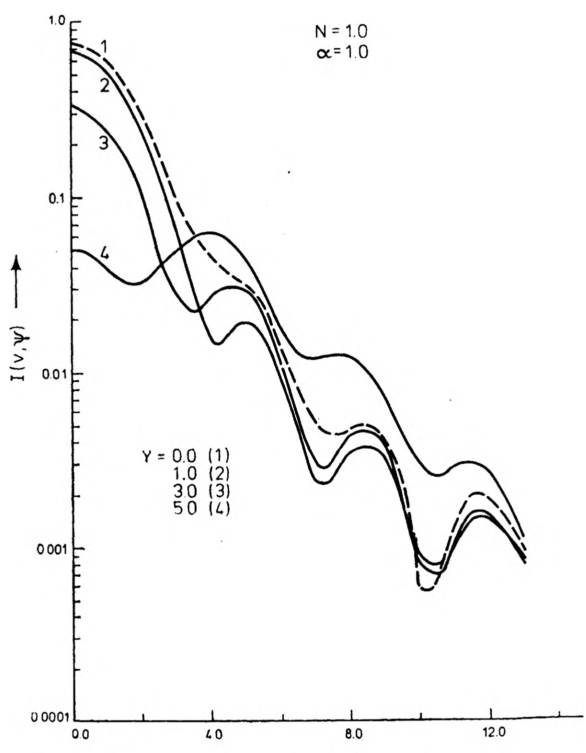

Fig. 11. Same as fig. 10, but for $a = 1.0$

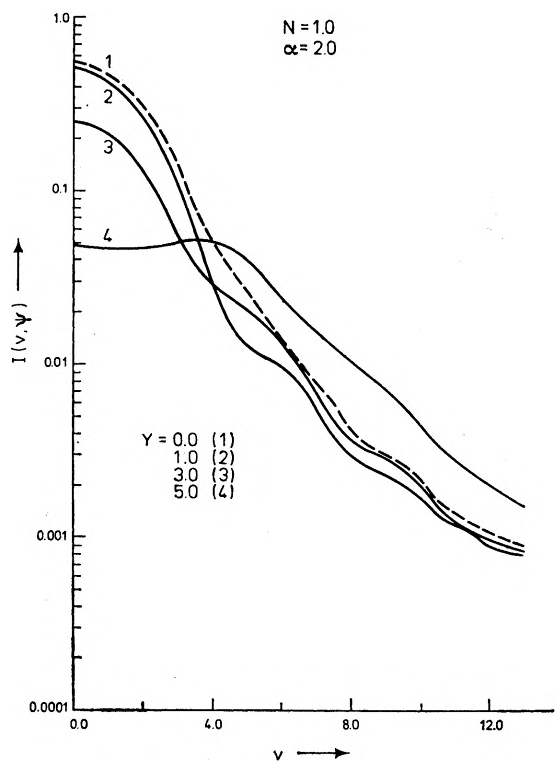

Fig. 12. Same as fig. 10, but for $\alpha = 2.0$

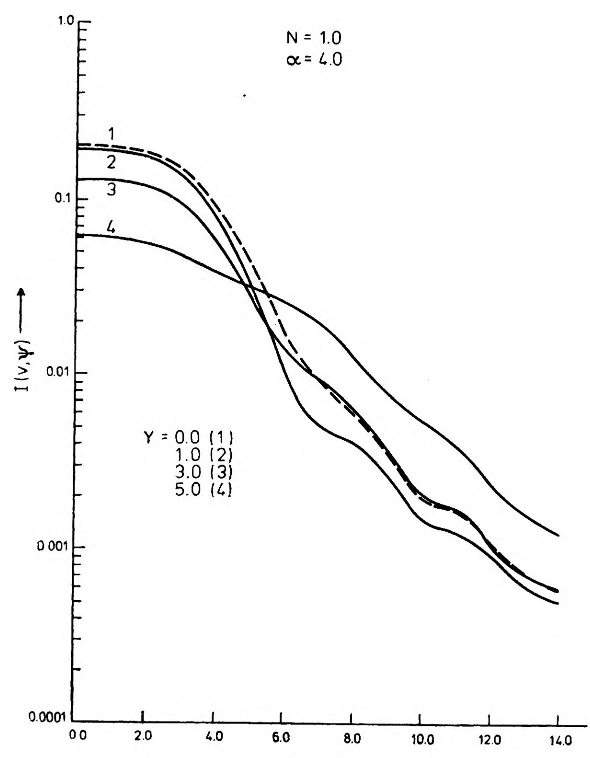

Fig. 13. Same as fig. 10 but for $\alpha = 4.0$

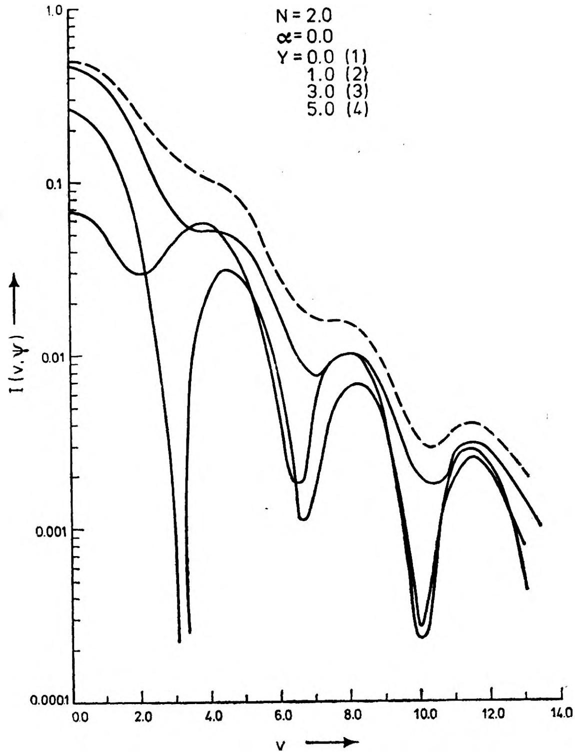

Fig. 14. Same as fig. 10 but for $N = 2.0$, $\alpha = 0.0$

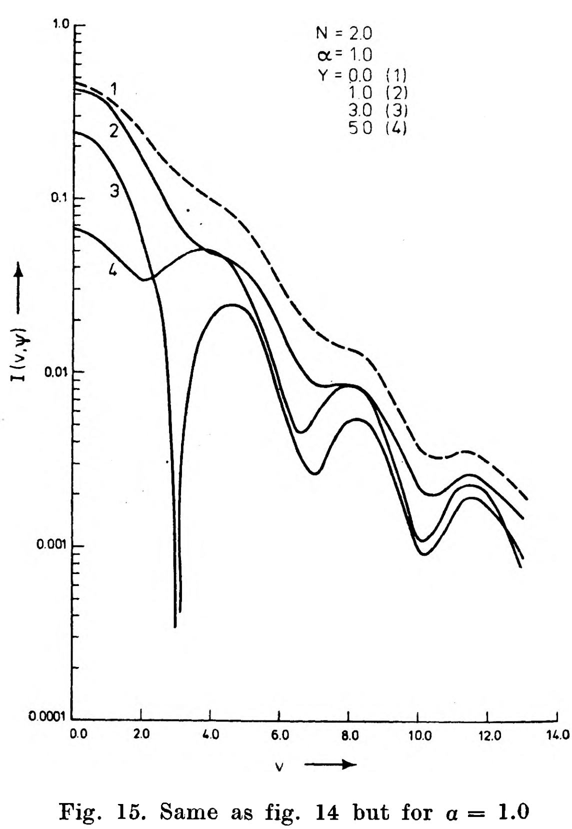

Fig. 15. Same as fig. 14 but for $\alpha = 1.0$

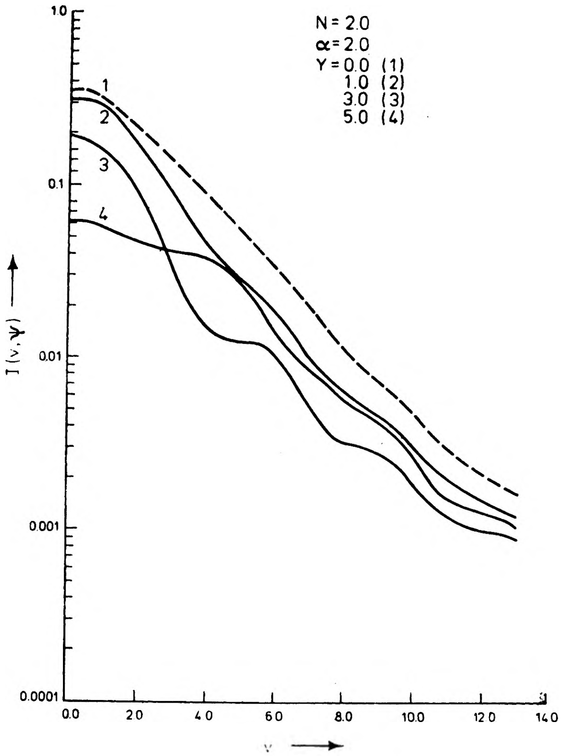

Fig. 16. Same as fig. 14, but for $\alpha = 2.0$

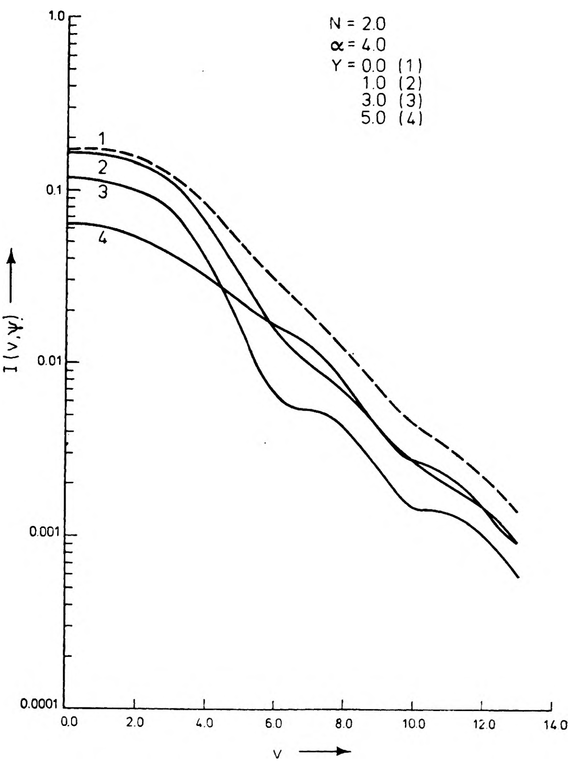

Fig. 17. Same as fig. 14, but for $\alpha = 4.0$

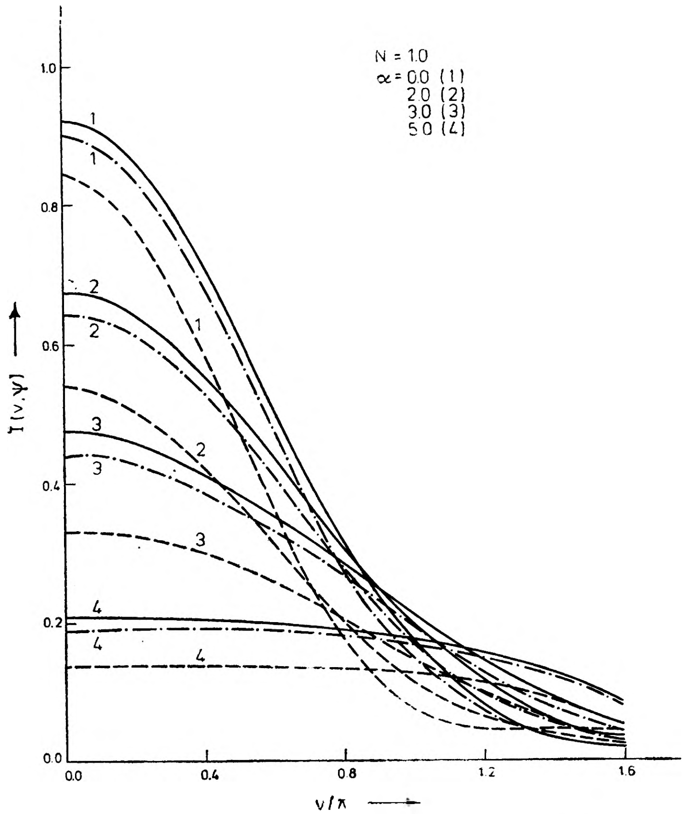

Fig. 18. Modified intensity distribution for $N = 1.0$, along $\psi = 0.0$ due to (i) shaded aperture (—), (ii) Straunel apodiser (— · — · —), (iii) uniform illumination (— — — —)

ance of $(1-r^2)$ type apodiser is slightly better as compared to that for Straubel apodiser. Our results for coherent case (i.e. $\alpha = 0.0$) agree with those of Biswas and Boivin [14] for various amounts of aberrations for Straubel apodiser. The effect of apodisation is less pronounced in the presence of aberration.

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Влияние коэффициента квадратной фазы на дифракцию далёкого поля в частично когерентном свете при наличии первичного астигматизма

Исследовано влияние первичного астигматизма на распределение интенсивности в дифракционном спектре Фраунхофера в оптической системе с круговой диафрагмой при частично когезионном освещении, когда функция взаимной когерентности содержит член пространственно квадратной фазы. Приведены результаты, иллюстрирующие влияние деградации при различных значениях астигматизма на качество изображения для нескольких значений интервала когеренции, а также фазового параметра. Для функции взаимной когерентности принят вид корреляции типа Бесинк. Степень улучшения распределения интенсивности проиллюстрирован для неравномерно освещённых отверстий в случае типичных аподазационных функций.