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Giant absorption of light in quasi-zero-dimensional structures

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We present a theory of the electromagnetic field interaction with one-particle localized charged states emerging near the small spherical semiconductor or dielectric particles of radius a placed in dielectric matrices. In the framework of the dipole approximation we predict theoretically giant magnitudes of: 1) transitions oscillator strengths, and 2) the resonance light absorption cross-section $\sigma_{abs}(\omega, a)$. We suggest that these peculiarities can be observed experimentally in the systems under consideration. We also discuss the dependence of $\sigma_{abs}(\omega, a)$ and the scattering cross-section $\sigma_{sc}(\omega, a)$ on light frequency and radius a of one-particle localized excitations under different physical circumstances.

1. Introduction

Optical properties of quasi-zero-dimensional structures (QZDS) which form systems consisting of small dielectric or semiconducting spherical particles of diameter a, being of the order of $1-10$ nanometers, distributed in dielectric material matrices [1]–[5] are the subject of intensive studies at the present time.

Large nonlinearity of the optical characteristics and short lifetimes of the photoexcited charge carriers in QZDS allow us to treat such heterophase systems as prospective materials for the production of new optic and optoelectronic devices controlling, in particular, light signals.

Linear dimension a of the semiconducting particles, the subject of our studies, will be comparable with the linear dimension of the characteristic quasiparticles in semiconductors. Under this condition the effect of the spherical boundary between the two materials, having different dielectric constants (DC), may lead to the size-quantization of carrier energy spectra due to both the pure boundedness in space of the quantization region [4], [5] and to the polarization interaction of charge carriers (CC) with the semiconducting particle surface [6]–[11].

It was proved in [6] that under the defined conditions the localized one-particle states (LOPS) of CC of different kinds might appear. Such states are, for example,

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the localized states of charge carriers in semiconductors near semiconductor or dielectric particle, called the localized surface states [7], [8], or the localization of CC inside the small spherical volume, called the volume localized states. The surface localization may appear near an outer or inner spherical interface between the two dielectric media, being called the outer surface states [7], [8] or the inner surface states [9], respectively.

At present there is available a sufficient amount of experimental data confirming the existence of LOPS's under consideration. For example, the localization of CC and excitons have been studied in the copper chloride [4] and cadmium sulphate [5], [12] small particles as well as in isoelectronic solid state solutions $A^{II}B^{VI}$ [13], [14]. In addition, the localization of the charge carriers on the antimony particle in n-type germanium and on the mercury particle placed in the dense mercury vapour have been observed in [15] and [16], respectively.

Meanwhile, the behaviour of QZDS placed in the field of light has not been practically studied theoretically so far. The main task of this paper is to develop a theory of light absorption and scattering on the one-particle local states in the quasi-zero-dimensional systems.

The paper is organized as follows. In Section 2 we briefly review the various types of LOPS and their energy spectra. In Section 3, we calculate the transition moments for the charge carriers using the dipole approximation. The light absorption and scattering in the systems considered are studied in Section 4. Finally, we compare the results obtained with the existing experimental data.

2. One-particle local states of charge carriers

The problem of the CC localization near the spherical interface between two dielectric media has been analyzed in [6] using the following simple model of QZDS: a neutral spherical dielectric particle (DP) of diameter a and dielectric permittivity (DPr) ε_2 is surrounded by medium with DPr ε_1, and a quasiparticle with charge e is moving either in the medium with DPr ε_1 and effective mass m_1 near the interface surface (the outer problem, OP) or in the medium with DPr ε_2 and effective mass m_2 inside spherical volume (the inner problem, IP). In addition, the polarization interaction $U(\vec{r}, a)$ ($\vec{r}$ denotes the distance between a charge carrier and the DP center) with the surface charges induced on the spherical interface has been taken into account. We point out that the magnitude of $U(\vec{r}, a)$ depends on the relative dielectric permittivity $\varepsilon = \varepsilon_1/\varepsilon_2$.

Let us write down the expression $U(\vec{r}, a)$ for the interaction of the charge carrier with the surface of dielectric particle [6] in the following forms:

– for the OP ($r > a$)

$$U(r,a) = \frac{e^2\beta}{2\varepsilon_1 a}\left[\frac{a^2}{r^2-a^2} - \left(\frac{a}{r}\right)^2 F\left(1,\alpha;\alpha+1;\left(\frac{a}{r}\right)^2\right)\right], \tag{1}$$

– for IP ($r < a$)

$$U(r,a) = -\frac{e^2\beta}{2\varepsilon_2 a}\left[\frac{a^2}{a^2-r^2}+\frac{1-\alpha}{\alpha}F\left(1,\alpha;\alpha+1;\left(\frac{r}{a}\right)^2\right]\right. \tag{2}$$

where $F(1;\alpha;\alpha+1;z^2)$ is the Gauss hypergeometric function, $\alpha = \varepsilon_1/(\varepsilon_1+\varepsilon_2)$ and $\beta = (\varepsilon_1-\varepsilon_2)/(\varepsilon_2+\varepsilon_2)$.

There are two possibilities of the charge carriers moving near the dielectric particle:

1. The polarization interaction $U(r,a)$ (*cf.* Eqs. (1) and (2)) leads to attraction of CC by the dielectric particle surface and formation of the outer (for $\varepsilon < 1$ [7], [8], *cf.* Fig. 1) and inner (for $\varepsilon > 1$ [9], *cf.* Fig. 2) surface states.

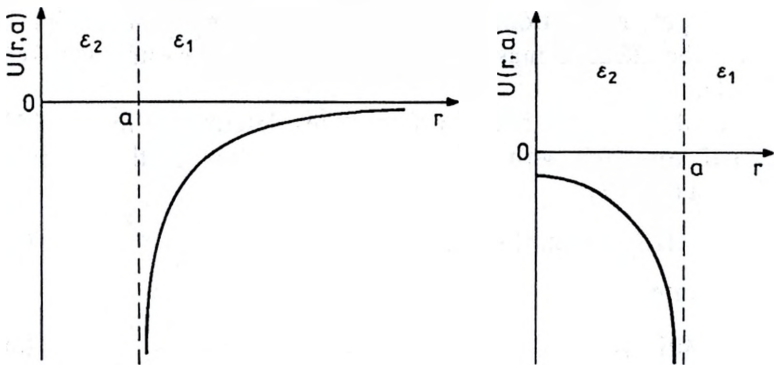

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Fig. 1. Potential $U(r,a)$ versus r for $\varepsilon = \varepsilon_1/\varepsilon_2 < 1$.

Fig. 2. Potential $U(r,a)$ versus r for $\varepsilon = \varepsilon_1/\varepsilon_2 > 1$.

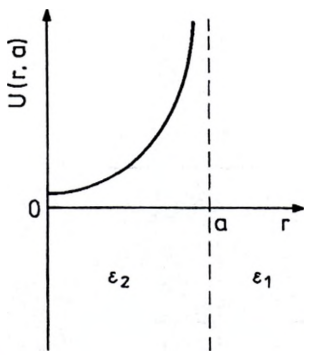

Fig. 3. Potential $U(r,a)$ versus r for the case of volume localized states.

2. For $\varepsilon < 1$ the polarization interaction (2) causes repulsion of CC from the inner surface of dielectric particle and formation of the volume localized states [10], [11] near the center of DP (see Fig. 3). This effect is called the volume localization.

We notice that in [2] the potential $U(r, a)$ for IP has been obtained in the form of infinite series with respect to variable $(r/a) < 1$. This series can be represented by the analytical form (2) derived in [6].

It has been shown in [6]–[11] that with a decrease of the DP diameter a, there appears the size-quantization effect, preventing CC localization because of the relative diminution of the potential energy (1) or (2) in comparison with the kinetic energy. The smallest critical DP diameter a_c, for which the local state is emerging, is approximately equal to b_i denoting the average distance of charge carrier localized in the ground state over the flat interface surface

$$a_c \sim b_i = \frac{6}{|\beta|} a_{Bi} \tag{3}$$

where $a_{Bi} = \varepsilon_i(m_0/m_i)(\hbar^2/m_0 e^2)$ is the Bohr diameter of CC in the medium with ε_i $(i = 1, 2)$; m_i and m_0 are the effective masses in the medium with ε_i, and the free carrier mass, respectively.

The energy spectrum of inner and outer surface states determined in [7], [9] employed the variational approach with the variational function of the forms:

– for inner surface states [9]

$$\chi_l(x, S_2) = A(S_2 - x)^{l+1} x(2S_2 - x)\exp(-\gamma_l(S_2)x), \tag{4}$$

– for outer surface states [7]

$$\chi_l(x, S_1) = Bx(S_1 + x)\exp(-\mu_l(S_1)x) \tag{5}$$

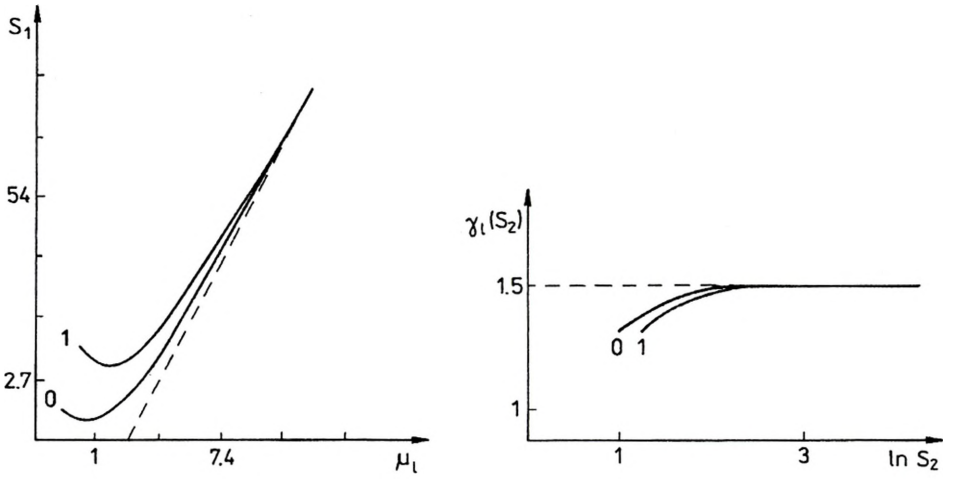

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Fig. 4. Dependence of the variational parameter $\mu_l(S_1)$ on the dielectric particle radius S_1. The curves labelled by 0 and 1 correspond to quantum number values $l = 0$ and $l = 1$, respectively. The dotted line corresponds to $S_1 = (2/3)\mu$.

Fig. 5. Variational parameter $\gamma_l(S_2)$ vs. $\ln S_2$, where S_2 denoted the dielectric particle radius S_2; the depicted numbers specify the magnitude of the quantum number l.

where l denotes the orbital quantum number of CC, $S_i = a/b_i$, x the CC distance from the DP surface in b_i units (*cf.* Eq. (3)). The dependences of the variational parameters $\gamma_l(S_2)$ and $\mu_l(S_1)$ on the radius S_i for $l = 0, 1$ are depicted in Figs. 4 and 5, respectively.

It has been shown in [10], [11] that the lowest-energy spectrum $E_{t,l}(S_2)$ of the CC volume local states in potential (2), for $\varepsilon < 1$ is given by

$$E_{t,l}(S_2) = \omega_l(S_2)(t+3/2),$$

$$\omega_l(S_2) = \frac{2\sqrt{3}}{S_2^{3/2}}\left[\frac{8\beta}{3(1+\alpha)} + \frac{2(7+5\alpha)}{2+\alpha}\sqrt{\frac{\beta L^2}{2S_2(1+\alpha)}} + \frac{(8+11\alpha+5\alpha^2)L^2}{2S_2(2+\alpha)}\right]^{1/2} \tag{6}$$

where: $t = 2n_r + l = 0, 1, 2, \ldots$ is the principal quantum number, $n_r = 0, 1, 2, \ldots$ is the radial quantum number, $\omega_l(S_2)$ denotes the CC vibration frequency measured in the units $R = \hbar^2/(2m_2 a_{B2}^2)$, and $L^2 = l(l+1)$.

In paper [17], the energy spectrum of CC localized near the interface of two different dielectric materials has been calculated theoretically under assumption that dielectric permittivities and potential $U(r, a)$ are continuous at the interface. It can be shown, that inclusion of noncontinuities of DPr's and potential $U(r, a)$ leads to the corrections of order of $\leqslant 7\%$ in spectra $E_{t,l}(s)$ (for $l = 0, 1$) for all CC localized states [7]–[11] mentioned above.

3. Dipole moments of the charge carrier transitions

In the range of frequencies corresponding to the energies of the considered states of CC localized near the spherical interface between two media, the electromagnetic wavelength is much longer than the linear dimension of all local states which are of order of b_i (*cf.* Eq. (3)). Therefore, their behaviour under electromagnetic field action can be well described in the framework of the dipole approximation. Moreover, operators of the dipole moments for the outer and inner problems have the following form [18]:

$$\vec{D}(\vec{r}) = \left[1 - \frac{\beta}{1+\alpha}\left(\frac{a}{r}\right)^3\right]e\vec{r}, \tag{7}$$

$$\vec{D}(\vec{r}) = \frac{3\varepsilon_1}{2\varepsilon_1 + \varepsilon_2}\, e\vec{r}, \tag{8}$$

respectively.

In order to estimate the magnitudes of the dipole moments for CC transitions induced by the external electromagnetic field, we will study the transitions between low-lying local states of all the types considered taking into account the parity of the corresponding states. For example, transitions between states s and p are permitted for the dipole transitions and changes $l \to l \pm 1$ of the orbital quantum number are accompanied by these transitions.

In order to calculate the matrix element of the dipole transition moment $D_{1,0}(S_i)$ for the charge carrier from state s to p, we assume that a monochromatic homogeneous electromagnetic wave $\mathscr{E}(\omega, t)$ is propagating parallel to the $0Z$ axis, where ω denotes the light frequency. As the external perturbation, causing the dipole transitions under study, we take for the OP and IP the dipole moments defined by Eqs. (8) and (9), which are induced by the external electromagnetic field $\mathscr{E}(\omega, t)$. Finally, the matrix elements of the transition dipole moments from state s to p of CC can be written in the form

$$\vec{D}_{1,0}(S_i) = \int_0^{2\pi} \mathrm{d}\varphi \int_0^{\pi} \sin\vartheta \,\mathrm{d}\vartheta \int_{r_1}^{r_2} r^2 \Psi_1(r, S_i) \vec{D}(r) \Psi_0(r, S) \mathrm{d}r \tag{9}$$

where: $\Psi_1(r, S_i)$ and $\Psi_0(r, S_i)$ are the wave functions of charge carrier in state s and p, respectively; ϑ and φ and denote the polar and azimuthal angles, respectively; $r_1 = a$, $r_2 = \infty$ for outer local states and $r_1 = 0$, $r_2 = a$ for inner local states.

In the case of volume states such dipole transitions correspond to the transition between the quantum states with $t = 0$ ($l = 0$) and $t = 1$ ($l = 1$). Using the formulae for wave functions of quantum oscillator and taking into account Eqs. (6), (8) and (9) we can obtain the magnitudes of the matrix elements of dipole transition moments $D_{1,0}$ between states with $t = 0$ and $t = 1$ localized in the dielectric particle center [19]

$$D_{1,0} = \frac{S_2^{3/4} e b_2 \dfrac{3\varepsilon_1}{2\varepsilon_1 + \varepsilon_2}}{3^{7/6} 2^{3/2} \left[\dfrac{8\beta}{3(1+\alpha)} + \dfrac{2(7+5\alpha)}{2+\alpha} \sqrt{\dfrac{\beta}{S_2(1+\alpha)}} + \dfrac{8+11\alpha+5\alpha^2}{S_2(2+\alpha)} \right]^{1/4}}. \tag{10}$$

In order to estimate the magnitudes of the matrix elements of dipole transitions corresponding to the inner $D_{1,0}(S_2)$ and outer $D_{1,0}(S_1)$ surface states we will use the variational wave functions (4) and (5), respectively. Taking into account the formulae (4), (8) and (5), (7), we can get the matrix elements of the dipole transitions (9) between the ground state with $l = 0$ and the first excited states with $l = 1$:

– for inner surface states [19]

$$D_{1,0}(S_2) = \frac{L_2 S_2 e b_2}{3\sqrt{2}},$$

$$L_2 = 2^5 \frac{\sqrt{3}\varepsilon_1}{2\varepsilon_1 + \varepsilon_2} \frac{\tilde{\mu}_0^{7/2} \tilde{\mu}_1^{9/2}}{(\tilde{\mu}_0 + \tilde{\mu}_1)^9} L_2^{(1)} L_2^{(2)} L_2^{(3)} \tag{11}$$

where:

$$L_2^{(1)} = (\tilde{\mu}_0 + \tilde{\mu}_1)^5 - 16(\tilde{\mu}_0 + \tilde{\mu}_1)^4 + 125(\tilde{\mu}_0 + \tilde{\mu}_1)^3 - 570(\tilde{\mu}_0 + \tilde{\mu}_1)^2 + 1470(\tilde{\mu}_0 + \tilde{\mu}_1) - 1680,$$

$$L_2^{(2)} = \frac{1}{\sqrt{8\tilde{\mu}_0^4 - 36\tilde{\mu}_0^3 + 78\tilde{\mu}_0^2 - 90\tilde{\mu} + 45}},$$

$$L_2^{(3)} = \frac{1}{\sqrt{4\tilde{\mu}_1^6 - 30\tilde{\mu}_1^5 + 123\tilde{\mu}_1^4 - 330\tilde{\mu}_1^3 + 585\tilde{\mu}_1^2 - 630\tilde{\mu}_1 + 315}},$$

$$\tilde{\mu}_0 = \gamma_0(S_2)S_2,$$

$$\tilde{\mu}_1 = \gamma_1(S_2)S_2,$$

– for outer surface states

$$D_{1,0}(S_1) = 2^{-1/2} L_1 S_1 e b_1,$$

$$L_1 = 2^{5/2} \frac{(\mu_0\mu_1)^{5/2}}{(\mu_0+\mu_1)^6} \frac{\varepsilon_2-\varepsilon_1}{2\varepsilon_1+\varepsilon_2} \frac{3(\mu_0+\mu_1)^2+12(\mu_0+\mu_1)}{\sqrt{3(\mu_0^2+3\mu_0+3)(\mu_1^2+3\mu_1+3)}}. \tag{12}$$

The variational parameters $\mu_0(S_1)$, $\mu_1(S_1)$, $\gamma_0(S_2)$, $\gamma_1(S_2)$ occurring in the trial wave functions of the inner and outer surface localized states are determined with the help of Fig. 4 and Fig. 5, respectively [7], [9].

We point out that numerical figures occurring in Eqs. (10)–(12) result in calculations of dipole moment transitions (9) using variational or oscillator-type wave functions [20] for the inner (4) and outer (5) surface states or the volume [19] local states, respectively.

Let us note that Eq. (11), defining the dipole transition moment $D_{1,0}(S_2)$ for the inner surface states, is valid provided both inequalities $\exp(-\gamma_1(S_2)) \ll 1$ and $\exp(-\gamma_0(S_2)) \ll 1$ are satisfied. These conditions are fulfilled for arbitrary values of the variational parameters $\gamma_0(S_2)$ and $\gamma_1(S_2)$ (*cf.* Fig. 5) and for all magnitudes of the dielectric particle radii $S_1 > S_1$ (*cf.* Eq. (3)).

As an example, let us write down the magnitudes of the dipole transition moments $D_{1,0}(S_1)$ (*cf.* Eq. (12)) for the outer problem and $D_{1,0}(S_2)$ (*cf.* Eq. (11)) for the inner problem in dielectric particle with radius $S_1 = S_2 = S = 4$ [19]:

$$D_{1,0}(S_1) = 7.6 \frac{\varepsilon_2-\varepsilon_1}{2\varepsilon_1+\varepsilon_2} e b_1, \tag{13}$$

$$D_{1,0}(S_2) = 4.2 \frac{\varepsilon_1}{2\varepsilon_1+\varepsilon_2} e b_2. \tag{14}$$

The magnitudes of $D_{1,0}(S_1)$ and $D_{1,0}(S_2)$ given by Eqs. (13) and (14) are practically constant with increasing DP radius and increase slightly as $S \to \infty$. This property follows from the fact that the radius a of the dipole has the linear dimension of the order of b_i radius state, which practically does not vary.

We should note that due to this property the variational functions (4) and (5) used for the estimation of the matrix element magnitudes of the transition moments (*cf.* Eqs. (10)–(15)) do not lead to a significant error and have an effect on the numeric prefactor being of the order of unity.

4. Light absorption and scattering

The results obtained so far and concerning the dipole moments of transitions for the volume states $D_{1,0}$ (*cf.* Eq. (10)) for the inner and outer states given by Eqs. (11) and (12), respectively, allow us to derive the light absorption and scattering in the QZDS under consideration in the electromagnetic wave frequency range corresponding to the energies of LOPSs discussed in the previous section.

The absorption cross-section $\sigma_{abs}(\omega, a)$ of the spherical DP with radius a is related to its polarization $A''(\omega, a)$ [20] in the form

$$\sigma_{abs}(\omega, a) = \frac{4\pi}{c} \omega A''(\omega, a) \tag{15}$$

where c is the light velocity in vacuum.

If the states of the charge carrier in DP are quantized, then their polarization $A''(\omega, a)$ can be easily found, assuming that the DP forms a single giant ion. In this case, the polarization $A(\omega, a)$ of the charged dielectric particle can be expressed by the matrix elements of dipole transition moments between quantum states [20], [21]. For sufficiently low temperatures fulfilling the inequality $k_B T < E_B = \hbar^2/(2m_i a_B^2)$, which are of the order of $1-10$ K at $a_B \sim 10-10^2$ Å, *i.e.*, for temperatures less than the binding energy E_B of the states considered, we can write down the explicit form of the polarization

$$A''(\omega, a) = \frac{e^2}{m_i} \sum_j \frac{f_{0j}(a)}{\omega_j^2(a) - \omega^2 - i\omega\Gamma_j(a)} \tag{16}$$

where

$$f_{0j}(a) = \frac{2m_i}{\hbar e^2} (\omega_j(a) - \omega_0(a)) |D_{j0}(a)|^2 \tag{17}$$

is the transition oscillator strength of the charge carrier with effective mass m_i from the ground to the j-th quantum state, $D_{j0}(a)$ denotes the matrix element of the dipole transition moment, $\hbar\omega_j(a)$ is the energy of the j-th quantum state, $\hbar\omega_0(a)$ – the ground state energy and $\Gamma_j(a)$ means the width of the j-th excited state [8], [10].

If the bound states are absent, then $D_{j0}(a)/e$ will be proportional to the linear dimension a of the delocalization range of the charge carrier (for large j we have $D_{j0}(a) = ae/j$ [20]). Moreover, the oscillator strength $f_{0j}(a) \sim |D_{j0}/e|^2 \sim a^2$. In this case, the polarization $A''(\omega, a)$ (*cf.* Eq. (16)) and the absorption cross-section (15) for DP of radius a have the following forms:

$$A''(\omega, a) = \omega f(\omega) a^2,$$

$$\sigma_{abs}(\omega, a) = \frac{4\pi}{c} \omega^2 f(\omega) a^2, \tag{18}$$

since $f(\omega)$ does not depend on a and $f(\omega)$ is weakly varying with ω [21]

$$\sigma_{abs}(\omega, a) \sim \omega^2 a^2. \tag{19}$$

If the bound states of CC were formed near the interface between two different dielectric media, then the main contribution to $A''(\omega, a)$, for $k_B T < E_B$, would come from the transitions between quantum states of a discrete spectrum. The contribution of a single resonance state into (16) can be written in the form

$$A''(\omega, a) = f_{01}(a) F(\omega_1, \omega) \tag{20}$$

where

$$F(\omega_1, \omega) = \frac{e^2}{m_i} \frac{1}{\omega_1^2 - \omega^2 - i\omega\Gamma_1(a)} \tag{21}$$

has the usual resonance form and it in the vicinity of the resonance is independent of a [21]. Let us note that we have isolated the single resonance contribution corresponding to a transition between the ground (s) and p states discussed above for which $f_{01}(a)$ is defined by Eqs. (10)–(12) and (17).

It is evident that restricting our studies to the consideration of the states s and p does not influence the obtained behaviour of $A''(\omega, a)$ and $\sigma_{abs}(\omega, a)$ as a function of the DP radius a, since the dependence of the oscillator strength $f_{01}(a)$ has the same character for other states (*cf.* Eq. (17)).

From Equations (15), (20) and (21), it follows that the resonance light absorption cross-section is given by

$$\sigma_{abs}(\omega, S_i) = \left(\frac{4\pi}{c}\omega\right) f_{01}(S_i) F(\omega_1, \omega) \tag{22}$$

where the oscillator strength $f_{01}(S_i^2)$, in accordance with Eqs. (10) and (17), for the volume states localized in the DP center with radius S_i takes the form

$$f_{01}(S_2) = \frac{\dfrac{m_2 b_2^2}{\hbar}\left(\dfrac{\varepsilon_1}{2\varepsilon_1 + \varepsilon_2}\right)^2 S_2^{3/2}\omega_1}{4\cdot 3^{1/3}\left[\dfrac{8\beta}{3(1+\alpha)} + \dfrac{2(7+5\alpha)}{2+\alpha}\sqrt{\dfrac{\beta}{S_2(1+\alpha)}} + \dfrac{8+11\alpha+5\alpha^2}{S_2(2+\alpha)}\right]^{1/2}}. \tag{23}$$

The oscillator strength transition $f_{01}(S_i)$ exhibits, for the surface and the inner states, the same dependence on DP radius S_i and is given by

$$f_{0i}(S_i) = \frac{m_i b_i^2}{\hbar} S_i^2 L_i^2 \omega_1 \tag{24}$$

where the parameters L_i for inner ($i = 2$) and outer ($i = 1$) surface states are defined by Eqs. (11) and (12), respectively.

We point out, that the comparison of Eqs. (22)–(24) shows the different character of the dependence of the light absorption cross-section $\sigma_{abs}(\omega, a)$ on frequency ω and radius a of the dielectric particle, under different physical conditions.

If the bound states near the spherical interface between two media do not exist, then, in accordance with Eq. (19), we have

$$\sigma_{abs}(\omega, a) \sim \omega^2 a^2.$$

In the case of the volume states of CC, the absorption cross-section $\sigma_{abs}(\omega, a)$ is defined by Eqs. (22), (23) and

$$\sigma_{abs}(\omega, a) \sim F(\omega_1, \omega)\omega a^{3/2}. \tag{25}$$

For the surface inner and outer states the absorption cross-section $\sigma_{abs}(\omega, a)$, in accordance with Eqs. (22) and (24), has the same dependence on the DP radius a

$$\sigma_{abs}(\omega, a) \sim F(\omega_1, \omega)\omega a^2. \tag{26}$$

Thus, the localization of CC on the spherical interface and inside the small dielectric particle leads to qualitatively different dependence of the absorption of electromagnetic field on the DP radius a and light frequency ω. This fact gives an additional experimental possibility for spectroscopic findings and studies of the localized states of charge carriers.

A similar possibility is also given by the elastic scattering of electromagnetic waves with frequency ω on the small spherical DP of radius a. The scattering cross-section $\sigma_{sc}(\omega, a)$ is given by [20]

$$\sigma_{sc}(\omega, a) = 2^7 3^{-3}\pi^3 |A''(\omega, a)|^2 \left(\frac{\omega}{c}\right)^4, \tag{27}$$

and, according to Eqs. (18) and (20)–(24), $\sigma_{sc}(\omega, a)$ exhibits different dependence on the radius a and frequency ω for various types of localized states under consideration.

In fact, for inner and outer surface states, according to Eqs. (20), (21), (24), the cross-section $\sigma_{sc}(\omega, a)$ of the elastic scattering of electromagnetic waves is given by the formula

$$\sigma_{sc}(\omega, a) = \frac{2^7}{3}\pi^3 \left(\frac{\omega}{c}\right)^4 F^2(\omega_1, \omega) L_i^2 \left(\frac{m_i b_i^2 \omega_1}{\hbar}\right)^2 S_i^4. \tag{28}$$

The scattering cross-section $\sigma_{sc}(\omega, S_2)$, for the volume local states, according to Eqs. (20), (21) and (23), has the following form:

$$\sigma_{sc}(\omega, S_2) = \frac{2^3 3^{-5/3}\pi^3 \left(\frac{\omega}{c}\right)^4 F^2(\omega_1, \omega) \left(\frac{\varepsilon_1}{2\varepsilon_1 + \varepsilon_2}\right)^4 S_2^3 \left(\frac{m_2 b_2^2 \omega_1}{\hbar}\right)^2}{\frac{8\beta}{3(1+\alpha)} + \frac{2(7+5\alpha)}{2+\alpha}\sqrt{\frac{\beta}{S_2(1+\alpha)}} + \frac{8+11\alpha+5\alpha^2}{S_2(2+\alpha)}}. \tag{29}$$

If the bound states near the DP do not occur, then, acording to Eq. (18), the dependence of the cross-section $\sigma_{sc}(\omega, S_i)$ (*cf.* Eq. (27)) on ω and S_i is given by

$$\sigma_{sc}(\omega, S_i) = 2^7 3^{-1}\pi^3 \left(\frac{b_i}{c}\right)^4 \omega^6 f^2(\omega) S_i^4. \tag{30}$$

The optical coefficient of attenuation $\eta(\omega, a)$, which takes into account the light absorption and cattering effects on the LOPSs near the small dielectric particles

of radius a and concentration N, may be written in the form

$$\eta(\omega, a) = N[\sigma_{abs}(\omega, a) + \sigma_{sc}(\omega, a)]. \tag{31}$$

The last expression may be used for a group of noninteracting dielectric particles. This condition is fulfilled if the inequality

$$a_c \ll \frac{1}{N^{1/3}} \tag{32}$$

holds true, where $a_c \sim b_i$ (*cf.* also Eq. (3)) denotes the DP radius and $1/N^{1/3}$ is the average distance between dielectric particles. The condition (32) is fulfilled for concentrations not exceeding $N \leqslant 10^{14}$ cm^{-3} of small cadmium sulphate or cadmium selenide particles with $a_c \sim b_i \sim 100$ Å (*cf.* refs. [6]–[11]), as documnted in experimental studies [1], [2], [5], [12].

5. Comparison of theory with experimental data

Finally, we discuss briefly physical situations for which the results obtained in this paper are of particular importance. An interesting feature of the states of charge carriers localized on the spherical interface is removal of degeneracy with respect to the orbital quantum number l. As a result, the distance between the energy sublevels will be sufficiently small [7]–[11].

If we consider that: i) the dipole moments of transitions for all the local states considered, according to Eqs. (10)–(12), have large magnitudes $D_{1,0}(a) \sim 1$ D (*i.e.*, $D_{1,0}(a)$) exceeds typical magnitudes for the dipole moment transition of the nonorganic semiconducting materials for which $D_{1,0} \sim 10^{-1}$ D [22]), and ii) the allowed dipole transitions induced by the electromagnetic field between the nearest energy sublevels correspond to a change in the quantum number l by 1, then it is evident that the QZDS discussed are strongly nonlinear media for the electromagnetic radiation. The last circumstance is of special interest in construction of novel nonlinear devices in a wide wavelength range. The nonlinearity magnitude depends on the nature of the contacting materials.

As an example, let us consider a quasi-zero-dimensional system which consists of a spherical semiconducting particle of CdS of radius $a \simeq 54$ Å placed in boric-silicate glass. In this system, an oscillator-like energy spectrum for the hole states localized near the dielectric particle centre with energies of ~ 100 meV has been found experimentaly [23]. In papers [24], [25], it was shown that the spectrum of the hole volume local states studied experimentally in [23] consists of a series of equidistant energy levels and the distance between those energy sublevels is proportional to $a^{-3/2}$. The dipole moments of hole transitions between such equidistant sublevels, according to Eq. (10), take high values, *i.e.*, $D_{1,0} \sim 10-20$ D.

Let us carry out a qualitative estimation of the cross-sections $\sigma_{abs}(\omega, a)$ and $\sigma_{sc}(\omega, a)$ of light on the above mentioned local states of CC for the isolated transition $|0\rangle \to |1\rangle$ under the experimental conditions of [1]–[5], [23]. Let us assume that

the light frequency is not comparable with the resonance frequency ω_1 of the local state in the dielectric particle and let us also suppose that the broadening Γ_1 of the energy level $E_1 = \hbar\omega_1$ is sufficiently small, *i.e.*, $\Gamma_1/\omega_1 \ll 1$. Then we can estimate the magnitudes of the cross-sections $\sigma_{abs}(\omega, a)$ and $\sigma_{sc}(\omega, a)$ using Eqs. (15), (23), (24) and (27) for which the DP polarization takes the form

$$A''(a) = \frac{e^2 f_{01}(a)}{m_i \omega_1^2}. \tag{33}$$

Table. The parameters of electron and hole bound states localized on a small dielectric particle, made from materials specified in the second column, with dielectric permittivity ε_2 (*cf.* the third column) placed in dielectric materials with dielectric permittivity ε_1 (a – the radius of DP, m_h – the effective hole mass in DP, E_1 – the electron or hole binding energy, f_{01} – the transition oscillator strength, A'' – the polarization of DP, σ_{abs} and σ_{sc} – the cross-sections of light absorption and scattering, respectively).

ε_1	DP	a[Å] (S)	ε_2	m_h/m_0	E_1 [meV]	f_{01} [10^{-1}]	A'' [10^{-24}cm^3]	σ_{abs} [10^{-24}cm^2]	σ_{sc} [10^{-24}cm^2]
1.5	GaAs	340 (4)	13.2	0.62	50	3.56	$2.6\cdot10^2$	$1.3\cdot10^8$	$2.4\cdot10^{-3}$
1.5	CdS	54 (12.65)	9.3	5	50	0.18	$1.6\cdot10^2$	$4\cdot10^4$	$6\cdot10^{-13}$
1	He	420	1.06		0.65	3.81	$4.8\cdot10^3$	$1.6\cdot10^4$	$1.6\cdot10^{-17}$

In the table, the estimated magnitudes of the oscillator strength $f_{01}(a)$ (*cf.* Eqs. (23) and (24)), the polarization $A''(\omega,a)$ (*cf.* Eq. (33)), the light absorption cross -sections $\sigma_{abs}(\omega, a)$ (*cf.* Eq. (15)) and $\sigma_{sc}(\omega, a)$ (*cf.*, Eq. (27)) corresponding to the one-particle localized states specified above, are listed for QZDS consisting of a dielectric matrix and dielectric (or semiconducting) particles (*cf.* Eq. (3)) of radius $a \geqslant a_c$ [7]–[11] dispersed in it.

From the estimations presented, it follows that the cross-section $\sigma_{abs}(\omega, a)$ takes giant magnitudes of $\sigma_{abs}(\omega, a) \simeq 1.3\cdot10^{-16}$ cm^2 for the light absortion on the hole volume local states in the semiconducting GaAs particle embedded in a boric-silicate glass matrix (or in another medium with $\varepsilon_1 \ll \varepsilon_2$). We point out that the magnitude of $\sigma_{abs}(\omega, a) \simeq 1.3\cdot10^{-16}$ cm^2 is 10^8 times larger than the typical atomic values of $\sigma_{abs}(\omega, a) \sim 10^{-24}$ cm^2 [22].

The observed values of the cross-section $\sigma_{abs}(\omega, a)$ take $\sim 10^{-20}$ cm^2 for the light absorption by the inner surface states of the electron in a bublon (a macroscopic spherical cavity in a liquid superfluid) embedded in superfluid helium [26] as well as by the hole volume local states in the cadmium sulphate particle placed in a boric-silicate glass matrix [23]. In addition, the elastic light scattering cross-section σ_{sc} by the above-mentioned one-particle local states will be sufficiently small in comparison with σ_{abs}, *i.e.*, $(\sigma_{sc}/\sigma_{abs}) \sim 10^{-11}$. This leads to the conclussion that the optical attenuation coefficient of light $\eta(\omega, a)$ (*cf.* Eq. (31)) is determined mainly by processes of light absorption by one-particle local states of charge carriers. The magnitude of $\eta(\omega, a)$, corresponding to light absorption by loal states of QZDS

described in the table, takes the values of $\eta \sim (10^{-2} - 10^{-6})\,\mathrm{cm}^{-1}$ for the dielectric particle of radius $a > a_c$ (*cf.* Eq. (3)) fulfilling condition (32) for concentration of $N = 10^{14}\ \mathrm{cm}^{-3}$.

In this way, the large absorption cross-section σ_{abs} of the electromagnetic field by one-particle local states of charge carriers occurring in the vicinity of the dielectric or semiconducting particle in QZDS offers an interesting possibility of applying such heterophase structures as new materials with strong electromagnetic wave absorption (in particular, light) in a wide wavelength range with its magnitude depending on the nature of the materials in contact.

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