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Research of chromatic dispersion by means of Fabry–Perot filter

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We report on the method of the chromatic dispersion determination of anisotropic refractive indices of homogeneously oriented layers of liquid crystals (LC). The experimental data have been obtained from transmission spectra of a plane parallel Fabry–Perot filter (FPF) filled with the liquid crystal under investigation. Physical basis, experimental techniques and mathematical procedures have been described. We assumed the perpendicular light incidence and the absence of absorption and scattering. The dispersion of both refractive indices has been determined over the visible spectrum. Values obtained for 5CB (K15 MERCK) during the test measurement appeared to be similar with those published in literature. Results for W602 and 1292 liquid crystalline mixtures made at the Institute of Chemistry Military University of Technology are given.

1. Introduction

Rapid advancement of the application of nematic liquid crystal (NLC) to sophisticated devices for optical processing makes knowledge of material parameters very important. To determine values of the dielectric tensor in liquid crystals several experimental methods have been used [1]–[6]. LAURENT and JOURNEAUX [7] observed differences between the indices measured at varying wavelength. A polarimetric and transmission method had been used in [8]. Horn employed interference fringe technique [9] to measure refractive indices as a function of pressure and temperature. Warenghem performed a detailed study of the chromatic dispersion in 5CB [10].

The light wave travelling through properly prepared FPF filled with homogeneously oriented birefringent liquid crystal (LC) layer is divided into two eigenwaves. One of them has parallel and the second perpendicular polarisation to LC optical axis, known as director, respectively. The transmission spectrum of polarized light gives two curves with peak transmission over a series of order k when

$$2nd\cos\theta = k\lambda, \tag{1}$$

symbol d denotes the effective FPF cavity thickness, λ is the wavelength in vacuum, and $n = n_o$ or n_e means ordinary and extraordinary refractive indices, respectively. An angle θ in formula (1) describes beam divergence. In calculations, it is assumed to be equal to zero. Because absorption in visible region is negligibly small, taking into account the reflectivity of mirrors the transmission spectrum is given in anisotropic case by the following formula (to isotropic case see, *e.g.* [11], [12]) describing proportion between transmitted and incident light intensities

$$\frac{I_t^{\mathrm{o,e}}}{I_i} = \frac{0.5(1-R^2)}{(1-R^2)+4R\sin^2\left(\frac{\delta_{\mathrm{o,e}}}{2}\right)}. \tag{2}$$

The true transmission spectrum of a Fabry–Perot system differs from that obtained from the above theoretical formula due to surface defects in the coatings, deviations from plate parallelism, and the diverging, rather than a parallel, incident beam. Usually the phenomenon mentioned is not modelled because of its stochastic nature. The shape of transmission curve depends on the reflectivity and phase change. Reflectivity R depends, in turn, on refraction indices in accordance with formula

$$R = \left(\frac{n_{\mathrm{e}}^{\mathrm{eff}} - n_m}{n_{\mathrm{e}}^{\mathrm{eff}} + n_m}\right)^2 \tag{3}$$

where n_m is the refractive index of mirrors covered with layers of indium-tin oxide (ITO) and polyimide. In formula (3), $n_{\mathrm{e}}^{\mathrm{eff}}$ is the refractive index of the LC layer. It is detected by the polarized light beam incident normally to the surface of the planar nematic liquid crystal layer, and is defined as

$$n_{\mathrm{e}}^{\mathrm{eff}} = \frac{n_{\mathrm{o}} n_{\mathrm{e}}}{\sqrt{n_{\mathrm{e}}^2 \sin^2\varphi + n_{\mathrm{o}}^2 \cos^2\varphi}}. \tag{4}$$

The angle φ denotes an angle between the direction of incident wave and the optical axis in the LC layer. The phase change $\delta_{\mathrm{o,e}}$ is the sum of the retardation during dual passage through the LC layer and phase changes upon reflection on both surfaces and is given for ordinary and extraordinary waves, respectively:

$$\frac{\delta_{\mathrm{o}}}{2} = \frac{2\pi d n_{\mathrm{o}}(\lambda)}{\lambda}, \tag{5}$$

$$\frac{\delta_{\mathrm{e}}}{2} = \frac{2\pi d n_{\mathrm{e}}^{\mathrm{eff}}(\lambda)}{\lambda}. \tag{6}$$

2. Theory of the chromatic dispersion

The refractive indices of a uniaxial LC are primarily governed by the LC constituents, wavelength and temperature considered. Ordinary index is slightly dependent on the molecular constituents. Usually it decreases as wavelength increases and weakly increases with temperature [13]. On the other hand, the extraordinary index depends very much on molecular constituents. As the temperature rises and the wavelength increases it decreases gradually. A couple of different models have been built to describe dispersion of refractive indices. We assumed polynomial description. The unknown coefficient in formula below would be determined by fitting procedure between theoretical and experimental characteristics of FP filter. Dispersion of both indices is described by the following formula [13]:

$$n_{o,e} = A_{o,e} + \frac{B_{o,e}}{\lambda^2} + \frac{C_{o,e}}{\lambda^4} \tag{7}$$

where

$$A_{o,e} = 1 + g_{o,e}\lambda^2_{\perp,\parallel},$$
$$B_{o,e} = g_{o,e}\lambda^4_{\perp,\parallel},$$
$$C_{o,e} = g_{o,e}\lambda^6_{\perp,\parallel}, \tag{8}$$

$\lambda_{\perp,\parallel}$ means resonance wavelength for ordinary and extraordinary waves, respectively. One can express

$$g_{o,e} \approx NZf_{o,e}(T) \tag{9}$$

where Z is effective number of electrons involved and $f_{o,e}$ the averaged oscillator strength. Formula (7) has been applied far above resonance absorption wavelengths. It used to be a rule that maxima of transmission were obtained as minima of sine function in formula (2) [12]. To find chromatic dispersion from zeroes of sine function one needs to know what the number of the peak observed is and which peak is observed. It may be estimated if weak chromatic dispersion is assumed. Unfortunately, such an assumption might be too rough for liquid crystals. Besides, the reflection R at the interface between LC and dielectric mirror may be highly affected by chromatic dispersion in spectrum range under consideration. So, in the case under consideration, looking for zeroes of derivative of the formula (2) is a more efficient way. The coefficients of dispersion from formula (7) are determined by fitting the positions of zeroes in theoretical spectrum derivative and the experimentally obtained peak positions. The fitting procedure is in fact looking for optimal chromatic dispersion like (7) for the exact accordance between theoretical and experimental resonance peak positions in FPF.

3. Sample preparation

The Fabry–Perot resonator consists of a pair of commercial glass plates used in LCD technology covered with 50 nm transparent indium-tin-oxide layers. Square samples with clear aperture of 20×20 mm^2 were separated by means of glass spacers with

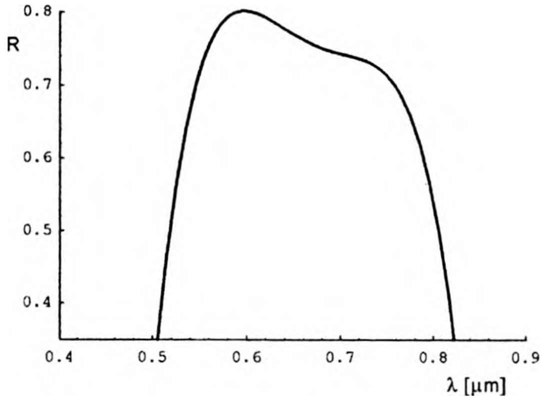

Fig. 1. Reflection coefficient for the mirrors used.

diameter from 6 to 15 μm. The inner surfaces which form the cavity are flat up to λ/20 at 632.8 nm. Onto these surfaces the multilayer dielectric mirrors which give a reflectivity of 75 to 80% over the middle part of the visible spectra have been deposited. Experimental data for mirror reflectivity are shown in Fig. 1. The range of the most flat part of reflection has been chosen for dispersion determination.

On the top of the mirror surface the 20 nm polyimide layer has been deposited by spinning method followed by mechanical rubbing. Glass plates have been assembled using special mounting devices and carefully separated with glass spacers and screwed to form plane mirror interferometer. The inner cavity has been filled with liquid crystal mixtures by capillary action giving a homogeneous orientation of the layer equivalent to plate with optical axis parallel to the surfaces of plates. Although the device is designed to be thermally stable by careful selection of materials used, in order to maximise the system performance it is still necessary to maintain a thermal control because of the properties of LC. The air temperature surrounding the interferometer is controlled to within 1.5–2.0 °C. A separate system keeps the FP etalon temperature constant within 0.5 °C.

4. Experimental procedure

The experimental procedure consists in measuring of transmitted spectra of mirror plates, empty interferometer cells and spectra measurement for two directions of light polarization with electric vector parallel and perpendicular to the director, respectively. In order to obtain dependence on the LC layer thickness in some cases spectrum for the sample tilted in the plane perpendicular to director has been measured. Placing a polarizer with parallel transmission axis in both measuring channels of Backmann spectrometer has changed light polarization plane. The transmission axis of polarizer has been turned through 90° for the extraordinary n_e refractive index measurement.

For W602 mixture an independent measurement provided n_e = 1.6494 and n_o = 1.5092 in 589.3 nm sodium line. That material has been applied to correction of

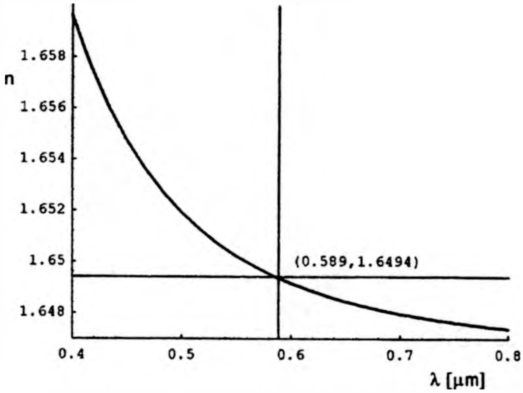

Fig. 2. Chromatic dispersion in W602 mixture fixed for sodium yellow line (extraordinary refractive index in W602 mixture, $T = 294$ K).

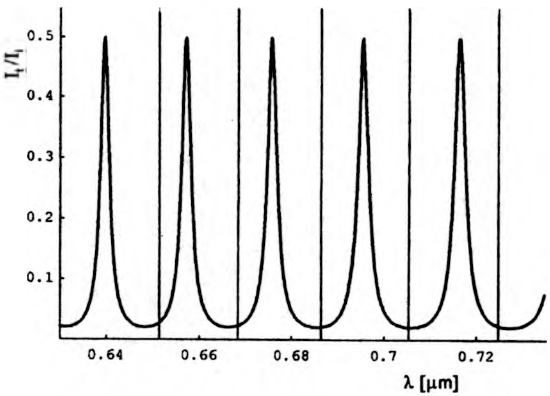

Fig. 3. Shift in FPF spectrum for chromatic dispersion fitted to yellow line of sodium. Ticks are placed at positions of measured peaks (extraordinary spectrum in W602 mixture, $T = 294$ K).

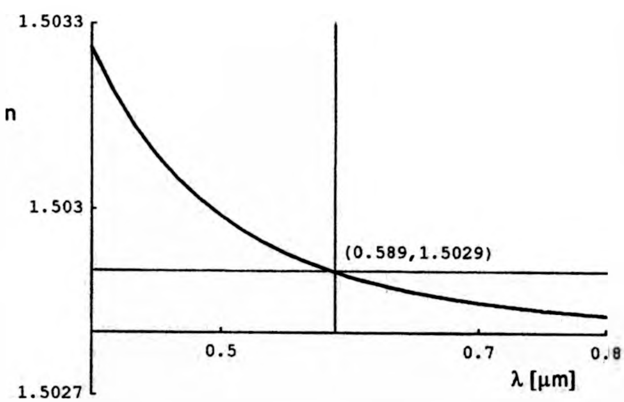

Fig. 4. Chromatic dispersion of the ordinary part of refractive index in W602 fitted for sodium yellow line ($T = 294$ K).

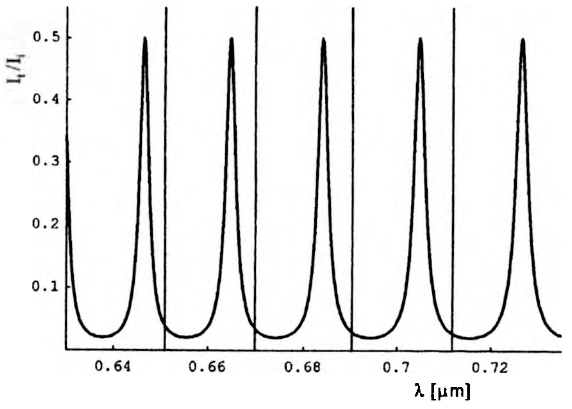

Fig. 5. Shift in ordinary spectrum of FPF with W602 for the case when chromatic dispersion is fitted to sodium yellow line (T = 294 K).

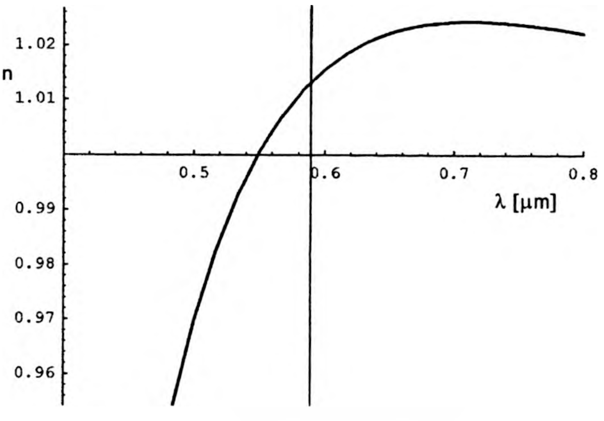

Fig. 6. Dispersion fitted for Cauchy formula (7) in the FPF with air gap.

FPF spectrum and to find measurement deviations. The fit results for dispersion in W602 is shown in Figs. 2–5.

It is easy to observe that exact fit in LC refraction coefficients on 589 nm results in a steady shift in the FPF spectrum. That shift is used to find measurement deviation because dispersion of ITO/mirror/polyimide layer has not been taken into account in theoretical model (2). This dispersion is the main source of measured deviation. The deviation has been applied as correction in fit procedures. The chromatic dispersion of the ITO/mirror/polyimide layer has been shown in Fig. 6 for FPF with air gap.

One can observe that with reference to air the ITO/mirror/polyimide layer manifests its chromatic dispersion strongly. That layer is relatively thin, so in FPF filled with LC its chromatic dispersion influences more weakly the whole result.

A short wavelength part of the refractive index dispersion in ITO/mirror/polyimide layer behaves as if it were quite near resonance. This is caused by the presence of conducting layer of ITO and polyimide.

5. Results and discussion

Below, the results obtained have been presented in the form of table and figures. The Table contains the obtained values of polynomial coefficient (8).

T a b l e. The Cauchy coefficients for the dispersion of refractive indices in three different LC.

	A_o [dimless]	A_e [dimless]	B_o [μm^2]	B_e [μm^2]	C_o [μm^4]	C_e [μm^4]
1292	1.5339	1.5915	0.0	0.0	0.00010	0.0013
5CB	1.5980		0.0		0.00024	
W602	1.5291	1.6465	0.0	0.0	0.000012	0.00033

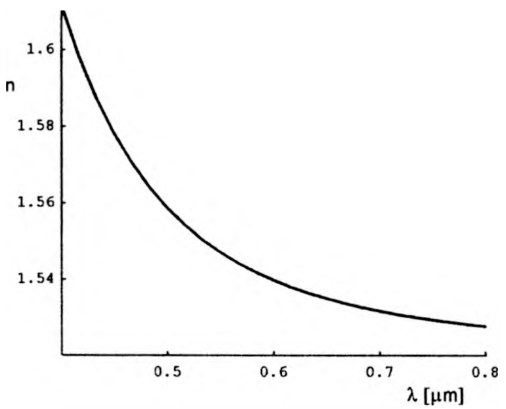

Fig. 7. Dispersion obtained for the ordinary refractive index in 5CB (T = 294 K).

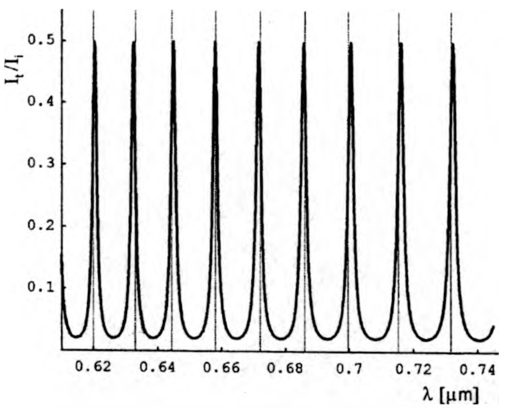

Fig. 8. Transmission calculated in FPR filled with 5CB and ticks at positions of measured transmission peaks for ordinary wave (T = 294 K).

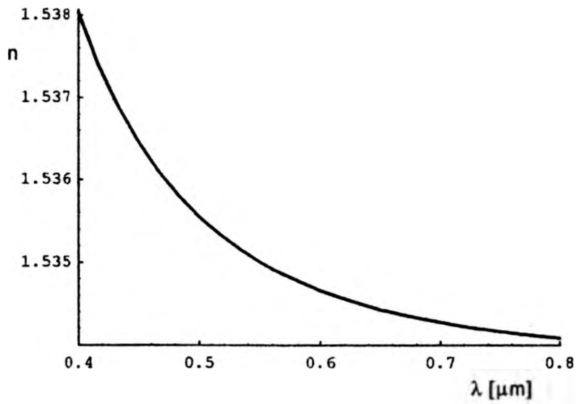

Fig. 9. Dispersion obtained for ordinary refractive index in 1292 nematic mixture (T = 293 K).

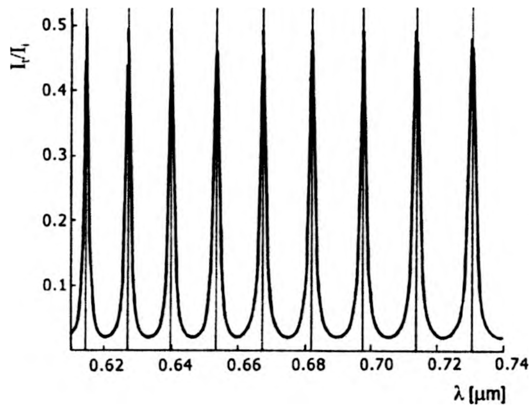

Fig. 10. Theoretical transmission in FPR filled with 1292 mixture and ticks at positions of measured transmission peaks for ordinary wave (T = 293 K).

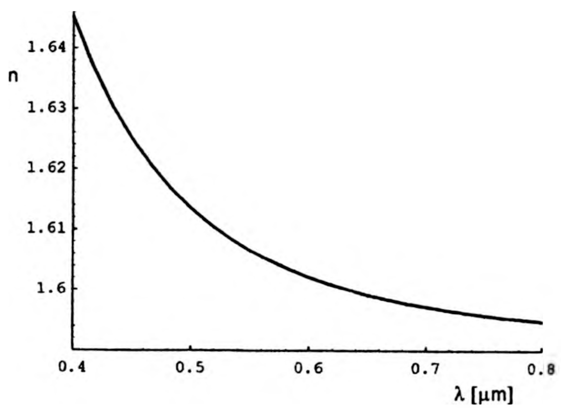

Fig. 11. Chromatic dispersion of the extraordinary refraction index of 1292 mixture (T = 293 K).

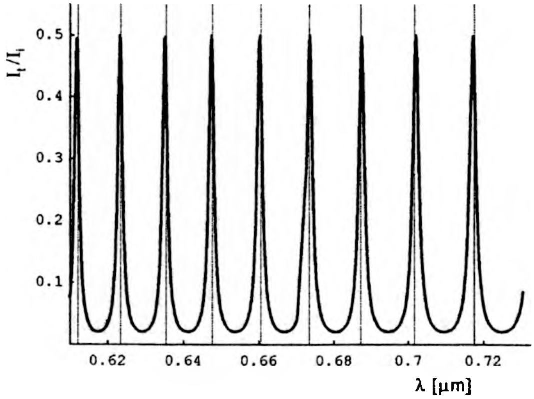

Fig. 12. Extraordinary peak position measured and calculated for the chromatic dispersion obtained (the 1292 mixture, $T = 294$ K).

In Figure 7 resulting dispersion curves for 5CB have been shown.

A good accordance between the theoretical and measured peak positions is obtained. It has been shown in Fig. 8. So, Cauchy coefficients from the Table are properly determined for 5CB. A similar accordance has been achieved for 1292 mixture. Dispersion curves for that material are placed in Figs. 9–12.

6. Conclusions

The spectrum of a typical FPF is presented as a tool for the chromatic dispersion determination in a uniaxial LC. Two different groups of the resonance peaks have been observed. The first one depends on extraordinary refractive index and the second depends on ordinary index. Positions of peaks within both groups have been measured and determined theoretically as well. The coefficients of the polynomial form for dispersion model for both refractive indices have been obtained by the fit between the experiment and the theory. The nonlinear optimisation method has been applied as a fit procedure.

The proposed way of calculations is especially effective when exact number of the observed peak of transmission is not known. Such a situation is always present in the FPF filled with liquid crystal.

The results obtained for 5CB are really similar to those presented in literature [10]. One can see that in the investigated FPF optical buffer layer ITO/mirror/polyimide layer causes big deviation in chromatic dispersion determination. That deviation can be as high as 10^{-2}. A real level of deviation depends on consistent thickness of that layer. Differences between results are caused by chemical purity of LC substance, too. Only ordinary wave spectrum has been compared because of usually present diversification in 5CB alignment data. The sample alignment highly influences the behaviour of extraordinary part of the transmission spectrum. In the literature, no method of 5CB aligning has been described, so ordinary refraction index dispersion is more proper for any comparison as non-affected by the LC alignment.

The results for other materials investigated are also presented. In both cases, peak positions are observed to be in good accordance. It means that the dispersion obtained is properly described.

It should be underlined that the glass plates used as well as other technology details could influence the measured peak position. It has not been included in the theoretical model of transmission.

It seems that transmission peak positions in FPF are enough to obtain proper shape of chromatic dispersion in liquid crystals even for a relatively simple model of interference. Nevertheless, it should be noted that the proposed model of light transmission in FPF even if adopted for anisotropic case is not enough.

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