Influence of waveguide parameters on the difference interference in optical planar structure

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The paper presents studies involving the influence of waveguide parameters on the sensitivity of planar sensor with the application of difference interference. Gradient waveguides and homogeneous waveguides have been taken into consideration. A new structure of difference interferometer has been proposed, constructed with the use of gradient waveguide and homogeneous dielectric layer. The suggested structure provides much higher sensitivity levels as compared to the currently designed structures.

1. Introduction

One of the branches of optoelectronics which has been showing the fastest development over the last decade is the branch of planar waveguide sensors. Their optical systems are made on flat substrates with the application of planar technology [1], [2]. The substrate in these sensors has two functions. It serves as an optical element, but it principally serves as a supporting structure for the whole system. As a result, planar waveguide sensors, in contrast to sensors made with the use of optical fibers, usually do not require the application of additional mechanical structures. Planar technologies have a number of advantages and are particularly useful in the production of intrinsic sensors [2], [3]. Waveguides to be used for planar sensors are produced with the application of various methods, such as: ion exchange in glass [4]—[6], sol-gel technology [7], and semiconductor technologies [3], [8]. Each of these methods has different advantages in sensor related applications [1], [2].

The paper discusses the influence of parameters of waveguides produced with the application of different technologies on the sensitivity of planar sensors built in the system of difference interferometer. The technology of ion exchange in glass is relatively cheap and is commonly applied in planar optics [4]—[6], [9]. Lateral dimensions of strip waveguides produced in that technology are comparable with the dimensions of fiber waveguide cores. Therefore they can be easily connected with fibers. In sol-gel technology the most commonly produced layers are dielectric ones with the application of oxide system SiO₂:TiO₂. The layers obtained in such a way
can have refractive indices of the value within the range $n \approx 1.20 - 2.30$ [7]. The layers $\text{SiO}_2:\text{TiO}_2$ of high values of refractive indices, deposited on glass substrate or silicon wafer, are used as waveguide layers in interference sensors. The highest refractive indices are characteristic for waveguides made in material system $\text{GaAs/AlGaAs}$. The maximum step of refractive index generated by this system is about 0.3 [8]. Due to the application of semiconductor substrates (GaAs), it is possible to produce on the same substrate the sources and detectors, as well as electronic signal processing systems. Ion exchange process can be applied for the production of both multi-mode waveguides, used in amplitude systems [9]–[11], and single-mode waveguides which are used in phase systems [9], [12]. The semiconductor technology or sol-gel technique is used principally for the production of single-mode waveguides, which are applied in interference systems [3], [13], [14]. Phase sensors are most commonly built in the Mach–Zehnder interferometer system, or in the system of difference interferometer. Due to some technological limitations (length of interaction range), the measurement scope in planar systems is very often limited to a single interference fringe. Therefore, with respect to these systems, they should be optimized to obtain possibly high sensitivity values.

Difference interferometer is technologically the least complicated system to produce, and as it was presented by LUKOSZ et al. [13], [14], it can be useful in many applications. In the planar difference interferometer, a single optical path is used, which is a planar waveguide. Both single- or multi-mode (several-mode) planar waveguides can be used in such interferometers. In the single-mode waveguide, two fundamental modes $\text{TE}_0$ and $\text{TM}_0$ are excited simultaneously, and their propagation constants, as it follows from dispersion equations of the waveguide, are different. This difference changes according to change of the parameters of waveguide cover. In multi-mode waveguides, we can also use the interference of modes of different order having the same polarization states [15], [16].

It is known that the sensitivity of planar interference sensors is increasing with an increase of the step of refractive index along the border waveguide layer–surroundings [1], [8]. For that reason, in order to obtain high sensitivity values, waveguides with high steps of refractive index should be used.

New potentials involving the acquisition of particular sensitivity values of planar sensors are provided here by the author's proposition to combine the ion exchange technology with the step-index waveguide technology, e.g., sol-gel technology. The problem has been theoretically discussed in the paper. The sensitivity values of sensors working in the system of difference interferometer are compared with respect to the change of refractive index of the cover, for three different refractive index profiles of the waveguide. The investigation involves the interference of modes of the same order having the orthogonal polarization states, as well as modes of different orders having the same polarization states. The discussion has been focused on the influence of waveguide layer parameters on the sensitivity of the meter measuring the changes of refractive index, and working in the system of difference interferometer.
2. Analysis of planar sensor

Planar waveguide sensors are built as monolithic multilayer systems, which have the form of an optical chip. The diagram of a typical structure of a planar waveguide sensor is presented in Fig. 1. Optical waveguide W was produced on the substrate B, and on the waveguide W one or several sensor layers C, S were deposited. In waveguide planar sensors, optical waveguides having constant refractive index are used (Fig. 1b), or waveguides having gradient distribution of refractive index (Fig. 1c). The electromagnetic wave propagating in the multilayer structure penetrates sensor layers with its evanescent field. Hence, the change of parameters of any sensor layer as a result of the influence of external factor (physical, chemical or biological) results in the change of propagation conditions of the wave, which in consequence results in the change of its parameters (amplitude, polarization, phase).

LUKOSZ et al. [13], [14] singled out three effects which can be applied in planar interference sensors. In planar sensors, both chemical and biochemical, as well as in physical intrinsic planar sensors, the most commonly used physical effects are the following: the change of refractive index of sensor layer, or the change of the thickness of sensor layer.

The theoretical analysis of the multilayer structure of planar sensor (Fig. 1) can be considerably simplified by introducing the effective refractive index of the cover \( n_{e,\text{eff}} \). It is the refractive index of the cover "seen" by a particular mode propagating in the sensor structure. The "seeing" process is effected by the evanescent field, which is penetrating this area. Such a simplification is often used in calculations, when multilayer cover is replaced with a single layer of the refractive index \( n_{e,\text{eff}} \) defined by [17]:

![Fig. 1. Diagram of a typical sensor structure: B — substrate, W — waveguide, C, S — layers of the cover (a). Refractive index profile of the sensor structure with homogeneous waveguide (b). Refractive index profile of the sensor structure with gradient waveguide (c).](image-url)
\[ n_{c,eff}^2 = \frac{2}{z_0} \int_{0}^{\infty} \exp \left( \frac{2z}{z_0} \right) n^2(z) dz \]  

where: \( z_0 \) — evanescent penetration depth, \( n(z) \) — refractive index profile of the cover. In this way, the multilayer structure of planar waveguide presented in Fig. 1 is replaced with three-layer structure (Fig. 2), where the waveguide \( W \) has infinitely thick cover of the refractive index \( n_{c,eff} \).

Fig. 2. Diagram of the substitute sensor structure.

It can be seen that, irrespective of the fact whether the physical effect is the change of refractive index of sensor layers \( C, S \) (Fig. 1) or the change of thickness of sensor layer \( C \), the analysis of the sensor is limited to the analysis of three-layer structure in which the effective refractive index of the cover \( n_{c,eff} \) is changed. The range of changes of \( n_{c,eff} \) with the application of particular physical effect is easy to estimate. For example, when we use as the physical effect the change of thickness of sensor layer \( C \), which is an air layer \( (n_c = 1.000) \), separating the dielectric \( S (n_a > n_c) \) from waveguide \( W \), the effective refractive index of the cover for the whole range \( d \) meets the condition \( 1 < n_{c,eff} < n_s \). When we use the change of refractive index of the sensor layer as the physical effect, then, in general, \( 1 < n_{c,eff} < N \), where \( N \) stands for effective refractive index of the mode under investigation. It follows from the above that if we want to analyze the usefulness of particular waveguide type and the influence of its parameters on the sensitivity of the sensor being designed, we can reduce our investigation to the analysis of three-layer structure. We should first of all analyze the influence of the changes of refractive index of the cover \( n_{c,eff} \) in the three-layer structure on effective refractive indices of the modes in the sensor structure. Considering the influence of waveguide layer on the sensitivity of planar sensor, independent of the fact on which of the physical effects its functioning has been based, the whole analysis can be reduced to the one involving the influence of the parameters of waveguide layer on the sensitivity of the sensor to changes of refractive index \( n_c \) of the infinitive cover.
In the following, we present the results of analysis of the influence of changes of refractive index of the cover on the sensitivity $S_\alpha$ of difference interferometer. The said sensitivity is defined as

$$S_\alpha = \left| \frac{\partial \Delta N}{\partial n_c} \right|$$

where $\Delta N$ stands for the difference of effective refractive indices of the modes being investigated and $n_c$ is a refractive index of the cover. The results given below have been obtained using the matrix method $4 \times 4$ [1], [18], [19].

3. Influence of the parameters of waveguide layer on the sensitivity of difference interferometer

The following part of the paper presents the results of analysis involving the dependence of sensitivity $S_\alpha$ on refractive index of the cover for different types of planar waveguides, for the system of difference interferometer. Gradient refractive index profiles, as produced in ion exchange in glass were allowed for, as well as step-index profiles, as produced with the use of sol-gel technology. The investigation has covered the cases involving the interference of fundamental modes $TE_0$-$TM_0$ as well as the modes of different orders having the same polarization states. A new planar structure suggested by the author is discussed, which makes use of gradient waveguides and homogeneous layers.

3.1. Diffusion waveguides

The production of optical waveguides with the ion exchange method consists in modifying the refractive index of glass substrate in the surface area [4]—[6].

The most commonly used ion exchange processes for the production of planar and strip waveguides are as follows: $K^+ - Na^+$ and $Ag^+ - Na^+$. The use of silver as admixture (ion exchange $Ag^+ - Na^+$) can result in an increase of refractive index that can reach 0.1. In the case of ion exchange $K^+ - Na^+$, the maximum values of an increase of refractive index obtained are by one order smaller ($\approx 0.01$).

3.1.1. Ion exchange $K^+ - Na^+$

Planar waveguides produced in the ion exchange $K^+ - Na^+$ exhibit birefringence [15], [16]. Birefringence results from mechanical stresses, which are generated after the potassium ions $K^+$ are introduced to the glass substrate. It is caused by big difference in the radii of the exchanged ions. As a result, the refractive profiles of the planar waveguides produced depend on the polarization of light. For the TM polarization, higher changes of the refractive index have been observed compared to the TE polarization. Figure 3 presents refractive profiles of the single-mode waveguide produced in the glass BK7 when melted potassium nitrate
**Fig. 3.** Refractive profiles of the waveguide produced in ion exchange $\text{K}^+–\text{Na}^+$ in glass BK7. For parameters of the process see the text.

$\text{KNO}_3$ has been used as the source of ions $\text{K}^+$ and the process is carried out at a temperature of $400\,^\circ\text{C} (673\,\text{K})$ for $0.5\,\text{h}$. The depth of the changes of refractive index is $2.9\,\mu\text{m}$.

**Fig. 4.** Influence of refractive index of the cover $n_c$ on effective refractive indices of fundamental modes of the planar waveguide having the refractive profiles as presented in Fig. 3.

**Fig. 5.** Dependence of the difference of effective refractive indices $\Delta N$ and sensitivity $S_n$ on the refractive index of the cover $n_c$ for the waveguide produced in ion exchange $\text{Na}^+–\text{K}^+$ in glass BK7. For parameters of the technological process see the text.
The influence of the refractive index of the cover $n_c$ on effective refractive indices $N$ of the fundamental modes of this waveguide is presented in Fig. 4. With an increase of the refractive index of cover $n_c$ the effective refractive indices of the modes being investigated also increase. The increment is different for polarizations TE and TM. The strongest changes of effective indices are observed when the effective index of the cover $n_c$ is approaching the refractive index of the waveguide, i.e., when the modes are near cut-off. The influence of refractive index of the cover $n_c$ on the difference of effective refractive indices $\Delta N$ of modes $TM_0$ and $TE_0$ is presented in Fig. 5. The difference of effective refractive indices $\Delta N$ is changing with an increase of the refractive index of cover $n_c$. For the waveguides produced with the ion exchange method $K^+-Na^+$ the difference of effective indices $\Delta N$ of modes $TM_0$ and $TE_0$ increases with the refractive index of the cover $n_c$. This is characteristic for waveguides which exhibit birefringence. The increase in the difference of effective refractive indices $\Delta N$ is slightly above $4 \cdot 10^{-4}$, when the refractive index of the cover $n_c$ changes within the range from 1.330 to 1.500. The same figure presents the sensitivity curve $S_n$ which changes from $S_n = 0.001$ for $n_c = 1.330$ to $S_n = 0.010$ for $n_c = 1.500$. For the sake of comparison, the refractometer in the interferometer system of Mach–Zehnder, described in [12], which was produced in the ion exchange $K^+-Na^+$ was characterized by sensitivity $S_n = 0.0003$ for refractive index of the cover $n_c = 1.330$ and sensitivity $S_n = 0.0140$ for refractive index of the cover $n_c = 1.510$, respectively.

### 3.1.2. Ion exchange $Ag^+-Na^+$

Waveguides produced in the ion exchange process $Ag^+-Na^+$ do not exhibit birefringence [4]. Hence, the refractive profiles of these waveguides do not depend on polarization. Figure 6 presents the refractive profile of planar waveguide which corresponds with the ion exchange $Ag^+-Na^+$ carried out at temperature $T = 300 ^\circ C (573 K)$ for 15 minutes, when the melted salt $AgNO_3$ has been used as admixture source. The depth of the changes of the refractive index is $1.4 \mu m$.

The influence of refractive index of the cover on effective refractive indices of fundamental modes $TE_0$ and $TM_0$ is presented in Fig. 7. In this case, along the whole range of changes of $n_c$, it is the effective refractive index corresponding to polarization TE that has higher value. Together with an increase of refractive index of the cover $n_c$, a faster growth of the effective refractive index corresponding to polarization TM is observed. Hence, as presented in Fig. 8, the difference of effective refractive indices of the modes being investigated $\Delta N = |N_{TM} - N_{TE}|$ decrease with refractive index of the cover $n_c$. The change of the difference $\Delta N$ within the range $n_c = 1.330 - 1.500$ is about $20 \cdot 10^{-4}$. The changes can be seen to be five times higher compared to the changes presented above for the ion exchange $K^+-Na^+$. The same Figure presents the dependence of sensitivity $S_n$ on refractive index of the cover $n_c$. 
Fig. 6. Refractive profile of the waveguide corresponding to ion exchange process $\text{Ag}^+ - \text{Na}^+$ in glass BK7. For parameters see the text.

Fig. 7. Influence of refractive index of the cover $n_c$ on effective refractive indices of fundamental modes of the planar waveguide having the refractive profiles as presented in Fig. 6.

Fig. 8. Dependence of the difference of effective refractive indices $\Delta N$ and sensitivity $S_n$ on refractive index of the cover $n_c$, for the waveguide produced in ion exchange $\text{Na}^+ - \text{Ag}^+$ in glass BK7. For parameters of the technological process see the text.

Sensitivity $S_n$ for this waveguide reaches the value $S_n = 0.0055$ for $n_c = 1.330$ and $S_n = 0.0120$ for $n_c = 1.500$. And it is higher within this range of $n_c$ than the sensitivity of the waveguide produced in ion exchange $\text{K}^+ - \text{Na}^+$. While com-
paring the relations $\Delta N(n_c)$ for waveguides produced in ion exchange $K^+ - Na^+$ (Fig. 5) and $Ag^+ - Na^+$ (Fig. 8), we can see qualitative difference between them. The increase of refractive index of the cover $n_c$ of the waveguide obtained as a result of ion exchange process $K^+ - Na^+$ leads to the rise of the difference of effective indices $\Delta N$, and in the case of ion exchange $Ag^+ - Na^+$ the difference $\Delta N$ is getting smaller. Also the level of sensitivity $S_n(n_c)$ is different for both waveguides. It was shown in [20] that the increase of diffusion times and heating of waveguides results in the lowering of sensitivity $S_n$ for both waveguide types.

3.2. Homogeneous waveguides

Homogeneous waveguides are produced with the use of sol-gel method, oxide system $SiO_2:TiO_2$ or semiconductor technologies, where $Si_3N_4$ layers or AlGaAs layers are formed. In the following, the author analyzes the influence of the parameters of homogeneous layer on sensitivity $S_n$ for different types of interference. Waveguide layers of the refractive index $n = 1.649$ were produced by the author with the use of sol-gel method [21]. Figure 9 presents modal characteristics of such

![Modal characteristics of homogeneous waveguide](image)

Fig. 9. Modal characteristics of homogeneous waveguide. Refractive indices of: waveguide layer $n_i = 1.649$, substrate $n_s = 1.511$, cover $n_c = 1.000$. Wavelength $\lambda = 677$ nm.

a waveguide. For the thickness of waveguide layer $d = 221.8 - 682.8$ nm, only fundamental modes $TE_0$ and $TM_0$ can propagate in the waveguide. With the thickness of the layer higher than $d = 682.8$ nm also the mode $TE_4$ can propagate, and when the thickness is higher than $t = 734.4$ nm the propagation of the mode $TM_4$ is possible. Further increase of the thickness $d$ brings about the conditions for the propagation of the next modes. It can be seen from the above that when we apply the interference of the modes having the same polarization state $TE_0$ and $TE_1$ or $TM_0$ and $TM_1$, the waveguide layer must be thicker than $d = 734.4$ nm. Making
use of the interference of the zero order modes, we can apply waveguide layers of smaller thickness, but not lower than 221.8 nm. In practice, however, the thickness of waveguide layers should be selected in such a way as to ensure that the modes applied are far from the cut-off.

3.2.1. Interference of modes $\text{TE}_0 - \text{TM}_0$

The character of the dependence of effective refractive indices on the refractive index of the cover $N(n_c)$ for homogeneous waveguides is the same as for the waveguide produced in the ion exchange process $\text{Ag}^+ - \text{Na}^+$ (Fig. 7). The influence of refractive index of the cover on the differences of effective indices $\Delta N$ of the 0-order modes $\text{TM}_0$ and $\text{TE}_0$ for a homogeneous waveguide layer is presented in Fig. 10. These relations correspond to the different thicknesses of waveguide layer, marked in the picture. Refractive index of waveguide layer $n_l = 1.649$ and refractive index of the substrate $n_b = 1.511$. The calculations were carried out for the wavelength $\lambda = 677$ nm.

![Fig. 10. Influence of refractive index of the cover on the differences of effective refractive indices. For parameters of the waveguides see the text.](image)

The results show that within the investigated range of changes of refractive index of the cover, maximum difference of effective indices $\Delta N$ grows with a decrease of the thickness $d$ of waveguide layer. The difference gets lower with an increase of refractive index of the cover $n_c$. Within a wide range of the refractive index of the cover, the difference $\Delta N$ increases with a decrease of the thickness of waveguide layer. The thickness of waveguide layer $d$ has a considerable influence on sensitivity $S_n$, as indicated by the calculation results in Fig. 11. The calculated relations show that with the drop in thickness $d$ of waveguide layer, an increase of sensitivity $S_n$ should be expected. However, the thickness of the layer
cannot be arbitrarily small, since, for modes near cut-off, these relations have slightly different character. It is visible for the run corresponding with the thickness $d = 260$ nm. For low refractive indices of the cover $n_c$, sensitivity $S_n$ corresponding with that thickness is lower than for higher values $d$. For higher refractive indices of the cover $n_c$, sensitivity $S_n$ for that layer grows and surpasses sensitivity values corresponding with the remaining thickness values. However, that range can be of little importance, since for modes near cut-off, strong attenuation of these modes is likely to occur.

The influence of refractive index of waveguide layer $n_1$ on sensitivity $S_n$ is illustrated by the calculation results in Fig. 12. The figure presents three families of characteristics corresponding to different thicknesses $d$ of waveguide layer. In each case of the relation $S_n(n_c)$, we can distinguish two ranges of $n_c$, where different dynamics of sensitivity increase is observed. It is very clear for $d = 350$ nm and $d = 250$ nm. In the first range, corresponding with lower values of refractive index of the cover $n_c$, lower increase dynamics of $S_n$ is observed, caused by an increase of refractive index of the cover $n_c$. In that range, higher values of refractive index of the waveguide layer correspond with higher sensitivity values $S_n$. This range gets longer with an increase of refractive index of waveguide layer $n_1$.

For higher refractive indices of the cover $n_c$, the influence of refractive index of waveguide layer $n_1$ on sensitivity $S_n$ is quite reverse. But, as indicated above, that range is of little value for practical applications.

3.2.2. Interference of modes $TM_0 - TM_1$

The dependence of the difference of effective refractive indices $\Delta N$ of modes $TM_0$ and $TM_1$ on the refractive index of the cover $n_c$, for different thicknesses $d$
Fig. 12. Influence of refractive index of the cover $n_c$ on sensitivity $S_n$ for different parameters of waveguide layer. Interference of fundamental modes $\text{TE}_0-\text{TM}_0$.

Fig. 13. Dependence of the difference of effective refractive indices of modes $\text{TM}_0$ and $\text{TM}_1$ on refractive index of the cover $n_c$ for homogeneous planar waveguide. Refractive indices: substrates $n_b = 1.511$, waveguide layer $n_i = 1.649$. Calculations for wavelength $\lambda = 677$ nm.

of waveguide layer is presented in Fig. 13. The relations correspond with the structure in which the refractive index of waveguide layer $n_i = 1.649$, and the refractive index of the substrate $n_b = 1.511$. The calculations were carried out
for the wavelength $\lambda = 677$ nm. The relation $\Delta N(n_c)$ is similar here in character to the same relation for waveguides produced in ion exchange Na$^+ - $Ag$^+$. Figure 14 presents the dependence of sensitivity $S_n$ on refractive index of the cover $n_c$. The highest sensitivity, as it was for the interference of substrate modes, corresponds to the thinnest waveguide layer, within the whole range of changes of refractive index of the cover $n_c$. The sensitivity $S_n$ for the interference of modes having polarization TM increases proportionally to a decrease of the waveguide layer thickness $d$. The same results are obtained for modes having the polarization TE. But the sensitivity $S_n$ is in that case smaller than the sensitivity corresponding with polarization TM. Comparing the sensitivity values $S_n$, which can be obtained with the application of homogeneous waveguide layers having the same refractive indices, we can see that for the interference of modes TM$_0$, TM$_1$ it is higher than for fundamental modes TE$_0$, TM$_0$. For the refractive index of the cover $n_c = 1.500$, the sensitivity $S_n$ which can be obtained for the interference of modes TM$_0$, TM$_1$, depending on the thickness $d$ of waveguide layer, may be even several times higher than the sensitivity available with the interference of fundamental modes being applied. However, practical realization of the structure of planar difference interferometer with the interference of modes having the same polarization state will be slightly difficult since for such cases the waveguide layer applied should have the thickness close to 1000 nm, while in the sol-gel technology, in a single technological process, it is possible to obtain two-component layers SiO$_2 - $TiO$_2$ of good optical parameters where thickness does not exceed 300 nm. Due to the stresses involved in the process, the layers of greater thickness get cracked.
3.3. Structure of the type gradient waveguide — homogeneous layer

The above considerations involved the difference interference in gradient waveguides and homogeneous waveguides. Comparing the results obtained it follows that the difference interferometer produced with the application of homogeneous waveguides is characterized by a considerably higher sensitivity than the interferometer produced with the use of gradient waveguides.

All results presented in the paper were obtained with the application of the $4 \times 4$ matrix method of waveguide analysis \cite{1}, \cite{18}, \cite{19}. However, in order to explain the problem of higher sensitivity of homogeneous waveguides to the changes of refractive index of the cover it is more convenient to apply characteristic equations obtained in the WKB approximation method of waveguide analysis. Therefore, let us consider two waveguides of comparable parameters: a homogeneous waveguide of the constant refractive index and a waveguide having a gradient profile of refractive index. The refractive profile of the gradient waveguide corresponds with polarization TM (Fig. 3). The profiles of the waveguides being investigated are presented in Fig. 15. These waveguides have the same refractive indices of the substrate $n_b = 1.5137$, and the refractive index of the homogeneous waveguide is equal to the refractive index of the gradient waveguide $n_1 = 1.5265$ on the surface $z = 0$. In addition, the thickness of homogeneous waveguide $d = 813.93$ nm has been selected in such a way as to ensure that both waveguides, for the refractive index of the cover $n_c = 1.310$, have the same effective refractive indices $N = 1.5138$. Characteristic equations for the polarization TM, respectively for homogeneous waveguide and gradient one (Fig. 15) are as follows:
where $z_m$ stands for the position of the turning point of the $m$-th mode.

Let us consider how an increase of refractive index of the cover $n_c$ influences the effective refractive indices $N$ of the modes in each of the waveguides. Let us assume for a moment that the effective refractive index $N$ is independent of the refractive index of the cover $n_c$. Furthermore, let us disregard for a while the third expression in Eq. (3). When the refractive index of the cover $n_c$ increases the second expression in both equations decreases. Hence, to ensure that both characteristic equations are still satisfied, their first expressions must decrease, as well. In the characteristic Eq. (3) of the homogeneous waveguide the value of the first expression may decrease only with an increase of the effective refractive index $N$. It can be easily seen that the changes in the value of the third expression in Eq. (3) are smaller than the changes involving the value of the second expression. Therefore, it seems that the changes of effective refractive index $N$ are slower than the changes of refractive index of the cover $n_c$. It can also be seen that in the homogeneous waveguide the changes of effective refractive index $N$ will be the bigger the smaller its thickness $d$ is. In the case of the waveguide with gradient distribution of refractive index, to effect the decrease of the first expression (4), the effective refractive index $N$ must also increase as in the case of homogeneous waveguide. Therefore, it results in shifting the position of the turning point to the waveguide's surface, which means lowering the limit of integration $z_m$. Hence, the required change of the value of the first expression in Eq. (4) is obtained with lower change of effective refractive index $N$ compared to the homogeneous waveguide. This is illustrated in Fig. 15. For the refractive index of the cover $n_c = 1.310$, effective refractive indices of modes $TM_0$ for both waveguides are the same and equal to $N = 1.51382$. The turning point of the mode $TM_0$ in the gradient waveguide is then in the position $z_a = 2.37 \mu m$. When the refractive index of the cover approaches the value $n_c = 1.480$, the turning point is shifted to the position $z_b = 1.87 \mu m$, and the effective index value increases to $N = 1.51451$. And in the homogeneous waveguide, the effective refractive index of the mode $TM_0$ increases to the value $N = 1.51505$. Therefore, it can be seen that the homogeneous waveguide is characterized by bigger increase of the effective refractive index than the gradient waveguide. The above considerations explain why the waveguides produced in the ion exchange process $K^+ - Na^+$ are characterized by the smallest dependence $N(n_c)$ of all the profiles discussed above. In these waveguides, considerable changes in the position of turning point $z_m$ correspond with small changes of the effective refractive index. Stronger dependence $N(n_c)$ is characteristic for waveguides produced in the ion exchange process $Ag^+ - Na^+$ since their refractive profiles are characterized by bigger changes of refractive index $n(z)$ than the profiles of waveguides produced in
the ion exchange process $K^+ - Na^+$. Therefore, slight changes in the position of the turning point will correspond here with bigger changes of the effective refractive index $N$. The same conclusions can be reached by the analysis of characteristic equations for the polarization TE. But in that case when the waveguides do not exhibit birefringence, the dependence $N(n_c)$ will be smaller than for the polarization TM. Based on the detailed calculation results for gradient and homogeneous waveguides presented above, also the dependence involving the difference of effective refractive indices $\Delta N(n_c)$ is stronger for homogeneous waveguides than for gradient ones.

We can conclude from the above that there is a possibility to produce the structure of planar difference interferometer in which, in order to obtain considerably higher values of sensitivity than obtained so far, the properties of gradient waveguides and homogeneous waveguides can be simultaneously applied.

![Diagram of the structure (type G—H) under investigation.](image)

The diagram of a new planar structure proposed by the author, which can be produced from the gradient waveguide (G) and homogeneous layer (H), is presented in Fig. 16. This structure will be further referred to as G—H structure. The picture is out of scale. In reality the depth of the gradient waveguide is at least a few times greater than the thickness $d$ of homogeneous layer. On the planar waveguide with the gradient profile of refractive index — the same as the one produced in ion exchange process in glass, there is a dielectric layer of the refractive index $n_f$ and thickness $d$. The refractive index of the layer is higher than the refractive index of the gradient waveguide ($n_f > n_1$) on the surface $z = 0$. When the parameters of the structure are selected in such a way as to ensure that two modes of the same polarization state can be excited in it, e.g., TM$_0$ and TM$_1$ (or TE$_0$ and TE$_1$), with the fundamental mode being excited in the homogeneous layer and the first order mode having its turning point in the area of gradient changes of refractive index, then the structure will be very sensitive to changes of refractive index of the cover $n_c$. The fundamental mode, since it propagates in the homogeneous waveguide, will be more sensitive to changes of refractive
index of the cover $n_c$ than the first order mode whose turning point is lying in the area of gradient changes of refractive index.

Below, the results of analysis for such a structure are presented for the interference of modes of TM polarization. The results were obtained for gradient profile, the same as that produced in glass BK7 in the ion exchange process $\text{Na}^+ - \text{K}^+$, carried out from pure potassium nitrate at 400 °C for 4 h ($a = 7.9 \, \mu\text{m}$). The refractive index of dielectric layer in the structure under investigation was assumed to be $n_f = 1.649$.

![Fig. 17. Influence of refractive index of the layer on effective indices of modes TM$_0$ (a) and TM$_1$ (b).](image)

The influence of refractive index of the cover $n_c$ on effective indices of modes TM$_0$ and TM$_1$ for the G—H structure investigated is presented in Fig. 17. The picture presents theoretical dependences $N(n_c)$ which correspond with various thicknesses of the dielectric layer. The horizontal broken line stands for the level of refractive index $n = 1.5265$ corresponding with the refractive index of gradient waveguide on the surface $z = 0$. The increase of the refractive index of the cover $n_c$ results in the increase of effective indices of the modes under investigation. For modes TM$_1$ an increase of effective index is small and comparable with an increase occurring for fundamental modes in gradient waveguides without dielectric layers as discussed above. A different character of the dependence $N(n_c)$ in the G—H structure is observed for the fundamental mode TM$_0$. On each of the characteristics presented in Fig. 17a we can see two ranges: the first one with small dependence $N(n_c)$ and the other one with big dependence. The range of small dependence $N(n_c)$ reflects the situation in which the turning point of the fundamental mode is still lying in the area of gradient distribution of refractive index. The range of strong dependence $N(n_c)$ corresponds with the position of the turning point on the surface $z = 0$. The refractive index of the cover $n_o$, where the second range of the characteristic begins, depends on the thickness of dielectric layer.
Fig. 18. Dependence of the difference of effective refractive indices of modes $TM_0$ and $TM_1$ on refractive index of the cover $n_c$.

Fig. 19. Dependence of sensitivity $S_n$ on refractive index of the cover $n_c$ for different thicknesses of dielectric layer.

Calculation results involving $\Delta N_{\text{eff}}(n_c)$ for modes $TM_0$ and $TM_1$, for different thicknesses $d$ of the dielectric layer are presented in Fig. 18. As one could expect, based on the results presented earlier (Fig. 17), we can also find two ranges on the characteristics which have different dynamics of changes $\Delta N(n_c)$. For the $G - H$ structure, the difference of effective refractive indices $\Delta N$ also increases with refractive index of the cover $n_c$. The increase is considerably bigger in the second range, which corresponds to the propagation of mode $TM_0$ in the homogeneous layer.

Figure 19 presents the calculated dependences of sensitivity $S_n$ on the refractive index of the cover $n_c$. The sensitivity $S_n$ for the $G - H$ structure which has the structure of the thickness $d = 220$ nm ($n_f = 1.649$) is almost by one order higher than the one which can be obtained with the interference of fundamental modes, using a homogeneous waveguide layer of the same refractive index as here ($n_f = 1.649$). It is also higher than sensitivities which can be obtained using homogeneous waveguides of high refractive indices (Fig. 12). Furthermore, this sensitivity changes within narrow range when $n_c$ is going up, as opposed to the cases of structures discussed earlier for which the sensitivity $S_n$ undergoes considerable changes with an increase of refractive index of the cover $n_c$. For $d = 220$ nm and $n_c \approx 1.330$, the sensitivity of $G - H$ structure, $S_n = 0.247$, is over twice as high than the sensitivity offered by the homogeneous waveguide of the refractive index $n_1 = 2.00$ and thickness $d = 180$ nm (Fig. 12). By decreasing the thickness of the homogeneous layer in the $G - H$ structure we can obtain an
increase of sensitivity even a few times for higher values of refractive index of the cover $n_c$. For smaller thicknesses of the homogeneous layer $d$, the sensitivity $S_n$ decreases with an increase of $n_c$.

It can be seen from the results presented that the proposed G—H structure of the planar difference interferometer offers very high sensitivity, considerably higher than sensitivity available for structures which only apply homogeneous waveguides, having the same refractive index as the dielectric layer deposited on the gradient waveguide. Another advantage of this structure is the possibility of adapting the sensitivity $S_n$ to a particular range of changes of refractive index of the cover. This sensitivity can be modified within wide range by a respective selection of the thickness of dielectric layer.

It results from the calculations carried out for the G—H structures with homogeneous layers of higher refractive indices that it is possible to obtain still higher values of sensitivity $S_n$ than the ones that have been presented here. For homogeneous layers of the refractive index $n_f = 1.750$ it is possible to obtain sensitivities twice as high as the ones presented here. Homogeneous layers of such a refractive index can be produced in the sol-gel technology. They are presently being investigated by the author and in the near future they will be applied to the G—H structure proposed in this paper.

We can conclude from the results presented in this paper that the application of the proposed structure in the systems of planar sensors will make it possible to obtain much higher values of sensitivity compared to the sensitivity to be found in systems produced now.

4. Summary

The paper presents the results of analysis of the influence of waveguide parameters on the sensitivity of planar sensors working in the system of difference interferometer. The studies covered both gradient waveguides produced in the ion exchange process in glass and homogeneous waveguides, which can be produced with the sol-gel technique. A new structure of planar difference interferometer has been proposed, which offers much higher values of sensitivity compared the ones to be found in systems produced so far. It has been demonstrated that in order to obtain big changes of effective refractive indices caused by the changes of refractive index of the cover, homogeneous waveguides of high refractive indices should be applied.

The modes in gradient waveguides are less sensitive to the changes of refractive index of the cover than the modes in homogeneous waveguides. This is effected by a simultaneous change of the position of the turning point of the mode in gradient waveguides when the refractive index of the cover is changed. For the waveguides being discussed, greater dependences $N(n_c)$ occur for waveguides characterized by profiles undergoing high changes $n(z)$. Hence, the modes in waveguides produced in ion exchange process $Ag^+ - Na^+$ are more sensitive to the changes of refractive
index of the cover than the modes in waveguides produced in the ion exchange process $K^+ - Na^+$. The paper has demonstrated that the sensitivity values $S_n$, which can be obtained for planar sensors working in the system of difference interferometer are to a considerable extent dependent on the parameters of planar waveguides. For gradient waveguides and homogeneous waveguides, the sensitivity $S_n$ increases with refractive index of the cover $n_c$. Making use of the interference of fundamental modes in planar waveguides produced in the ion exchange technique $K^+ - Na^+$ we can obtain sensitivity values ranging from $S_n = 0.001$ for $n_c = 1.330$ to $S_n = 0.010$ for $n_c = 1.500$. By the application of waveguides produced in ion exchange technique $Ag^+ - Na^+$ the sensitivity $S_n$ can be increased five times for $n_c \approx 1.330$. Considerably higher sensitivity values $S_n$ can be obtained by the application of homogeneous waveguides having high refractive indices. The sensitivity values they offer are higher even by two orders than the ones that can be obtained with the application of waveguides produced in the ion exchange technique $K^+ - Na^+$.

The paper puts forward a proposition of a new structure of planar difference interferometer (G — H) built from a gradient waveguide and homogeneous waveguide. Due to the application of the interference of mode $TM_0$, which propagates in the homogeneous layer, and of the mode $TM_1$, whose turning point is lying in the area of gradient changes of refractive index, the proposed structure offers very high values of sensitivity $S_n$ to the changes of refractive index of the cover. For the polarization TE the values of sensitivity are lower. By the application of gradient waveguides produced in the ion exchange technique $K^+ - Na^+$ as well as homogeneous layers of a relatively low refractive index $n \approx 1.65$ we can obtain the sensitivity $S_n > 0.2$ within the whole range of changes of the refractive index of the cover. This sensitivity is several times higher than the sensitivity that can be obtained with the application of only homogeneous layers of high refractive indices and the interference of fundamental modes. By selecting an appropriate thickness of homogeneous layer in the G — H structure we can obtain almost flat sensitivity characteristics within a wide range of refractive index of the cover. By the application of homogeneous layers of higher refractive indices in the G — H structure we will be able to obtain still higher values of sensitivity than the ones presented in the paper. The new structure of planar difference interferometer can be applied in planar chemical and biochemical sensors.

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References

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