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Influence of the length of a uniform fiber Bragg grating on the accuracy of measuring an impulsive strain

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The influence of the length of a uniform, unapodized and apodized fiber Bragg grating on the accuracy of converting an impulsive strain is discussed. Using the transfer matrix description of the grating, derived from the coupled mode theory, the reflectivity spectrum changes of the grating are calculated, which are caused by the strain pulses propagating along it. On the basis of the introduced effective central frequency of the grating, the rise-time error (RTE) and the amplitude error of the frequency change (AFCE) were calculated as a function of the ratio of the strain pulse leading front width to the grating length. These errors were calculated for different waveforms of the strain pulses. Charts presenting results of the calculations allow to select the proper length of the uniform fiber grating, when the converting error is established, and the waveform of the pulse is identified.

Keywords: optical fiber Bragg grating sensor, impulsive strain, converting error.

1. Introduction

In the applications of a fiber Bragg grating as a sensor to measure dynamic strains in a structure, where the strains are produced by mechanical impact, one must take into consideration the grating length. The disturbance at the point of loading will propagate into the structure in the form of strain waves propagating at speeds defined by the mechanical properties of the material of the structure. If the length of any strain pulse is small enough to be comparable with the grating length, then at any given instant there are likely to be appreciable differences in strain magnitudes along the grating. Under these circumstances, the grating output signal is not likely to be truly representative of the impressed strain at the center of the grating. Therefore a fiber Bragg grating with a finite length does not convert the impulsive strains accurately. The distortions of the output signal of the grating, caused by its finite length, and arising during the converting of the strain pulse are shown in Fig. 1. In the quantitative assessment of the converting accuracy of the grating, errors are used:

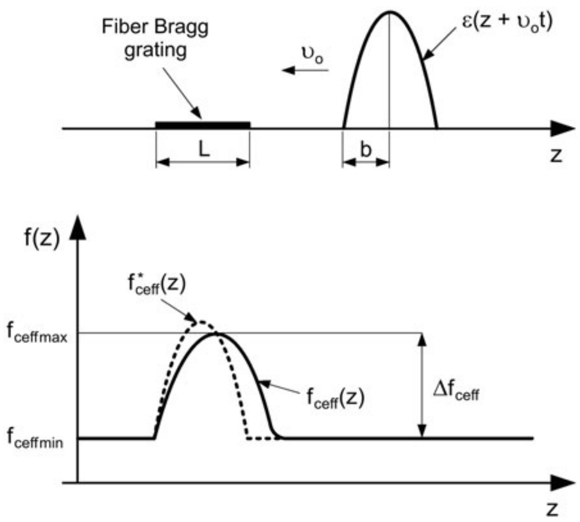


Fig. 1. The distortions of the output signal of the grating, caused by its finite length, and arising when converting the strain pulse, which propagates at the speed of υ_o. As the output signal of the grating, one assumes the effective central frequency: $f_{\text{ceff}}(t)$ for a finite-length grating, and $f^*_{\text{ceff}}(t)$ for a zero-length grating. Δf_{ceff} is the amplitude of the effective central frequency change – the deviation of the effective central frequency.

the RTE and the AFCE. These errors were calculated as a function of the ratio of the strain pulse rise-length to the grating length, for pulses with different waveforms, typical of mechanical impact.

The periodic perturbation to the effective index of refraction n_{eff} of the optical fiber core, that produces fiber Bragg gratings, is given by the following expression [1, 2]

$$\delta n_{\text{eff}}(z) = \overline{\delta n_{\text{eff}}}(z)\left[1 + s\cos\left(\frac{2\pi}{\Lambda}z + \varphi(z)\right)\right] \tag{1}$$

where s is the fringe visibility, Λ is the grating period, $\varphi(z)$ accounts for the grating chirp, and $\overline{\delta n_{\text{eff}}}$ is the "dc" index change spatially averaged over the grating period.

The model of the fiber Bragg grating is usually based on the coupled mode theory. This theory can be summarized as follows: the dominant interaction lies near the wavelength for which reflection occurs from a mode of amplitude $A(z)$ and an identical counter-propagating mode of amplitude $B(z)$. This leads to a set of the coupled mode equations [1–3]

$$\frac{du}{dz} = j\sigma u(z) + j\kappa v(z) \tag{2}$$

$$\frac{dv}{dz} = -j\sigma v(z) - j\kappa^* u(z) \tag{3}$$

where $u(z) = A(z)\exp(j\delta z - \varphi/2)$, $v(z) = B(z)\exp(-j\delta z + \varphi/2)$, $\sigma(z)$ is the general dc self coupling coefficient and $\kappa(z)$ is the ac coupling coefficient. These coefficients are defined as

$$\sigma(z) = \delta + \frac{2\pi}{\lambda}\overline{\delta n_{\mathrm{eff}}} - \frac{1}{2}\frac{\mathrm{d}\varphi}{\mathrm{d}z} \tag{4}$$

$$\kappa(z) = \frac{\pi}{\lambda}\overline{\delta n_{\mathrm{eff}}}\, s\, g(z) \tag{5}$$

where δ is the detuning, expressed as

$$\delta = 2\pi n_{\mathrm{eff}}\left(\frac{1}{\lambda} - \frac{1}{\lambda_B}\right) \tag{6}$$

where $\lambda_B = 2n_{\mathrm{eff}}\Lambda$ is the design wavelength for Bragg reflectance. The function $g(z)$ is the function of apodization. The derivative term $(1/2)\mathrm{d}\varphi/\mathrm{d}z$ describes the chirp of the grating period.

For a uniform grating, where σ and κ are constant, Eqs. (2) and (3) simplify into coupled first-order ordinary differential equations with constant coefficients. These equations can be solved analytically given the appropriate boundary conditions. The solution of the coupled-mode Eqs. (2) and (3), in a closed-form expression, in the case of non-uniform gratings, is more difficult to obtain. There is a variety of methods to compute the reflection and transmission spectra for non-uniform gratings. The most extensively used are: the direct numerical integration method and the transfer matrix method. This second method was used to calculate the reflection spectra of the grating, which converts a strain pulse. In this method the grating, with a total length L, is divided into a sufficiently large number M of sections with length Δz so that each section can be approximated as a uniform grating. The method is based on identifying a square matrix of the dimension 2×2 for each of the M uniform sections of the grating, and multiplying all of them together to obtain a single matrix 2×2 that describes the whole grating [1, 2]. In the case of reflection gratings, with boundary conditions $u(L) = 1$ and $v(L) = 0$, the propagation through each uniform section k is described by a matrix T_k such that

$$\begin{bmatrix} u_k \\ v_k \end{bmatrix} = T_k \begin{bmatrix} u_{k-1} \\ v_{k-1} \end{bmatrix}, \quad k = M, M-1, ..., 1 \tag{7}$$

where the matrix T_k for a Bragg grating is given by

$$T_k = \begin{bmatrix} \cosh(\gamma\Delta z) - j\dfrac{\sigma}{\gamma}\sinh(\gamma\Delta z) & -j\dfrac{\kappa}{\gamma}\sinh(\gamma\Delta z) \\ j\dfrac{\kappa}{\gamma}\sinh(\gamma\Delta z) & \cosh(\gamma\Delta z) - j\dfrac{\sigma}{\gamma}\sinh(\gamma\Delta z) \end{bmatrix} \tag{8}$$

where σ and κ are the local coupling coefficients for the k-th section, and $\gamma =$ $= (\kappa^2 - \sigma^2)^{1/2}$. The total grating structure can be described as

$$\begin{bmatrix} u_M \\ v_M \end{bmatrix} = T \begin{bmatrix} u_0 \\ v_0 \end{bmatrix} \tag{9}$$

where $T = T_M \cdot T_{M-1} \cdot ... \cdot T_k \cdot ... \cdot T_1$ is the overall transfer matrix. As a result, T is a 2×2 matrix with the elements

$$T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \tag{10}$$

Once T is found, the amplitude reflection coefficient r and the power reflection $R = |r|^2$ coefficient are calculated by the relations

$$r(\delta) = -\frac{T_{21}}{T_{22}}, \qquad R(\delta) = \left| -\frac{T_{21}}{T_{22}} \right|^2 \tag{11}$$

obtained by substitution of the appropriate boundary conditions into (9). The number of sections M cannot be made arbitrarily large, since the coupled-mode theory is no longer valid when a uniform grating section is only a few grating periods long [1, 2]. This condition can be stated as

$$M \ll \frac{2 n_{\mathrm{eff}} L}{\lambda_B} \tag{12}$$

2. Method

For strain-sensing applications, when the grating is subjected to a non-uniform strain, different parts of the grating will contribute to different wavelengths according to the local state of the strain. As a consequence, its reflected spectrum will not only be shifted, but also distorted due to non-uniform changes of the local index of refraction and the grating period. For the uniform grating subjected to an axial strain $\varepsilon(z)$, these two effects can be taken into consideration through an effective grating period described by [4]

$$\Lambda(z) = \Lambda_0 \left[1 + (1 - p_e)\,\varepsilon(z) \right] \tag{13}$$

where Λ_0 is the period of the grating without the strain and p_e is the effective strain-optic constant.

The adaptation of the transfer matrix method to the modeling of a uniform grating structure along which a strain pulse propagates, is based on calculating the average period for each grating section due to the strain pulse using (13) and putting the calculated period directly into the local coupling coefficient according to the relation

$$\sigma(z) = \frac{2\pi}{\lambda}(n_{\text{eff}} + \overline{\delta n_{\text{eff}}}) - \frac{\pi}{\lambda(z)} \tag{14}$$

For the consideration of the influence of the length of a uniform grating on the accuracy of converting an impulsive strain one defines a non distorting grating and an effective central frequency of the grating. By "the non distorting uniform grating" one means a grating, whose reflective spectrum will be shifted, when the strain pulse propagates along it. At any given instant the uniform grating is subjected to a uniform strain with a value equal to the instantaneous value of the strain pulse acting on the beginning of the grating. A so-defined non-distorting grating replaces a zero-length grating in the simulation. By the effective central frequency of the grating one means the abscissa of the reflective spectrum centroid, defined as [5]

$$f_{\text{ceff}} = \frac{\int_0^{\infty} f R(f) \mathrm{d}f}{\int_0^{\infty} R(f) \mathrm{d}f} \tag{15}$$

Assuming that the effective central frequency of the grating is the output signal, the converting error was calculated as a function of the ratio of the leading edge length of the strain pulse to the grating length, for different waveforms of the strain pulses. In the calculations, as the converting errors of the strain pulses one assumes: the RTE δ_{rt}, and the AFCE $\delta_{A\Delta f}$. The RTE is defined as

$$\delta_{rt} = \frac{t_{ro} - t_{ri}}{t_{ri}} \tag{16}$$

where t_{ri}, t_{ro} are the rise times of the input signal, and the output signal of the finite--length grating, respectively. The AFCE $\delta_{A\Delta f}$ is described by

$$\delta_{A\Delta f} = \frac{\Delta f_{\text{ceff}} - \Delta f^*_{\text{ceff}}}{\Delta f^*_{\text{ceff}}} \tag{17}$$

where Δf_{ceff}, Δf^*_{ceff} are the amplitudes of the effective central frequency change of the finite-length grating and the zero-length grating, respectively. There are many rise time definitions. The rise time of any waveform is usually defined as the time taken

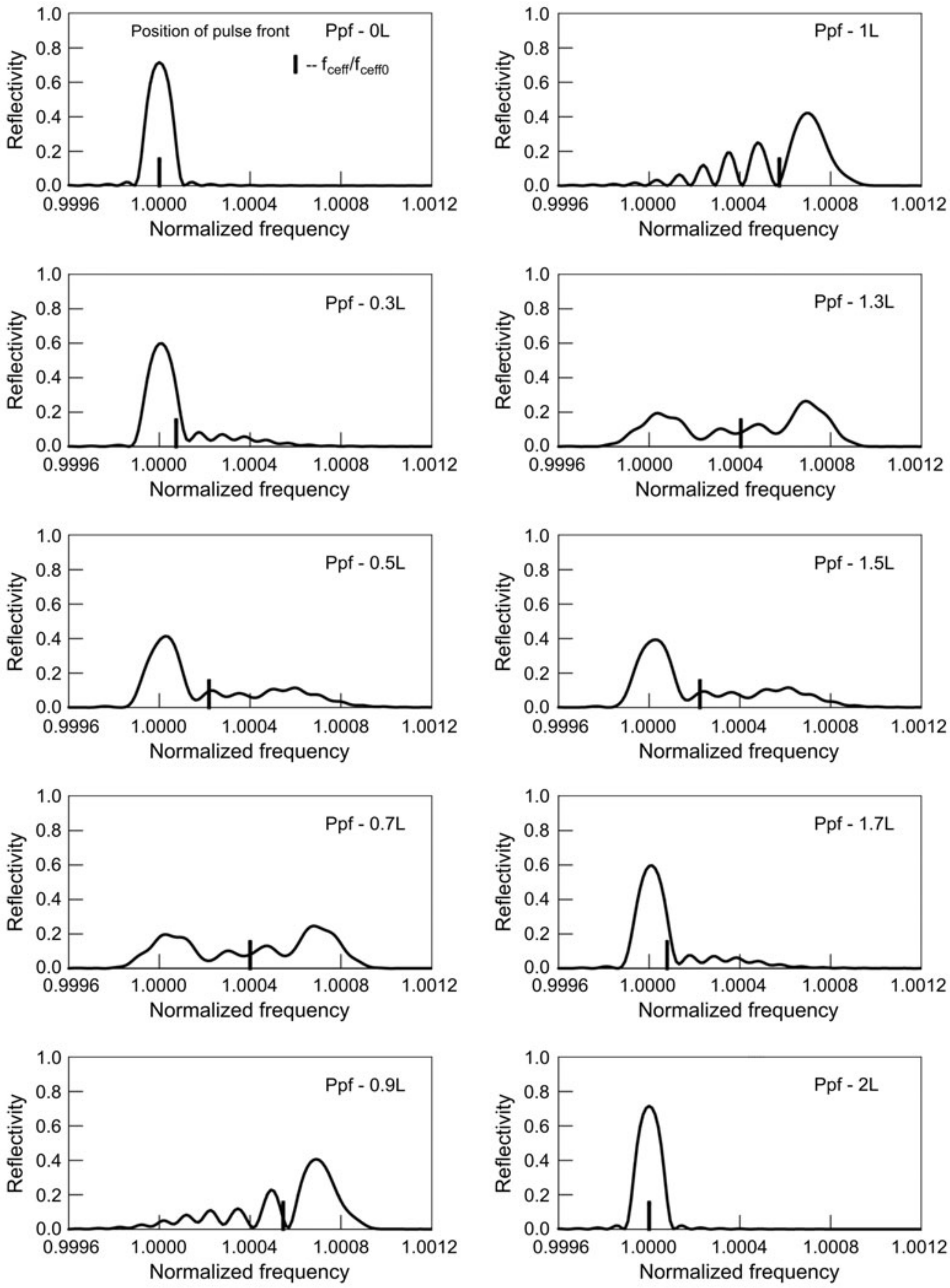


Fig. 2. The reflected spectrum and the corresponding effective central frequency of the apodized grating subjected to the single half-sine compressive strain pulse with an amplitude of $A = 1000\,\mu\varepsilon$ and a rise length of $b = L/2$, for the different position of the pulse front with respect to the beginning of the grating.

for the leading edge of the waveform to rise from 10 to 90 per cent of its peak value. Less frequently the rise time is defined as the time between the waveform reaching 5% and 95% or 0% and 100% of its final value. The rise time error was calculated for these three rise time definitions.

3. Results

In the calculations, the uniform unapodized and apodized grating were considered. The modeled gratings have the following parameters: $L = 6$ mm, $n_{\text{eff}} = 1.46$, $\lambda_B = 1550$ nm, $\overline{\delta n_{\text{eff}}} = 1.25\times10^{-4}$, $\overline{\delta n_{\text{eff}}} = 2.2\times10^{-4}$, $p_e = 0.22$, FWHM = 0.205 nm for the unapodized grating and FWHN = 0.200 nm for the apodized grating. These two different $\overline{\delta n_{\text{eff}}}$ values determine a range. Within this range lie the $\overline{\delta n_{\text{eff}}}$ values of the majority of uniform weaker gratings used as sensors. To model the apodized grating, the Gaussian function $g(z)$ in the formula (5) was used. For each transfer matrix simulation 200 grating sections were used.

The calculations of the RTE as a function of the ratio of the strain pulse leading front width to the grating length b/L were carried out in three stages. In the first stage, on the basis of the transfer matrix method, the reflected spectrum due to the propagating strain pulse is determined. In the second stage, using the determined spectrum and formula (15), the effective central frequency of the grating is calculated. In the third stage, the RTE and the AFCE are determined using formulas (16) and (17), respectively.

The strain pulses $\varepsilon(z)$ produced by impact can be approximated reasonably well by the shapes: the single half-sine pulse $\varepsilon(z) = A\sin(\pi z/2b)$, the whole-sine pulse $\varepsilon(z) = (A/2)[1 - \cos(\pi z/b)]$ and the constant slope front step function $\varepsilon(z) = (A/b)z$ for $z \le b$. The propagation of strain pulses of these forms along the grating will

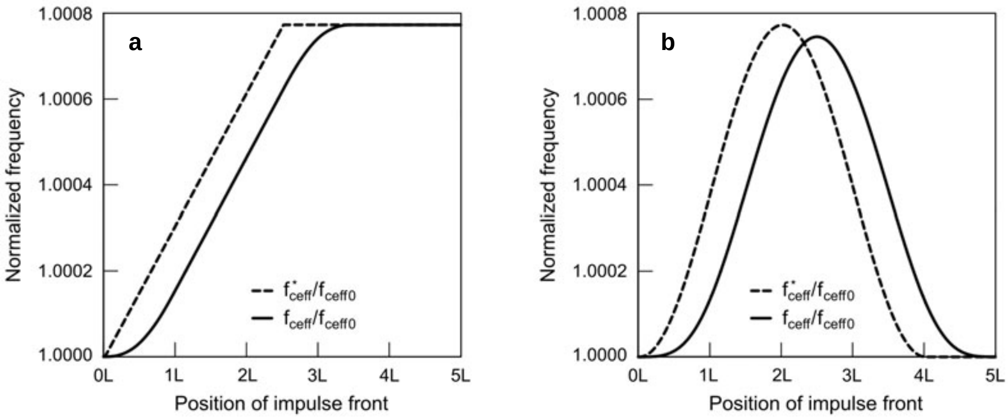
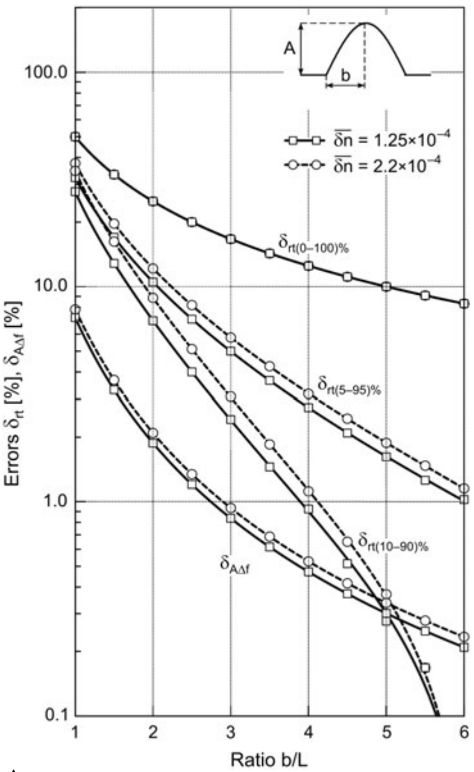


Fig. 3. The normalized effective central frequency of the apodized gratings: non-distorting one (dashed line) and distorting one (solid line), as a function of the front position of the acting compressive strain pulses. These pulses have an amplitude of $A = 1000\,\mu\varepsilon$ and the shapes: constant slope front step function with a rise length of $b = 2.5L$ (**a**), single whole-sine with a rise length of $b = 2.0L$ (**b**).

be considered. Each of the pulses has an amplitude A and leading edge width b. As an example, the reflected spectrum changes of the grating and the corresponding effective central frequency, due to the single half-sine compressive strain pulse propagating along the grating, are shown in Fig. 2. The amplitude of the strain pulse equals $1000\,\mu\varepsilon$, the strain pulse length is equal to the grating length $2b = L$.

Figure 3 shows the selected effective central frequency response of the real and the non-distorting grating subjected to strain pulses with the waveforms: the single half-sine and the constant slope front step function. This frequency response was calculated from relation (15). The converting errors: the RTE and the AFCE as functions of the ratio of the strain pulse leading front width to the grating length b/L, calculated by means of relations (16) and (17), respectively, for the uniform apodized grating, are shown in Figs. 4–6. The RTE was calculated for the following rise times: 10–90, 5–95 and 0–100 per cent. Figure 4 was plotted for the case of the half-sinusoidal strain pulse. Figure 5 represents similar results for the whole-sinusoidal strain pulse and Fig. 6 represents the results for the constant slope front step function.



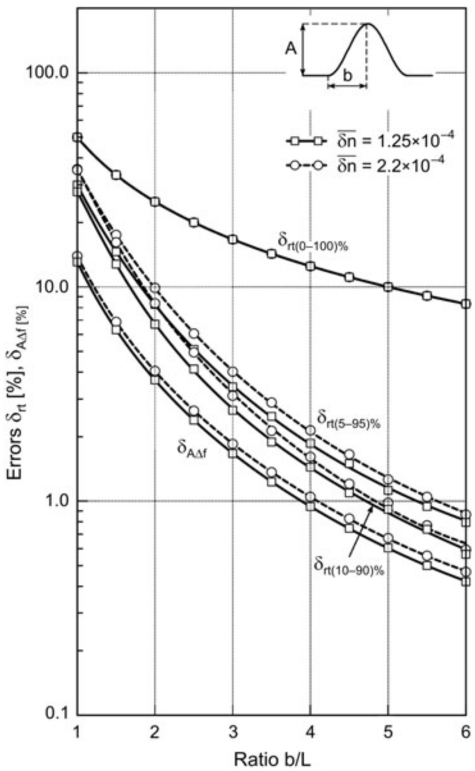


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Fig. 4. The converting errors of the apodized gratings as functions of the ratio b/L, for the half-sinusoidal strain pulse.

Fig. 5. The converting errors of the apodized gratings as functions of the ratio b/L, for the whole-sinusoidal strain pulse.

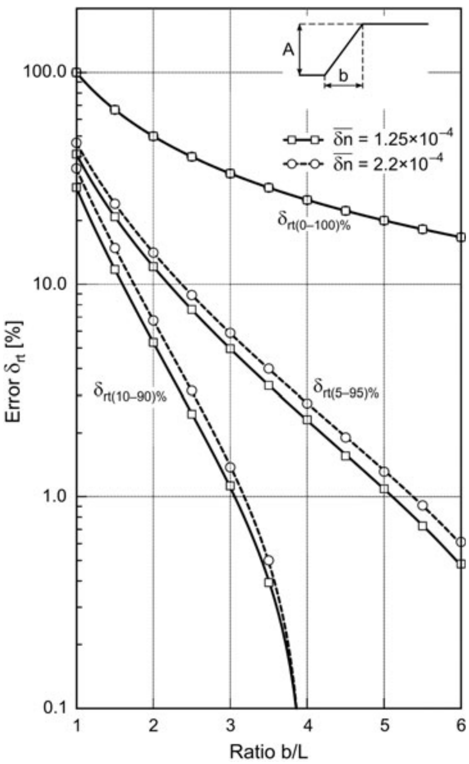

Fig. 6. The rise-time error of the apodized gratings as functions of the ratio b/L, for the constant slope front step function.

These results presented in Figs. 4–6, are in force for both compressive and tensile strain pulses. This follows from the fact that the reflected spectrum of the uniform grating caused by a tensile strain pulse is a mirror image of the grating spectrum caused by a compressive strain pulse. The RTE and the AFCE versus the ratio b/L characteristics for the unapodized grating are similar to the corresponding characteristics for the apodized grating.

4. Computational example

An electrical discharge in water allows to generate impulsive pressures of very short duration and very high amplitude, which can be measured by the fiber Bragg grating sensor [6]. These pressure pulses have a half-sinusoidal waveform of a few microseconds duration. These pulses have a measured 10–90 per cent rise time of 1.4 μs, a 5–95 per cent rise time of 1.6 μs and a 0–100 per cent rise time of 2.0 μs [7]. For these rise times, the corresponding strain pulse leading front widths are equal to $b_{(10-90)\%} = 8.0$ mm, $b_{(5-95)\%} = 9.0$ mm and $b_{(0-100)\%} = 11$ mm, for the strain pulse propagation velocity in silica glass of $\upsilon_o = 5700$ m/s.

If we assume that the grating length is equal to $L = 2$ mm, then the ratio b/L equals: $b_{(10-90)\%}/L = 4$, $b_{(5-95)\%}/L = 4.5$ and $b_{(0-100)\%}/L = 5.5$. On the basis of the charts in Fig. 4, the conversion errors of the single half-sin pulse, caused by the apodized grating, are equal to: $\delta_{A\Delta f}$ = (0.47–0.52)%, $\delta_{rt(10-90)\%}$ = (0.9–1.1)%, $\delta_{rt(5-95)\%}$ = = (2.1–2.4)%, $\delta_{rt(0-100)\%}$ = 9.1%. These errors for the unapodized grating are equal to: $\delta_{A\Delta f}$ = (0.8–0.9)%, $\delta_{rt(10-90)\%}$ = (1.6–2.0)%, $\delta_{rt(5-95)\%}$ = (3.6–4.5)%, $\delta_{rt(0-100)\%}$ = = 9.1%.

This example shows that a small uniform grating can be used in the measurement of very rapid strain transients. In such measurements apodized gratings should be used, because their converting errors are smaller than those of unapodized gratings.

The converting errors of the gratings are little dependent on the $\overline{\delta n_{\text{eff}}}$ index. The RTE criteria of the grating length selection are more demanding than the AFCE criterion. If it is assumed, however, that the RTE values are not smaller than 2 to 3 per cent, then the RTE criteria are not likely to lead to impracticably small gratings. Currently it is easy to make a uniform grating with a length smaller than 1 mm.

5. Conclusions

The numerical analysis of the converting errors of the uniform Bragg gratings presented in the paper, new in the context of the grating length influence on the converting accuracy, is very useful. It allows to select the proper length of the uniform grating, which is to be used in the measurement of impulsive strain of very short rise time. It was shown that generally the AFCE is likely to be smaller than the RTE, and that consideration of the RTE develops a "worst case" design.

In the measurement of impulsive strain, apodized gratings should be used, because their conversion errors are smaller than those of unapodized gratings.

The worked example suggests that the RTE criteria of the grating length selection are not likely to lead to impracticably small gratings for the majority of applications.

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