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Resonant Faraday effect in a Fabry–Perot cavity

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A standard approach to Faraday effect in a Fabry–Perot cavity is extended to resonant absorbing media. The analysis indicates that application of the cavity improves a magnetometric sensitivity despite the resonant absorption. The degree of this improvement depends strongly on the medium's density and is maximal for diluted media. The best effect is expected with rarified media and/or with weak optical transitions.

Keywords: magneto-optical effects, Faraday rotation, optical cavity.

1. Introduction

We report on our study of the Faraday effect (FE) in a resonantly absorbing sample placed in a Fabry–Perot (FP) cavity. This study extends previous work on nonabsorbing Faraday media in FP resonators [1–3] onto the systems where large resonant enhancement may be expected when the light frequency is tuned to the resonance line of the sample. Along with the resonance enhancement of the dispersive response, there occurs corresponding enhancement of the medium's absorption. While such an enhancement finds numerous applications, *e.g.*, in sensitive detection of paramagnetic species [4], for the studies of Faraday rotation it is an adverse effect which makes it necessary to compromise between the expected rotation gain and lowering of the cavity finesse. The present study addresses this compromise for the case of linear FE and aims at establishing the conditions for future experiments on nonlinear Faraday rotation [5] in FP cavities. In the same way as FP cavity enhances the linear magneto-optical rotation, it should also enhance the nonlinear Faraday effect (NFE) in an intracavity sample.

While for purely dispersive, nonabsorbing media it was shown that FP cavity indeed enhances the Faraday rotation by approximately the cavity finesse [1, 2], for resonantly absorbing media this increase is reduced by inserted cavity losses and related lowering of the effective cavity finesse. This issue is particularly important for NFE, where the sample excitation close to optical resonance is necessary for reaching sizeable nonlinearity. The results of the present work are thus useful for applications of FE and NFE in magneto-optic sensors and magnetometry [6].

2. Fabry–Perot cavity with resonant Faraday medium

The layout of a typical experiment devoted to studying of FE in a FP cavity is depicted in Fig. 1. The Faraday sample S is placed inside a cavity constituted by mirrors $M_{1,2}$ and within a homogenous magnetic field B directed along z axis. The effect is studied with a linearly polarized light beam (incident electric field E_i) which propagates along B. The transmitted light beam is analyzed by a balanced polarimeter consisting of a polarizing beam splitter P and two detectors D_x and D_y which monitor the beam components polarized along e_x and e_y, respectively. The polarimeter is balanced when E_i is at 45° relative to x and y axes.

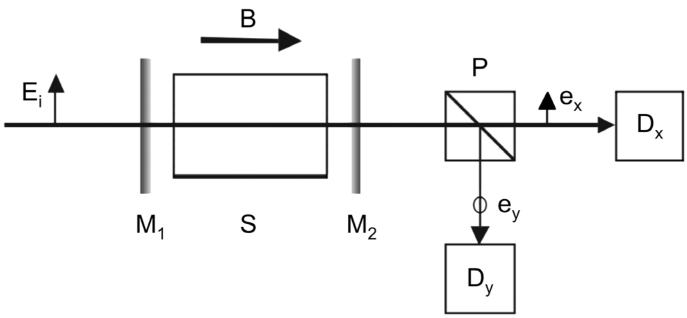

Fig. 1. Setup for measurements of FE. Sample S is placed inside a cavity (mirrors $M_{1,2}$) in a magnetic field B. The Faraday rotation is measured with a polarimeter made of a polarizing beam splitter P and detectors D_x and D_y which measure transmitted light components polarized along e_x and e_y, respectively.

For optically isotropic sample at $B = 0$, there is no Faraday rotation of the polarization plane of the transmitted beam, so the two polarimeter outputs have equal intensities, $I_x = I_y$, but for $B \neq 0$, the Faraday effect results in different output signals. For small rotations, the Faraday angle ϑ can be easily determined as

$$\vartheta = \frac{I_x - I_y}{2(I_x + I_y)} \tag{1}$$

The light beam frequency is tuned to the center of the absorption line of the sample or to its proximity.

2.1. Modeling of the Faraday effect in the cavity

We model the effect of resonant Faraday medium in a FP cavity by extending a standard method of calculation of the FP transmission to the case of optically anisotropic medium, such that the anisotropy (circular dichroism and circular birefringence) depends on external magnetic field, light beam detuning from the sample's resonance frequency and sample density. The medium is characterized by a single resonance frequency ω_0 which splits in a magnetic field into two components corresponding to two opposite circular polarization components $\sigma^{\pm}$. Each of the components is associated with a complex refractive index which for the case of linear FE, considered here, has a particularly simple form

$$\eta_{\pm} = 1 + \frac{\rho}{\omega - \omega_0 \mp \omega_L - i\Gamma/2} \equiv n_{\pm} + i\kappa_{\pm} \tag{2}$$

where ω is the light beam frequency, ω_L is the Larmor frequency, *i.e.*, the magnetic--field induced shift of the resonance frequency, Γ is the damping constant (the width of the resonance line), ρ and is the parameter responsible for the optical density of the medium at $B = 0$ which is proportional to the medium's density and its line strength. In the following calculations we have assumed $\Gamma = 6$ MHz and $\hbar\omega_L = g\mu_B B$ ($\hbar$, g, and μ_B being the Planck's constant divided by 2π, the Lande factor, and the Bohr magneton, respectively). For the sake of simplicity, we take $g = 1$. The quantities $n_{\pm}$ and $\kappa_{\pm}$ represent the regular index of refraction and absorption coefficient of the medium. To get most fundamental results on the FE in FP cavity, the medium does not need to be specified in more detail. It can be a solid, liquid or a gas sample, provided its spectrum can be approximated with a single resonance function (2) in a given spectral range. For more detailed treatment, however, the nature of line broadening needs to be considered more accurately. In particular, for atomic gas samples, one needs to take into account the effects of atom movement and related Doppler broadening. Also, for the future analysis of NFE, the role of the cavity-enhanced light intensity causing possible saturation and power broadening need to be considered in more complex expressions than Eq. (2).

Calculation of the light transmission across a FP cavity with resonant medium yields the following expressions for the electric field amplitudes $E_{\pm}$ of two circularly polarized principal waves exiting the cavity

$$E_{\pm} = E_0 \frac{t^2 T_{\pm} \exp(i\varphi_{\pm})}{1 - (r^2 T_{\pm})^2 \exp\left[i(\delta + 2\varphi_{\pm})\right]} \tag{3}$$

where E_0 is the amplitude of the incident light wave, t and r are the amplitude reflection and transmission coefficients of the cavity mirrors, respectively, $\delta = 2(\omega l/c)$, $\varphi_{\pm} = (n_{\pm} - 1)(\omega l/c)$, and $T_{\pm} = \exp[-\kappa_{\pm}(\omega l/c)]$ (l being the separation of the cavity mirrors and the Faraday medium length).

By projecting the exiting field with amplitudes (3) onto the x and y directions of the polarimeter axes, the intensities of the two polarimeter outputs can be found, and the net Faraday rotation signals can be analyzed as functions of all relevant parameters.

2.2. Faraday spectra and signals

Faraday spectrum is the dependence of the rotation angle on the light frequency $\vartheta(\omega)$. In Figure 2 we depict typical Faraday spectra calculated for various magnetic fields for a single pass of a light beam across the sample, *i.e.*, without any cavity.

Fixing the light frequency and varying the intensity of the magnetic field, one can study the Faraday signal $\vartheta(B)$. Figure 3 presents a typical single-pass Faraday signal calculated for resonance light frequency, $\omega = \omega_0$.

In Figure 4 we depict a 3D plot showing the $\vartheta(\omega, B)$ dependence of the single--pass rotation calculated for positive detunings $\Delta\omega = \omega - \omega_0$ for the same sample as in Figs. 2 and 3.

To assess possible practical applications of FP cavities for enhancing the sensitivity of the Faraday magnetometry, it is instructive to compare the Faraday rotation with and without the cavity. In Figure 5 we compare the single-pass Faraday signal from

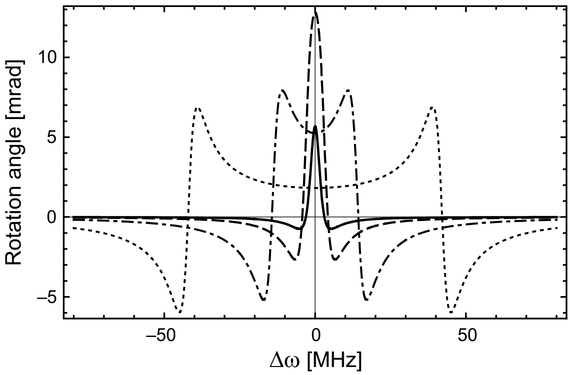

Fig. 2. Faraday rotation spectra $\vartheta(\omega)$ calculated for various magnetic field intensities: 0.5 G (solid), 2.15 G (dashed), 10 G (dot-dash), and 30 G (dotted). Assumed sample density corresponds to 5% maximum absorption.

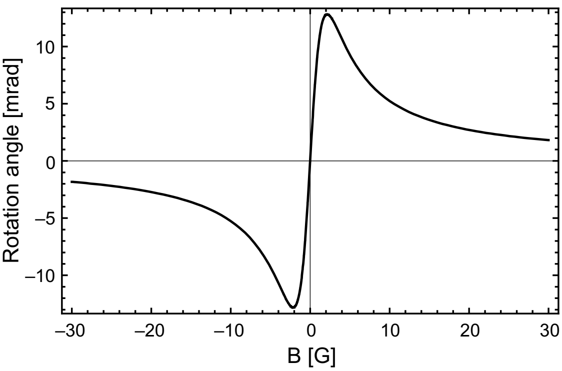

Fig. 3. Faraday rotation signal $\vartheta(B)$ for resonance excitation of a transition. Assumed sample density corresponds to 5% maximum absorption.

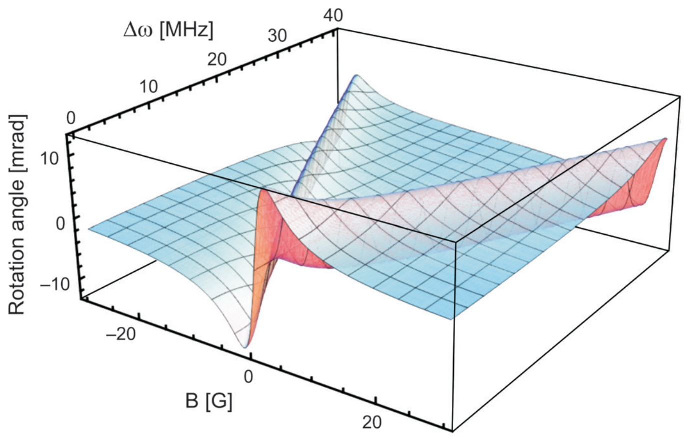

Fig. 4. Dependence of the rotation angle on laser detuning $\Delta\omega$ and magnetic field intensity B.

Fig. 3 (solid line) with the one calculated for a sample placed within a cavity with $|r|^2 = 0.95$ corresponding to *finesse* 61.5 (dotted line). As is well visible, the cavity significantly enhances the rotation of a given sample. However, the degree of the enhancement is not constant. It is maximal for samples with very low optical densities. For higher densities, a strong reduction of the enhancement occurs, which we attribute to lowering of the cavity finesse by an absorbing resonant medium. This effect is seen in Fig. 6 which depicts the rotation caused by a sample with 90% single-pass resonance absorption and other parameters the same as in Fig. 5. Much higher rotation angles seen with and without the cavity are caused by higher optical density of the sample. Maximal rotation is only slightly increased by the cavity. At the same time, there is a significant increase of the rotation at the wings of the Faraday signal. This effect demonstrates selective reduction of the cavity finesse by the resonant sample's absorption which is maximal close to $B = 0$. As the magnetic

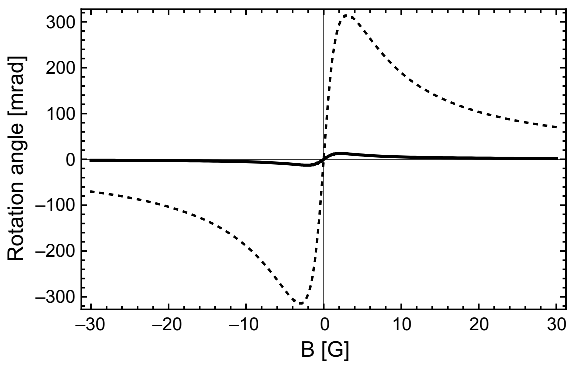

Fig. 5. A comparison of the signal obtained from the same sample with (dotted) and without (solid) a cavity. The sample's single-pass absorption is assumed to be 5% and the cavity mirrors have an intensity reflection coefficient of 0.95.

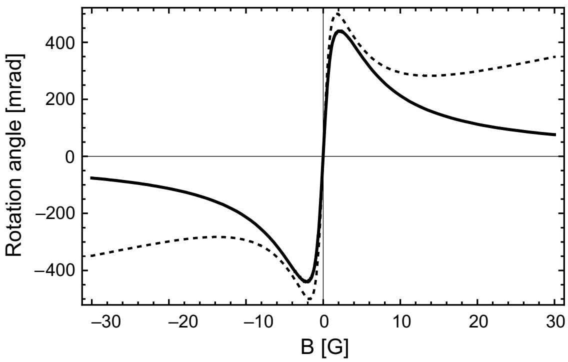

Fig. 6. A comparison of the signal obtained from the same sample with (dotted) and without (solid) a cavity. The sample's single-pass absorption is assumed to be 90% and the cavity mirrors have an intensity reflection coefficient of 0.95.

field increases, the absorption at the line center drops because of the Zeeman splitting of the resonance line which restitutes the cavity finesse and results in rotation increase. This effect might be useful for practical applications of FP in magneto-optical modulators as it relaxes strong magnetic-field dependence of the rotation angle beyond a given magnetic-field value.

3. Conclusions and outlook

We have extended the standard approach to FE in a FP cavity for resonant absorbing media. The analysis indicated that application of the cavity can improve magnetometric sensitivity despite the resonant absorption, but the degree of this improvement depends strongly on the medium's density and is maximal for diluted media. Thus, the best improvement is expected with rarified media and/or with weak optical transitions. Similarly as in Ref. [2], for small rotation angles, the Faraday rotation is enhanced

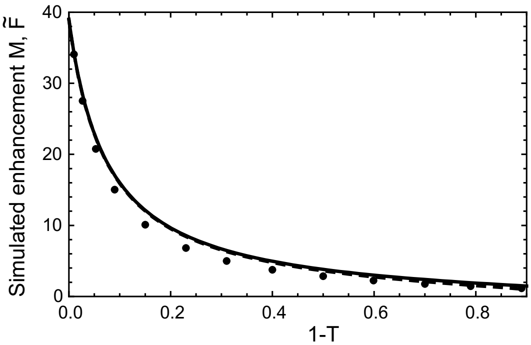

Fig. 7. Dependence of the enhancement factor of the maximum simulated Faraday rotation (at 1.4 G) – dots, the magnification factor M defined in text – solid line, and the effective finesse normalized to $2/\pi$ – dashed line, on the single-pass absorption $1 - T$.

by the cavity by factor $M = (1 + \tilde{R})/(1 - \tilde{R})$, where $\tilde{R} = r^2 T^2$ is an effective reflectivity coefficient accounting for intracavity losses ($T = \exp[-\kappa(\omega l/c)]$). Figure 7 depicts the enhancement factor obtained from our analysis as a function of the single--pass absorption $1 - T$ (dots). This dependence can be compared with the absorption dependences of the M factor and also of the effective finesse of the cavity with the Faraday medium, defined as $\tilde{F} = \pi\sqrt{\tilde{R}}/(1 - \tilde{R})$ (solid and dashed lines, respectively). As seen from the figure, the effective finesse gives a good account of the density effect. While it is impossible to make a formal criterion for distinguishing between the low- and high-density ranges, Fig. 7 shows that already absorption on the order of about 15% (optical densities about 0.15) results in about fourfold reduction of the enhancement. Thus, significant enhancement of the rotation by the cavity can be expected only in weakly absorbing media.

The analysis is very promising for future works on NFE, although for nonlinear magneto-optical effects, the role of intracavity power enhancement and related power--broadening of the NFE signals needs yet to be considered.

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