Optica Applicata, Vol. XLVI, No. 2, 2016
DOI: 10.5277/oa160212

The effect of cladding geometry on the absorption efficiency of double-clad fiber lasers

MOSLEM JAVADIMANESH, SAEED GHAVAMI SABOURI, ALIREZA KHORSANDI*

Department of Physics, University of Isfahan, 81746-73441 Isfahan, Iran

*Corresponding author: a.khorsandi@phys.ui.ac.ir

In the present work, the inner cladding geometries of a typical double-clad fiber laser are studied and numerically simulated for different cladding shapes to obtain the maximum absorption efficiency for the pump beam. This is performed by using the ray tracing approach and dislocating the fiber core from the center to impose the asymmetry on the investigated geometries. The high absorption efficiency of ~94.5% was obtained for the optimized offset D-shaped double-clad fiber. The hexagonal shape is proposed as a new geometry for the inner cladding to attain higher absorption efficiency. It was found that the absorption efficiency of ~68% for a symmetrical hexagonal can be improved to ~95% for an asymmetrized hexagonal-shaped double-clad fiber laser. Eventually the genetic algorithm was used to enhance the performance of the investigated geometries. This resulted in the further increasing of pump beam absorption efficiency of 99.5% in the genetic algorithm optimized asymmetrized hexagonal shape.

Keywords: double-clad fiber, absorption efficiency, cladding geometry.

1. Introduction

Double-clad fibers (DCFs) offer an efficient means of coupling the partially coherent light of diode lasers into a single transverse mode radiation. Because of the high power, efficiency and favorable thermal management, rare-earth-doped fiber lasers are one of the most attractive laser devices which have been designed for many applications, for example in the fabrication of optical sensors, medicine, optical communications and industrial processing [1, 2]. The optical efficiency of a DCF laser is strongly dependent on the coupling of the pump light into the core as well as the geometry of the core and cladding of the fiber. Yet, several cladding geometries are proposed to enhance the pump absorption efficiency. Those include circular offset [3], rectangular [4], D-shaped [5], hexagonal [6] and flower [7] shapes. Subsequently, sets of modeling approach and experimental works have been reported to investigate the effect of the described geometries on the efficiency of pump absorption. However, a straightforward technique is the use of ray tracing in order to estimate the absorption coefficient of a typical DCF laser. Because of the cost and technical issues, it is desirable to defer the fiber fabrication until after looking at the necessary modeling and simulating the physical parameters affecting the fiber outcomes. In this regard, Anping Liu and Kenichi Ueda [7] have provided a theoretical model to investigate the absorption characteristics of circular-, offset- and rectangular-shaped DCF lasers. On the basis of 2D model and 3D ray tracing calculation, it was found that the absorption efficiency of circular DCFs is lower than that of rectangular and offset ones. This has led to a great number of path rays that cross the asymmetry core and increase the absorption of the pump up to 80%. Thereafter, a comprehensive theory is represented by Leproux *et al.* [8] to improve the pumping of a DCF amplifier by modeling a chaotic propagation for the pump rays. In order to increase the reflection of the rays at the interface, it is desirable to investigate a particular geometry for the cladding to increase pump absorption efficiency with minimum reflection at the interface. This implies that it is possible by reducing the fiber length to enforce the pump rays passes the core many times and absorbs by the core. For this propose, D-shaped DCFs have been innovated and extensively used in many high power fiber laser devices because they promise cost effective and high pump absorption efficiency. In such configuration, the launched pump light propagates in a chaotic form and hence most of the coupled power is absorbed by the doped core as it does lots of passes through the core. Recently, a hexagonal cladding shape has been developed for DCFs to increase the absorption efficiency of the launched pump beam. However, it is found that such geometry is identical to the D-shaped configuration, but it provides lowest splice loss [9]. In the present work by using the ray tracing approach five cladding geometries as circular-, triangle-, rectangular-, D- and hexagonal-shaped are theoretically studied and numerically simulated for using in DCF lasers in order to obtain an optimum circumstance in which the absorption of pump power is maximized. This is performed by imposing an asymmetry on the utilized geometries. This has led to new conditions and criterions for the ratio of side sizes and angles, specifically for the D-shaped DCF where by calculating a new offset, absorption efficiency of ~94.5% was obtained. Then the study is focused on the chaotic feature of the pump rays propagating in the hexagonal-shaped DCF because such geometry can be approximated to other cladding shapes when the angles and side sizes are accordingly manipulated. By the calculation we found four independent parameters in the hexagonal -shaped DCF which can significantly affect the outputs of the fiber laser. By using the genetic algorithm (GA), a multi-objective optimization is performed to attain maximum efficiency for pump absorption in the investigated geometries. Eventually, from the results the particular asymmetrized hexagonal geometry is proposed for the cladding such that the absorption of the pump increased up to ~99.5% while the reflections at the interfaces are significantly decreased.

2. Numerical model

In order to trace out a ray through its propagation direction, we used a simple mathematics according to the method reported in [9]. By this method an arbitrary point in the inner clad is taken as a starting point for tracing the route of a single ray after reflecting many times from the inner surface. Based on the accuracy of our calculation about 10^4 rays have been traced by turning the starting point from zero to 360 degree on a certain circle. Subsequently, the numbers of reflections from the inner surface as well as the direction of propagation are recorded then. Next, the same procedure is repeated for other rays inside the cross-section area of the inner clad. This is performed by moving the starting point to the next circle until the whole area of the inner clad is completely scanned.

The length of a typical DCF laser may reach up to several tens of meters. Generally, it is uncritically bent on a soft mount to make the device compact. However, 3D analysis of wave propagation through such bent fiber is not an easy task. Because the bend radius is significantly larger than the radius of the inner cladding, thus by using a 2D ray tracing approach a good accuracy can be provided to consider the absorption efficiency of a pump beam in the core region. Such assumption requires that: the propagating rays are independent of each other, the diameter of the inner cladding is much larger than the pump wavelength, and the refractive index of the inner cladding is uniform. The next important parameter that has to be involved in the simulation is the number of reflections N from the inner surface of the inner cladding which determines the number of

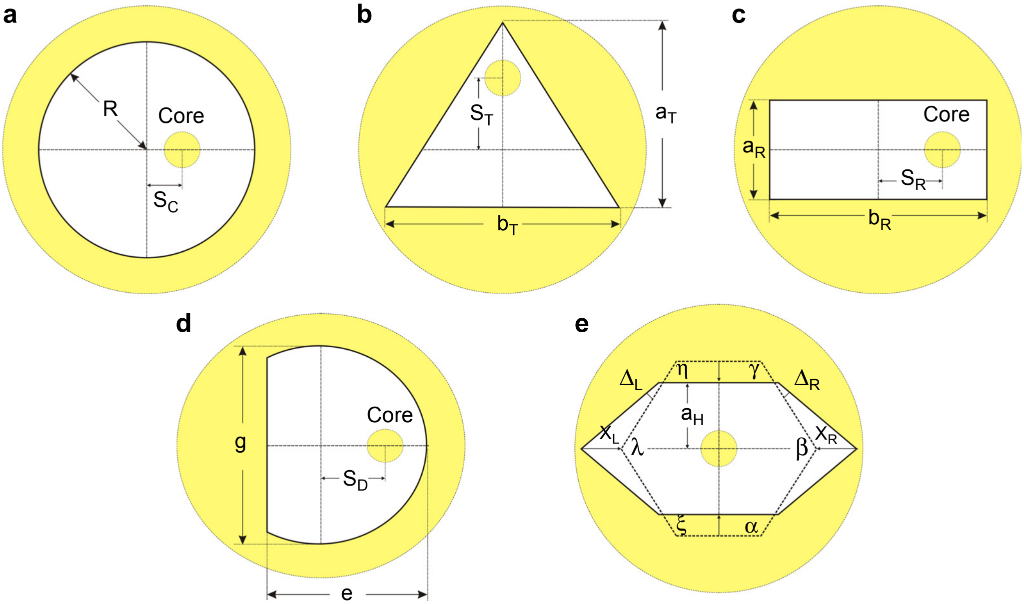

Fig. 1. A schematic demonstration of the geometries utilized for the cross-section of inner cladding as used in our investigation: circular (**a**), triangular (**b**), rectangular (**c**), D-shaped (**d**) and hexagonal (**e**). The white zone is the inner cladding. The core of the fibers is dislocated in order to break the symmetry and to increase the number of passes through the core and hence to improve the pump absorption efficiency. Variation of γ and η angles in the hexagonal shape are shown by Δ_R and Δ_L, respectively. The x_R and x_L are the extensions in opposite directions along the x coordinate which may impose an offset on the structure.

passes across the core for reaching the maximum absorption. Under this assumption, as it is contracted in [9], we consider the case of $N = 110$. Hereupon, the objective is considering the different shapes for the inner clad while the propagating rays have a maximum absorption with the lowest value of N. A further assumption has to be made before we can meaningfully compare the results, that is, all the cladding shapes have an equal cross-section area of about 0.13 mm^2. In Figure 1 a schematic illustration of the utilized cladding geometries along with the variables used in the simulation and optimization is demonstrated. The required geometrical deformation for optimization can be accomplished by changing the variables indicated in Fig. 1. The longitudinal variation can be performed by x_R and x_L in the hexagonal-shaped and by two parameters defined as $R_T = b_T/a_T$ and $R_R = b_R/a_R$ in triangular- and rectangular-shaped, respectively, while the angular variations in hexagonal-shaped are obtained by Δ_R and Δ_L. However, the location of the core in the inner cladding is very important because most of the core volume can be hit by the propagating rays and absorption of the launched pump will be increased.

To indicate the significance of the core location, a ray tracing approach is used to trace out the rays in the quintet geometries while the core is symmetrically placed at the center. Following this investigation, first we study the effects of the cladding geometry on the absorption efficiency and eventually the laser output power. Accordingly, the location of the core in the optimized cladding geometry will be studied in order to further increase the efficiency of the laser.

3. Simulation results

As it could be expected, in a circular double-clad structure due to the symmetry, absorption of the pump beam is very low. This is largely avoided by making use of offset structure in which the core is slightly moved from the center of the inner cladding by a radial displacement of S_C as specified in Fig. 1**a**. The practical value of the S_C parameter is determined based on the criteria given in [8]. Therefore, by the variation of S_C in the described range, the efficiency of pump absorption ζ which is defined as

$$\zeta = \frac{\text{Number of pump rays absorbed by core}}{\text{Number of total pump rays propagating in inner clad}} \tag{1}$$

can be calculated for a range of S_C values and number of reflections from the curved surfaces of the inner cladding. The results are shown in Figs. 2 and 3.

As can be seen from Fig. 3, for $N = 110$ at $S_C = 143$ μm, the efficiency of pump absorption is increased to ~80%. Thus, the circular geometry indicates to be very susceptible to the core offset value. However, the calculated efficiency shows a saturation feature for a certain N number and corresponding S_C value, implying that beyond an optimum point, the increase of the number of reflections does not lead to the further

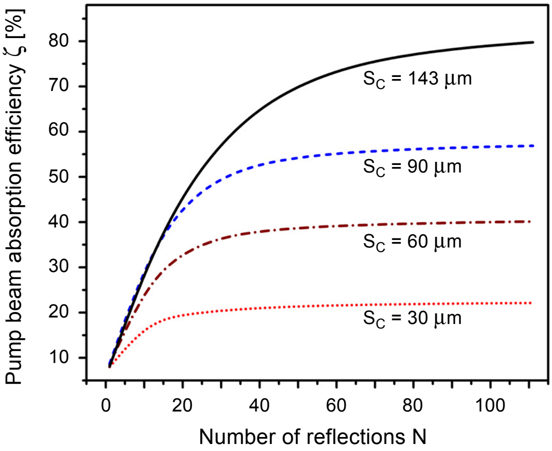

Fig. 2. Variation of pump absorption efficiency ζ with the number of reflections N forming from the clad –clad interface for different S_C offset values in the circular double-clad fiber. The radius of inner clad and core are 200 and 6 µm, respectively. To make a meaningful comparison, the area of inner clad is assumed ~0.13 mm^2.

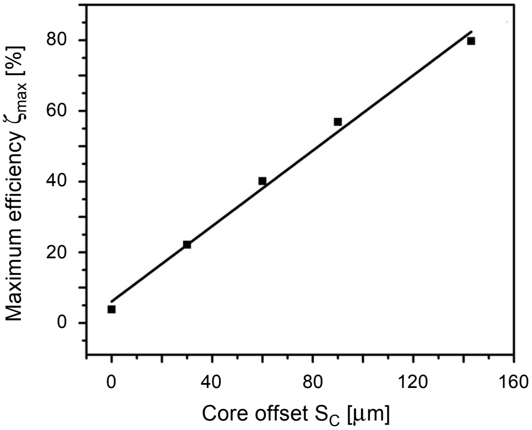

Fig. 3. The maximum absorption efficiency at corresponding core offset values for $N = 110$ reflections from the internal surface of the inner cladding. The solid line is the linear fit to the calculated data.

growth of the absorption efficiency. Similarly, the dependence of absorption efficiency on the $R_T = b_T/a_T$ and S_T variables shown in Fig. 1**b** can be investigated in the triangular geometry. The obtained results have been demonstrated in Figs. 4 and 5.

It is clearly indicated that the pump absorption efficiency is a decreasing function of R_T ratio. Nevertheless, for all values of S_T parameter, the maximum attainable efficiency occurs at about $R_T = 0.86$ which is equivalent to an equilateral triangle. In such case, the maximum efficiency of ~89% is obtained at $S_T = -100$ µm, corresponding to

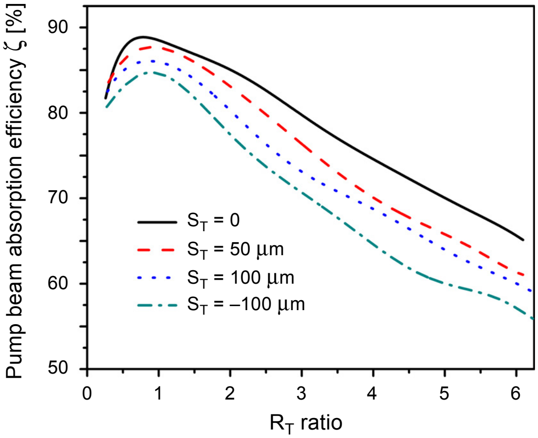

Fig. 4. Variation of pump absorption efficiency, simulated for a triangular geometry with a range of R_T and several values of core offsets. The cross-section area of the inner cladding is assumed ~0.13 mm^2 and core radius is 6 µm.

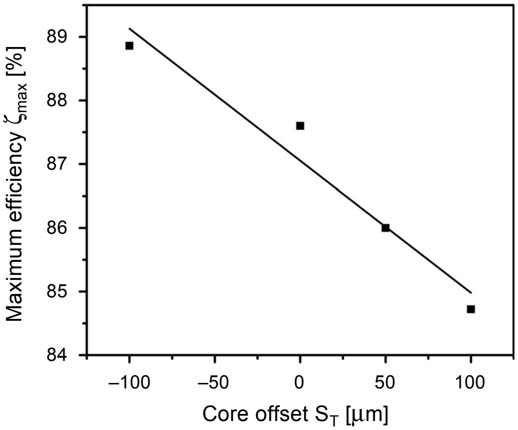

Fig. 5. Variation of the maximum efficiency for each core offset at corresponding reflections. The trend of variation is clarified by a linear fit.

a core displacement toward the base of the triangle. However, regardless of core location, for an asymmetrical triangles where $R_T > 0.86$, the pump absorption efficiency is dramatically dropped. This can be explained by the development of chaotic rays due to the irregularity. This has turned this geometry into an inefficient candidate for the design and fabrication of a double-clad fiber laser. Beside the characterized disadvantages of using the triangular-shaped fiber, it has a simplicity in the mathematical analysis of optimization. Therewith, by manipulating the R_T ratio and by further increase of the chaotic pattern for reflected rays, it is then possible to duplicate the number of the rays crossing the core before approaching the contracted N value. The next choice for enhancing the absorption efficiency is the rectangular shape. The simulation is per-

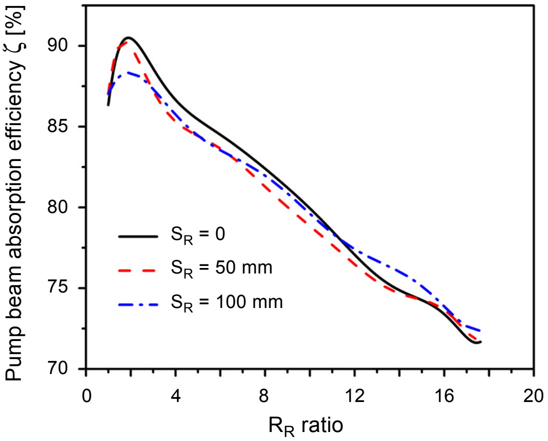

Fig. 6. Variation of pump absorption efficiency in a rectangular-shaped DCF laser for a range of R_R ratio and three values of core offsets. The cross-section area of the inner cladding and the core radius are similar to those used in Fig. 4.

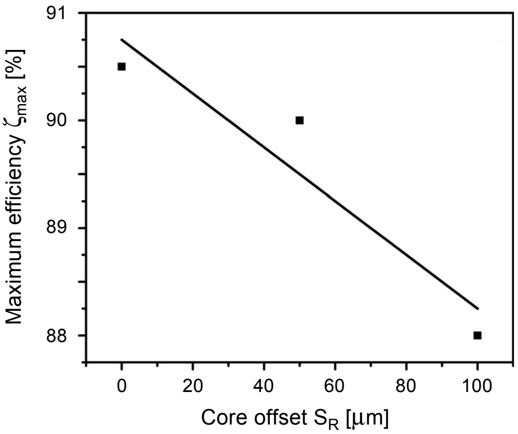

Fig. 7. The maximum obtainable efficiency is shown for particular core offsets in the *x* direction. Solid line shows a linear fit to the data obtained from Fig. 6.

formed by simultaneous variation of $R_R = b_R/a_R$ and S_R variables as specified in Fig. 1**c**. As shown in Figs. 6 and 7, the increase of efficiency is quite remarkable.

As it is clearly shown in the above plot, in $1.6 \le R_R \le 2$ range, the maximum pump absorption efficiency of 90% is obtained for $S_R = 0$, corresponding to a symmetrical case. However, the displacement of the core from the center in the *x* direction by 100 µm results in the declining of efficiency to 88%. A significant decrease in the efficiency is also observed for the drawn rectangles. Next, we investigate the D-shaped geometry which nowadays is extensively used to enhance the output characteristics of a DCF laser and amplifier. Due to the intrinsic asymmetrical structure of the D shape, the fiber core encounters a bundle of propagating rays, producing an appreciable increase in the

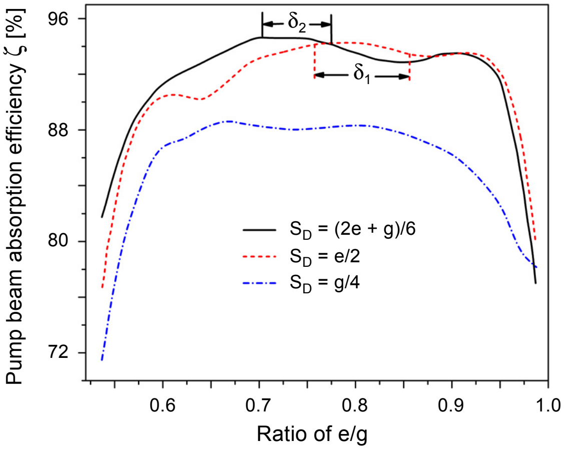

Fig. 8. Variation of pump beam absorption with the *e*/*g* ratio in a D-shaped DCF for $S_D = g/4$, $S_D = e/2$ and $S_D = (2e + g)/6$. Here δ_1 shows the optimum region of *e*/*g* for $S_D = e/2$ as specified by [11]. The δ_2 is the new interval based on using the $S_D = (2e + g)/6$ calculated in the present work. In varying of *e*/*g* ratio, the cross-section area of the inner cladding is fixed at ~0.13 mm^2.

efficiency of the DCF device. As specified in Fig. 1**d**, a D-shaped geometry is characterized by two interdependent parameters *e* and *g* as cut and bezel diameters, respectively, and by S_D as core offset. Thus, the ratio of *e*/*g* can be used to optimize the efficiency of pump absorption at corresponding offset value. Consequently, the geometry imposes a limitation on the variation range of the *e*/*g* ratio within $0.5 < e/g < 1$. Therefore, if for instance $e/g = 1$ then the structure converts to a circle. According to scientific reports released in [10, 11], the optimum location of the core in a typical D-shaped geometry is obtained at $S_D = g/4$ and $S_D = e/2$, respectively. However, the results of our simulation are in contrast with those in which for the maximum of the absorption efficiency we concluded that the optimum core offset occurs at $S_D = (2e + g)/6$. In order to indicate the performance of such induction, the pump absorption efficiency is calculated for the proposed S_D relation and relevant graphs are generated in Fig. 8.

Referring to [11] for a D-shaped DCF, the optimum efficiency of pump beam absorption is obtained when $0.75 < e/g < 0.85$ which is specified by δ_1 in Fig. 8. In this interval the maximum pump absorption efficiency obtained is ~94.1%. While based on the calculation performed in the present work, the highest efficiency of ~94.5% is obtained for the new interval of $0.67 < \delta_1 < 0.76$ which is obtained through the optimization of core location in the D-shaped inner cladding.

4. A specific example: hexagonal-shaped DCF

Due to the advanced modern technology, different inner cladding shapes have been developed for further enhancement of pump absorption efficiency in the DCF devices. A recent example is the use of hexagonal shape which shows a very complicated geometry because a number of variables such as those indicated in Fig. 1**e** are involved in the optimization. Therefore, a multi-objective method like GA is required to obtain the optimum parameters. The multi-variable optimization can be performed by proper choice of a merit function being compatible with the GA technique. According to our strategy, in order to improve the absorption efficiency, it is desirable to break the symmetry of the hexagonal through changing the angles and sides [6]. However, to reduce the independent parameters and making an appropriate comparison with the previous results, the cross-section area of the hexagonal shape is assumed to be fixed at ~0.13 mm^2 during a certain variation of angles and sides. In this regard, new angles are:

$$\begin{aligned} \alpha' &= \frac{2\pi}{3} + \frac{\Delta_R}{2}, \qquad \eta' = \frac{2\pi}{3} + \frac{\Delta_L}{2} \\ \beta' &= \frac{2\pi}{3} - \Delta_R, \qquad \lambda' = \frac{2\pi}{3} - \Delta_L \\ \gamma' &= \frac{2\pi}{3} + \frac{\Delta_R}{2}, \qquad \xi' = \frac{2\pi}{3} + \frac{\Delta_L}{2} \end{aligned} \tag{2}$$

where Δ_R and Δ_L are respectively the right- and left-hand variation of the hexagonal angles shown in Fig. 1**e**. By the proper choice of those angular variables from –60 to 100 degrees, the hexagonal can be changed to other cladding geometries. Therefore, with the aim of obtaining the maximum absorption efficiency the hexagonal shape can be optimized such that at the lower number of reflections the fiber core meets a greater number of propagating rays. Before using the GA optimization, Fig. 9 shows the calculated absorption efficiency of symmetrical and asymmetrical hexagonal-shaped as

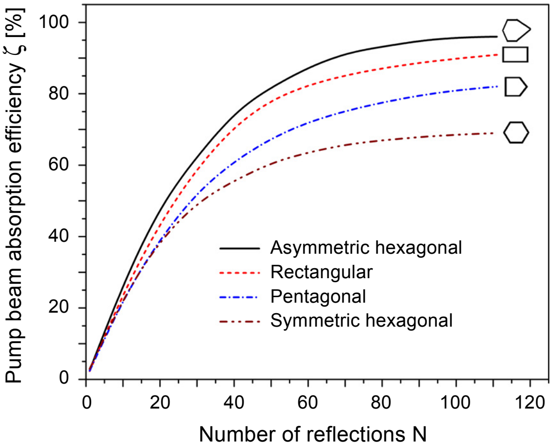

Fig. 9. Variation of pump beam absorption efficiency with the number of reflections for symmetrical hexagonal, reduced pentagonal ($\Delta_R = -60°$, $\Delta_L = 0$) and rectangular ($\Delta_R = \Delta_L = -60°$) and asymmetrical hexagonal ($\Delta_R = 50°$, $\Delta_L = 0$). The cross-section area of all the shapes is assumed to be ~0.13 mm^2.

well as reduced rectangular and pentagonal shapes reached by selecting $\Delta_R = \Delta_L = -60°$ and by selecting $\Delta_R = -60°$ and $\Delta_L = 0$, respectively, as analogous to hexagonal.

As it can be confirmed by the above plot, at the number of reflections of $N = 110$ forming the curved surfaces of inner cladding, the pump beam absorption efficiency in symmetrical hexagonal, pentagonal, rectangular and asymmetrical hexagonal is obtained 68%, 82%, 90% and 95%, respectively. This indicates a significant increase of efficiency for the optimized and asymmetrized hexagonal geometry. Therefore, it suggests that the optimization of the associated variables illustrated in Fig. 1**e** is very valuable. Thus, we start by using the GA technique which is known as a simple, powerful and direct method for multi-variable optimization, particularly in optical fiber devices. The GA is a well established method to find a global minimum of multi-variable functions [12] and obtaining the optimum parameters. In general, in order to reduce the nonlinear effect, it is desirable to shortening the fiber length. This in turn resulted in the reduction of the number of reflections and hence the number of rays crossing the core. Therefore, the main task of using the GA method is to increase the efficiency of pump absorption along simultaneously with lower number of reflections. As can be followed by the above discussion and the schematic indicated in Fig. 1, the hexagonal parameters under our investigation are x_L, x_R, Δ_L, Δ_R which can be independently changed as long as the a_H is accordingly changed to preserve the cross-section area at a constant value of ~0.13 mm^2. The best generations of the optimized parameters are then used to find the maximum absorption efficiency. This can be achieved by a definition of a merit function (MF) as [13]

$$\mathrm{MF} = \left[\frac{1}{M} \sum_{i=1}^{M} W_i \cdot \left(\frac{D_i}{\delta Q_i} \right)^k \right]^{1/k} \tag{3}$$

where M is the number of effective parameters being involved in the GA optimization and W_i is a weight factor of each parameter, $D_i = Q_i^T - Q_i$ is the difference between the i-th target and the i-th parameter of interest for which the MF approaches a minimum point, δQ_i is a tolerance factor that determines by which accuracy Q_i reaches to its target Q_i^T, and k is the constant parameter which is often set to two [13]. Eventually, the GA ends up when a minimum accuracy of δQ_i is reached. Here in our case the absorption efficiency ζ is the only parameter of interest for optimization and, hence, $M = 1$ and $W_i = 1$. Therefore, by substituting $Q_i^T \equiv \zeta^T$, $Q_i \equiv \zeta$, $D_i \equiv \zeta^T - \zeta$ into Eq. (3) we find

$$\mathrm{MF} = \frac{\zeta^T - \zeta}{\delta \zeta} \tag{4}$$

to obtain the maximum efficiency of absorption $\zeta_{\max}$, where ζ^T corresponds to the optimum target which is ideally set to 100%. This can be attained by selecting a pre-determined value of ~0.1% for the tolerance factor $\delta\zeta$. Therefore, GA starts to change the ζ reaching very close to ζ^T, until the MF would be minimized. Subsequently, the

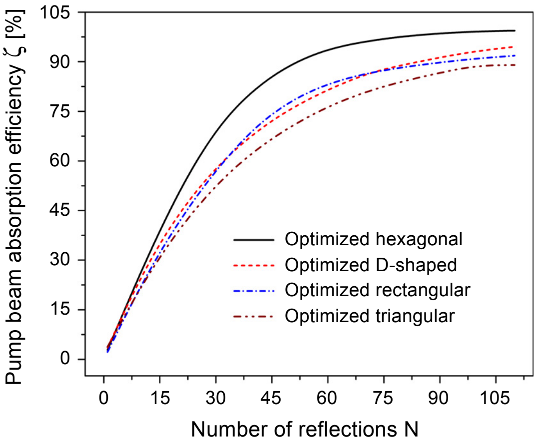

Fig. 10. Variation of the optimum efficiency at the corresponding number of reflections obtained using the GA method for the hexagonal-shaped DCF device. The optimum efficiency of pump absorption obtained for other geometries like triangular, rectangular and particularly D-shaped as simulated through Figs. 2 to 8 are also brought into the plot for comparison.

optimum a_H can be evaluated using the Newton–Raphson method. Figure 10 indicates the optimized absorption efficiency for the GA-optimized hexagonal shape. In order to verify the results and making an intuitive comparison, the optimum value of absorption efficiency for triangular-, rectangular- and D-shaped cladding demonstrated through Figs. 2 to 8 has also been brought into the plot. As it can be seen in Fig. 10, in the optimized hexagonal shape, the efficiency of the pump beam absorption is significantly increased such that for $N = 50$ the efficiency is raised up to ~90%. Whereas, for triangular, rectangular and D-shaped geometries the efficiency is 71%, 76% and 78%, respectively. This reflects the fact that the equivalent length of the fiber required for the maximum absorption of propagating rays is significantly reduced. On the other hand, in the GA-optimized hexagonal-shaped DCF, for the launched pump power being completely absorbed by the core, the number of reflections can be considerably decreased and hence shorter length of the fiber is needed. This is because the fiber core is encountered by a large amount of crossing rays at lesser z values. However, the advantage of using shorter length is the reduction of the interfering nonlinear effect which is the subject of long fibers. As a result, by the reduction of N in the hexagonal-shaped DCF down to fifty, the utmost absorption of a pump can be achieved while the disturbing effects are suppressed. Further increasing of the efficiency up to 88%, 91%, 94.5% and 99.5% for respectively triangular, D-shaped, rectangular and optimized hexagonal is also possible when the number of reflections approaches $N = 110$.

This shows an effective increase of ~10% in the absorption efficiency for the optimized hexagonal-shaped. In summary, the exceptional performance of the GA-optimized hexagonal shape is characterized by the increase of the pump absorption efficiency up to 99.5% and 90% for $N = 110$ and 50, respectively, indicating that the length of the utilized fiber can be effectively decreased in an optimized hexagonal

-shaped double-clad fiber and hence the nonlinear effects owing to the long fibers can be avoided.

5. Conclusions

The effect of very recent geometries used for the inner cladding on the pump beam absorption efficiency of a double-clad fiber has been studied and simulated. Those include circular, rectangular, triangular and D shapes which are extensively used in either theory or experiment. By using the ray tracing approach and breaking the symmetry of the investigated devices, we found that making the offset structure can improve the absorption efficiency and enhance the fiber laser characteristics and output. By tolerating the geometrical parameters and manipulating the location of the core in each shape, the optimum condition attained by assuming that the launched rays into the inner cladding will be completely absorbed after 110 reflections at the clad–clad interface. It is found that for the circular geometry the absorption efficiency can be increased up to ~80% for the offset value of ~143 μm. Moreover, as the core is displaced by ~100 μm toward the base of a triangular geometry, the maximum absorption efficiency of ~89% can be achieved when $R_T > 0.86$. It is clearly shown that in a rectangular -shaped DCF structure, the core dislocation has no significant benefits where by moving the core from the center by ~100 μm the pump beam absorption efficiency decreases down to ~88%. In a D-shaped DCF device we introduced a new criteria for the offset value as $S_D = (2e + g)/6$ in which the absorption efficiency up to ~94.5% is reached, indicating a meaningful increase compared to the [11] where 94.1% of conversion efficiency was reported for $0.75 < e/g < 0.85$ interval. Subsequently, a new geometry is proposed for the inner cladding of a DCF structure to further increase the efficiency of pump beam absorption. This has let to the efficiency of ~68% for symmetrical hexagonal shape and of ~95% for asymmetrical one. The obtained results confirm the beneficial effect of asymmetrization of the geometries on the DCF characteristics. The valuable performance of the introduced hexagonal geometry is finally indicated by approaching ~99.5% for absorption efficiency. This is performed by using the genetic algorithm for multi-objective optimization of hexagonal angles and sides in order to make the offset structure. The exceptional advantage of such investigation is that the represented calculation and simulation is essentially independent of the type of fiber laser and can be extended over different core-doped of the fiber laser and amplifiers. Indeed, the results obtained for the particular hexagonal geometry indicated that the maximum absorption efficiency can be achieved while the number of reflections at the clad–clad interface is decreased by half.

References

[1] NARRO-GARCÍA R., RODRÍGUEZ E., PONCE L., DE POSADA E., FLORES T., ARRONTE M., *Description of a ray trace algorithm for the evaluation of pump power absorption in double-clad fibers*, Optica Applicata **42**(3), 2012, pp. 571–585.

[2] Jianqiu Xu, Junhua Lu, Jianren Lu, Ueda K., *Influence of cross-sectional shape on absorption characteristics of double-clad fiber lasers*, Optical Engineering **42**(9), 2003, pp. 2527–2533.

[3] Kouznetsov D., Moloney J.V., *Efficiency of pump absorption in double-clad fiber amplifiers. II. Broken circular symmetry*, Journal of the Optical Society of America B **19**(6), 2002, pp. 1259–1263.

[4] Liu A., Song J., Kamatani K., Ueda K., *Rectangular double-clad fibre laser with two-end-bundled pump*, Electronics Letters **32**(18), 1996, pp. 1673–1674.

[5] Konieczny P., Świderski J., Zając A., Skórczakowski M., *Analysis of activation of active double -clad optical fibers*, Optica Applicata **35**(4), 2005, pp. 955–968.

[6] Yahua Li, Jackson S.D., Fleming S., *High absorption and low splice loss properties of hexagonal double-clad fiber*, IEEE Photonics Technology Letters **16**(11), 2004, pp. 2502–2504.

[7] Anping Liu, Kenichi Ueda, *The absorption characteristics of circular, offset, and rectangular double-clad fibers*, Optics Communications **132**(5–6), 1996, pp. 511–518.

[8] Leproux P., Février S., Doya V., Roy P., Pagnoux D., *Modeling and optimization of double-clad fiber amplifiers using chaotic propagation of the pump*, Optical Fiber Technology **7**(4), 2001, pp. 324 –339.

[9] Jun Zhou, Qihong Lou, Tiejun Li, Zhijiang Wang, Jingxing Dong, Runrong Wei, *New inner cladding shape for high-power double-clad fiber lasers*, Proceedings of SPIE **4914**, 2002, pp. 146–150.

[10] Michel C., Tascu S., Doya V., Legrand O., Mortessagne F., *Gain-controlled wave chaos in a chaotic optical fibre*, Journal of the European Optical Society – Rapid publications **4**, 2009, article 09020.

[11] Narro-García R., Arronte M., Guerrero-Contreras J., Rodríguez E., *Study of the pump absorption efficiency in D-shaped double clad optical fiber*, Optica Applicata **42**(3), 2012, pp. 587–596.

[12] Popov A., *Genetic Algorithms for Optimization*, User Manual, Hamburg, Vol. 2013, 2005.

[13] Dobrowolski J.A., Ho F.C., Belkind A., Koss V.A., *Merit functions for more effective thin film calculations*, Applied Optics **28**(14), 1989, pp. 2824–2831.

Received June 24, 2015
in revised form November 4, 2015