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Polarization properties of Gaussian–Schell model quantization field in a turbulent marine-atmosphere

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Polarization properties of Gaussian–Schell model quantization field propagating through the Kolmogorov turbulence of a marine-atmosphere channel are studied based on the degree of quantum polarization. The effective photon annihilation and creation operators of Gaussian–Schell model quantization field propagation in a marine-atmosphere are developed by making use of the extended Huygens–Fresnel integral of quantum field. The effects of the outer scale on the degree of polarization can be neglected. As the source transverse coherent width, the number of received photons, the inner scale of turbulent eddies, and the source transverse radius decrease or the refractive index structure parameter increases, the degree of polarization decreases. In theory, we find that the polarization fade of marine-atmosphere turbulence channels is larger than that of terrene -atmosphere turbulence channels under same transport parameters and the channel with a stronger turbulence strength will possess a larger detection area of a polarization signal, which have potentially important implications for free-space quantum key distribution.

Keywords: polarization fluctuation, Gaussian–Schell model, marine-atmosphere, Kolmogorov turbulence.

1. Introduction

In recent years, there has been increasing interest in studying the polarization fluctuations of Gaussian–Schell model (GSM) beams propagating in turbulent atmosphere [1–12]. Polarization fluctuations of the GSM beams and twist anisotropic GSM beams in turbulent atmosphere have been investigated [1–10]. The cross-spectral density matrix for a GSM beam truncated by a slit aperture was derived [10]. The generalized analytical expressions of the spectral degree of polarization of a two-dimensional rectangular array beam, composed of GSM sources, propagating through turbulent atmosphere have been derived [11], and the analytical expressions for the polarization transverse distribution of multi-Gaussian–Schell photon beams propagating through the modified von Kármán turbulence channel are given, in consideration with the effects of the outer and inner scale of turbulence [12]. Based on the extended Huygens–Fresnel principle, the analytical expressions for the generalized Stokes parameters of random electromagnetic GSM beams with or without vortex [13] and for the cross-spectral density function of partially coherent sine-Gaussian vortex beams and partially coherent sine-Gaussian non-vortex beams [14] propagating through atmospheric turbulence have been derived. It is shown that the vortex affects the evolution behavior of spectral Stokes parameters in atmosphere [13], the creation of the coherent vortex depends on the vortex beams, and the coherent vortices can be grouped into three classes according to the creation [14]. To the best of our knowledge, there is no report on the polarization fluctuations of Gaussian–Schell photon beams propagating in a turbulent channel of marine-atmosphere.

The aim of this paper is to develop a theoretical model for the polarization fluctuations of GSM quantization field in a marine-atmosphere channel with the Kolmogorov turbulence. In Section 2, the quantum Stokes operator of GSM quantization field in a marine-atmosphere is derived. In Section 3, the model for the effects of the transverse size and transverse coherent width of a source on the degree of polarization of GSM quantization field in a marine-atmosphere is investigated in detail. Numerical results and discussions are given in Section 4. Conclusions are presented in Section 5.

2. Quantum Stokes operator of GSM quantization field

The amplitude operator of a linearly polarized quantum field in a turbulent and paraxial marine-atmosphere (m-a) channel can be expressed as [15, 16]

$$\hat{E}_x^{\dagger}(\boldsymbol{\rho}, z) = -\frac{ik\exp(ikz)}{2\pi z}\sqrt{\tau}\int \hat{E}_x^{\dagger}(\boldsymbol{\rho}', 0)\exp\left[\frac{ik}{2z}(\boldsymbol{\rho}-\boldsymbol{\rho}')^2 + \psi_{\text{m-a}}(\boldsymbol{\rho}, \boldsymbol{\rho}', z)\right]\mathrm{d}^2\boldsymbol{\rho}' \tag{1}$$

where τ is the transmittance of the channel, $\boldsymbol{\rho}'$ denotes the transverse coordinate of the photon at the source plane, $\boldsymbol{\rho}$ denotes the transverse coordinate of the photon in the z plane, $k = 2\pi/\lambda$ is the optical wave number, λ is the wavelength, the function $\psi_{\text{m-a}}(\boldsymbol{\rho}, \boldsymbol{\rho}', z) = \chi_{\text{m-a}}(\boldsymbol{\rho}, \boldsymbol{\rho}', z) + is_{\text{m-a}}(\boldsymbol{\rho}, \boldsymbol{\rho}', z)$ describes the turbulent effects of the marine-atmosphere (m-a) on the propagating spherical wave and $\chi_{\text{m-a}}(\boldsymbol{\rho}, \boldsymbol{\rho}', z)$ and $s_{\text{m-a}}(\boldsymbol{\rho}, \boldsymbol{\rho}', z)$ terms account for the stochastic log-amplitude and phase fluctuations, respectively, imposed by marine-atmosphere turbulence. For the approximation of passive medium, which implies that the self-radiation of medium is not taken into account, the field operator at $z = 0$ is $\hat{E}_x(\boldsymbol{\rho}', 0)$ and can be given by [16, 17],

$$\hat{E}_x^{\dagger}(\boldsymbol{\rho}', 0) = (2\pi)^{-1}\int \hat{a}_x(\mathbf{q}, \boldsymbol{\rho}')\exp(i\mathbf{q}\cdot\boldsymbol{\rho}')\mathrm{d}^2\mathbf{q} \tag{2}$$

where $\hat{a}_x(\mathbf{q}, \boldsymbol{\rho}') = \hat{a}_{0x}(\mathbf{q})u(\boldsymbol{\rho}')$ is the effective photon annihilation operator in the model $(\mathbf{q}, x)$, $u(\boldsymbol{\rho}')$ is the transverse beam amplitude function for beam modes, $\mathbf{q}$ is the momentum of photon and x is its polarization along x axis, $\hat{n}(\mathbf{q}) = \hat{a}_x^{\dagger}(\mathbf{q})\hat{a}_x(\mathbf{q})$

is the number operator, $\hat{n}_0(\mathbf{q}) = \hat{a}^\dagger_{0x}(\mathbf{q})\hat{a}_{0x}(\mathbf{q})$ is the initial number operator and $\hat{n}_0(\mathbf{q})|\xi\rangle = n_0(\mathbf{q})$, $|\xi\rangle$ is a coherent state of polarization modes. The operators $\hat{a}_x(\mathbf{q})$ and $\hat{a}^\dagger_x(\mathbf{q})$ obey well-known communication relations $[\hat{a}_x(\mathbf{q}), \hat{a}^\dagger_y(\mathbf{q})] = \hat{\delta}_{xy}$, with $x, y = 1, 2$.

Using Eqs. (1) and (2), we can express the photon annihilation operator of linearly polarized quantum fields in the turbulence of a marine-atmosphere channel as

$$\hat{a}_x(\mathbf{q}, \boldsymbol{\rho}, z) = -\frac{ik\exp(ikz)}{2\pi z}\sqrt{\tau}\int \hat{a}_x(\mathbf{q}, \boldsymbol{\rho}')\exp[i\mathbf{q}\cdot(\boldsymbol{\rho}'-\boldsymbol{\rho})]\times$$
$$\times \exp\left[\frac{ik}{2z}(\boldsymbol{\rho}-\boldsymbol{\rho}')^2 + \psi_{\text{m-a}}(\boldsymbol{\rho}, \boldsymbol{\rho}', z)\right]\mathrm{d}^2\boldsymbol{\rho}' \tag{3}$$

and the corresponding photon creation operator $\hat{a}^\dagger_x(\mathbf{q}, \boldsymbol{\rho}, z)$ is expressed as follows:

$$\hat{a}^\dagger_x(\mathbf{q}, \boldsymbol{\rho}, z) = \frac{ik\exp(-ikz)}{2\pi z}\sqrt{\tau}\int \hat{a}^\dagger_x(\mathbf{q}, \boldsymbol{\rho}')\exp\left[-i\mathbf{q}\cdot(\boldsymbol{\rho}'-\boldsymbol{\rho})\right]\times$$
$$\times \exp\left[-\frac{ik}{2z}(\boldsymbol{\rho}-\boldsymbol{\rho}')^2 + \psi^*_{\text{m-a}}(\boldsymbol{\rho}, \boldsymbol{\rho}', z)\right]\mathrm{d}^2\boldsymbol{\rho}' \tag{4}$$

By Equations (3) and (4), we obtain the quantum Stokes operator expression

$$\hat{S}_0(\boldsymbol{\rho}, \mathbf{q}, z) = \hat{a}^\dagger_x(\boldsymbol{\rho}, \mathbf{q}, z)\,\hat{a}_x(\boldsymbol{\rho}, \mathbf{q}, z) =$$
$$= \left(\frac{k}{2\pi z}\right)^2 \hat{n}(\mathbf{q})\iint u^*(\boldsymbol{\rho}')u(\boldsymbol{\rho}'')\mathrm{d}^2\boldsymbol{\rho}'\mathrm{d}^2\boldsymbol{\rho}''\times$$
$$\times \exp\left[ik\frac{(\boldsymbol{\rho}-\boldsymbol{\rho}')^2-(\boldsymbol{\rho}-\boldsymbol{\rho}'')^2}{2z}\right]\exp[\psi^*_{\text{m-a}}(\boldsymbol{\rho}, \boldsymbol{\rho}', z)+\psi_{\text{m-a}}(\boldsymbol{\rho}, \boldsymbol{\rho}'', z)] \tag{5}$$

where $\hat{n}(\mathbf{q}) = \hat{a}_x(\mathbf{q})\hat{a}^\dagger_x(\mathbf{q})$ is the photon number operator.

3. Degree of polarization of GSM quantization field

As the discussion in [16], the degree of polarization of GSM and linearly polarized quantization field propagation in marine-atmosphere can take the form

$$P = \frac{\sqrt{\langle\langle\xi|\hat{S}_0(\boldsymbol{\rho}, \mathbf{q}, z)|\xi\rangle\rangle_{\text{m-a, s}}}}{\sqrt{\langle\langle\xi|\hat{S}_0(\boldsymbol{\rho}, \mathbf{q}, z)|\xi\rangle\rangle_{\text{m-a, s}}+2}} \tag{6}$$

where $\hat{S}_0(\boldsymbol{\rho}, \mathbf{q}, z) = \hat{a}_x^\dagger(\boldsymbol{\rho}, \mathbf{q}, z)\,\hat{a}_x(\boldsymbol{\rho}, \mathbf{q}, z)$ is the quantum Stokes operator of the field, $\langle \xi|\hat{S}_0(\boldsymbol{\rho}, \mathbf{q}, z)|\xi\rangle = s_0(\boldsymbol{\rho}, \mathbf{q}, z)$ is the Stokes parameter, $\langle \ldots \rangle_{\text{m-a}}$ and $\langle \ldots \rangle_{\text{s}}$ denote the ensemble average of turbulent atmosphere and the field source, respectively.

In the paraxial propagation channel $(\mathbf{k} \cdot (\boldsymbol{\rho}'' - \boldsymbol{\rho}') \approx 0)$ and by Eq. (5), we obtain the average photon number of linearly polarized quantum fields in marine-atmosphere at the receiving plane

$$\langle\langle \xi|\hat{S}_0(\boldsymbol{\rho}, \mathbf{q}, z)|\xi\rangle\rangle_{\text{m-a, s}} = n\left(\frac{k}{2\pi z}\right)^2 \iint \langle u^*(\boldsymbol{\rho}')u(\boldsymbol{\rho}'')\rangle_{\text{s}}\, \mathrm{d}^2\boldsymbol{\rho}'\mathrm{d}^2\boldsymbol{\rho}'' \times$$

$$\times \exp\left[ik\frac{(\boldsymbol{\rho} - \boldsymbol{\rho}')^2 - (\boldsymbol{\rho} - \boldsymbol{\rho}'')^2}{2z}\right]\langle \exp[\psi^*_{\text{m-a}}(\boldsymbol{\rho}, \boldsymbol{\rho}', z) + \psi_{\text{m-a}}(\boldsymbol{\rho}, \boldsymbol{\rho}'', z)]\rangle_{\text{m-a}} \tag{7}$$

where $n = \tau n_0$ denotes the number of received photons.

The angular bracket $\langle\exp[\psi^*_{\text{m-a}}(\boldsymbol{\rho}, \boldsymbol{\rho}', z) + \psi_{\text{m-a}}(\boldsymbol{\rho}, \boldsymbol{\rho}'', z)]\rangle_{\text{m-a}}$ in Eq. (7) can be approximated as [18]

$$\langle\exp[\psi^*_{\text{m-a}}(\boldsymbol{\rho}, \boldsymbol{\rho}', z) + \psi_{\text{m-a}}(\boldsymbol{\rho}, \boldsymbol{\rho}'', z)]\rangle_{\text{m-a}} =$$

$$= \exp\left[-\frac{\pi^2 k^2 z}{3}(\rho'^2 + \rho''^2 - 2\boldsymbol{\rho}' \cdot \boldsymbol{\rho}'')\int_0^\infty \kappa^3 \phi_n(\kappa)\mathrm{d}\kappa\right] =$$

$$= \exp\left[-\frac{\rho'^2 + \rho''^2 - 2\boldsymbol{\rho}' \cdot \boldsymbol{\rho}''}{\rho_H^2}\right] \tag{8}$$

where ρ_H is the lateral coherence length of the spherical wave in marine-atmosphere channels.

For the Kolmogorov turbulence of marine-atmosphere channels, the turbulence spectrum is represented as [19]

$$\phi_n(\kappa) = 0.033C_n^2\left[1 - 0.061\left(\frac{\kappa}{\kappa_H}\right) + 2.836\left(\frac{\kappa}{\kappa_H}\right)^{7/6}\right]\exp\left(-\frac{\kappa^2}{\kappa_H^2}\right)(\kappa^2 + \kappa_0^2)^{-11/6} \tag{9}$$

where C_n^2 denotes the refractive index structure parameter of the marine-atmosphere, $\kappa_H = 3.41/l_0$ and $\kappa_0 = 2\pi/L_0$ (while l_0 – the inner scale of turbulence, and L_0 – the outer scale of turbulence).

Also, in the Kolmogorov turbulence of terrene-atmosphere (t-a) channels, the turbulence spectrum is represented as follows [19]:

$$\phi_n(\kappa) = 0.033C_n^2\left[1 + 1.802\left(\frac{\kappa}{\kappa_l}\right) - 0.254\left(\frac{\kappa}{\kappa_l}\right)^{7/6}\right]\exp\left(-\frac{\kappa^2}{\kappa_l^2}\right)(\kappa^2 + \kappa_p^2)^{-11/6} \tag{10}$$

where $\kappa_l = 3.3/l_0$ and $\kappa_p = 1/L_0$.

By the marine-atmosphere turbulence spectrum (Eq. (9)), the lateral coherence length of the spherical wave is given by

$$\rho_H^{-2} = 3.257\,k^2 z\int_0^\infty \kappa^3 \phi_n(\kappa)\,\mathrm{d}\kappa =$$

$$= 0.109\,C_n^2 k^2 z\int_0^\infty \left[\kappa^3 - 0.061\frac{\kappa^4}{\kappa_H} + 2.836\frac{\kappa^{25/6}}{\kappa_H^{7/6}}\right]\frac{\exp(-\kappa^2/\kappa_H^2)}{(\kappa^2+\kappa_0^2)^{11/6}}\,\mathrm{d}\kappa \tag{11}$$

Using the following integral [20]:

$$\int_0^\infty \kappa^{2\mu}\frac{\exp(-\kappa^2/\kappa_H^2)}{(\kappa^2+\kappa_0^2)^{11/6}}\,\mathrm{d}\kappa = \frac{1}{2}\kappa_0^{2\mu-8/3}\,\Gamma\left(\mu+\frac{1}{2}\right)U\left(\mu+\frac{1}{2};\ \mu-\frac{1}{3};\ \frac{\kappa_0^2}{\kappa_H^2}\right)$$

we find that

$$\rho_H^{-2} = 0.054\,\kappa_0^{1/3}k^2 zC_n^2\left[U\left(2;\ \frac{7}{6};\ \frac{\kappa_0^2}{\kappa_H^2}\right) - 0.082\frac{\kappa_0}{\kappa_H}U\left(\frac{5}{2};\ \frac{5}{3};\ \frac{\kappa_0^2}{\kappa_H^2}\right) + \right.$$

$$\left. + 2.619\left(\frac{\kappa_0}{\kappa_H}\right)^{7/6}\Gamma\left(\frac{7}{12}\right)U\left(\frac{31}{12};\ \frac{7}{4};\ \frac{\kappa_0^2}{\kappa_H^2}\right)\right] \tag{12}$$

where $U(a;\,b;\,z)$ is the confluent hypergeometric function of the second kind.

Similarly, for terrene-atmosphere (t-a) turbulence channels, we get

$$\rho_{\text{t-a}}^{-2} = 0.054\,\kappa_p^{1/3}k^2 zC_n^2\left[U\left(2;\ \frac{7}{6};\ \frac{\kappa_p^2}{\kappa_l^2}\right) + 2.396\frac{\kappa_p}{\kappa_l}U\left(\frac{5}{2};\ \frac{5}{3};\ \frac{\kappa_p^2}{\kappa_l^2}\right) + \right.$$

$$\left. - 0.235\left(\frac{\kappa_p}{\kappa_l}\right)^{7/6}\Gamma\left(\frac{7}{12}\right)U\left(\frac{31}{12};\ \frac{7}{4};\ \frac{\kappa_p^2}{\kappa_l^2}\right)\right] \tag{13}$$

For the Gaussian–Schell photon beams, the two-point correlation function $\langle u^*(\boldsymbol{\rho}')u(\boldsymbol{\rho}'')\rangle_\mathrm{s}$ of the transverse field amplitude can be expressed by [11]

$$\langle u^*(\boldsymbol{\rho}')u(\boldsymbol{\rho}'')\rangle_\mathrm{s} = \exp\left[-\frac{\rho'^2+\rho''^2}{4w_0^2} - \frac{|\boldsymbol{\rho}'-\boldsymbol{\rho}''|^2}{2\rho_{s_0}^2}\right] \tag{14}$$

where w_0 represents the source transverse size, ρ_{s_0} represents the source transverse coherent width, and $l_{s_0} = \sqrt{2}\,\rho_{s_0}$ is the transverse coherent radius of sources.

Substituting Eqs. (8) and (14) into Eq. (7), we have the average Stokes parameter of a turbulent ensemble

$$\langle s_0\rangle_{\text{m-a, s}} = n\left(\frac{k}{2\pi z}\right)^2 \iint \exp\left[-\frac{\rho'^2+\rho''^2}{4w_0^2} - \frac{|\boldsymbol{\rho}'-\boldsymbol{\rho}''|^2}{2\rho_{s_0}^2} - \frac{(\boldsymbol{\rho}'-\boldsymbol{\rho}'')^2}{\rho_H^2} + \right.$$
$$\left. + ik\frac{(\boldsymbol{\rho}-\boldsymbol{\rho}')^2-(\boldsymbol{\rho}-\boldsymbol{\rho}'')^2}{2z}\right] \mathrm{d}^2\boldsymbol{\rho}'\,\mathrm{d}^2\boldsymbol{\rho}'' \tag{15}$$

Using the integral formula (3.323) in page 333 of [20], we obtain the analytical expression of the Stokes parameter $\langle s_0\rangle_{\text{m-a, s}}$ for a linearly polarized quantum field in turbulence marine-atmosphere

$$\langle s_0\rangle_{\text{m-a, s}} = n\left(\frac{k}{2\pi z}\right)^2 \left(\frac{\pi}{\alpha_H\beta_H}\right)^2 \times$$
$$\times \exp\left\{-\left[\left(\frac{1}{2\alpha_H z}\right)^2 - \left(\frac{1}{4z\beta\alpha_H^2\rho_{s_0}^2} + \frac{1}{2z\beta_H\alpha_H^2\rho_H^2} - \frac{1}{2z\beta_H}\right)^2\right]k^2\rho^2\right\} \tag{16}$$

where:

$$\alpha_H = \sqrt{\frac{1}{4w_0^2} + \frac{1}{2\rho_{s_0}^2} + \frac{1}{\rho_H^2} - \frac{ik}{2z}}$$

$$\beta_H = \sqrt{\frac{1}{4w_0^2} + \frac{1}{2\rho_{s_0}^2} + \frac{1}{\rho_H^2} + \frac{ik}{2z} - \left(\frac{1}{2\alpha_H\rho_{s_0}^2} + \frac{1}{\alpha_H\rho_H^2}\right)^2}$$

By Equations (6) and (16), we have the polarization model of GSM quantization field in the Kolmogorov turbulence of marine-atmosphere channels

$$P = \sqrt{A}\,/\sqrt{A+2} \tag{17}$$

where

$$A = n\left(\frac{k}{2\pi z}\right)^2 \left(\frac{\pi}{\alpha_H\beta_H}\right)^2 \times$$
$$\times \exp\left\{-\left[\left(\frac{1}{2\alpha_H z}\right)^2 - \left(\frac{1}{4z\beta_H^2\alpha_H^2\rho_{s_0}^2} + \frac{1}{2z\beta_H\alpha_H^2\rho_H^2} - \frac{1}{2z\beta_H}\right)^2\right]k^2\rho^2\right\}$$

Similarly, we can obtain the degree of polarization in the Kolmogorov terrene-atmosphere turbulence

$$P_{\text{t-a}} = \frac{\sqrt{\dfrac{1}{\alpha^2\beta^2}\exp\left\{-\left[\left(\dfrac{1}{2\alpha z}\right)^2 - \left(\dfrac{1}{4z\beta\alpha^2\rho_{s_0}^2} + \dfrac{1}{2z\beta\alpha^2\rho_{\text{t-a}}^2} - \dfrac{1}{2z\beta}\right)^2\right]k^2\rho^2\right\}}}{\sqrt{\dfrac{1}{\alpha^2\beta^2}\exp\left\{-\left[\left(\dfrac{1}{2\alpha z}\right)^2 - \left(\dfrac{1}{4z\beta\alpha^2\rho_{s_0}^2} + \dfrac{1}{2z\beta\alpha^2\rho_{\text{t-a}}^2} - \dfrac{1}{2z\beta}\right)^2\right]k^2\rho^2\right\} + \dfrac{8z^2}{nk^2}}} \tag{18}$$

where:

$$\alpha = \sqrt{\frac{1}{4w_0^2} + \frac{1}{2\rho_{s_0}^2} + \frac{1}{\rho_{\text{t-a}}^2} - \frac{ik}{2z}}$$

$$\beta = \sqrt{\frac{1}{4w_0^2} + \frac{1}{2\rho_{s_0}^2} + \frac{1}{\rho_{\text{t-a}}^2} + \frac{ik}{2z} - \left(\frac{1}{2\alpha\rho_{s_0}^2} + \frac{1}{\alpha\rho_{\text{t-a}}^2}\right)^2}$$

4. Numerical calculation and analysis

To analyze and compare the effects of source transverse widths, the outer- and inner -scale of turbulence, the received photon number, and the transverse coherent width of sources on the polarization of GSM quantization field in turbulence channels of the

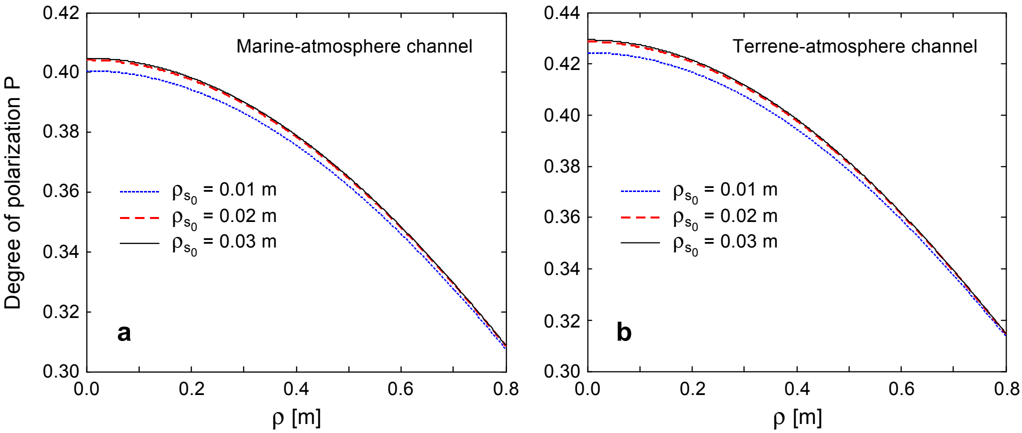

Fig. 1. The degree of polarization as a function of the transverse coordinate for different coherent width of source $\rho_{s_0} = 0.01, 0.02, 0.03$ m with parameters: $C_n^2 = 3 \times 10^{-13}$ m$^{-2/3}$, $L_0 = 10$ m, $l_0 = 1$ mm, $n = 21$, and $w_0 = 0.1$ m. Marine-atmosphere channel (**a**), and terrene-atmosphere channel (**b**).

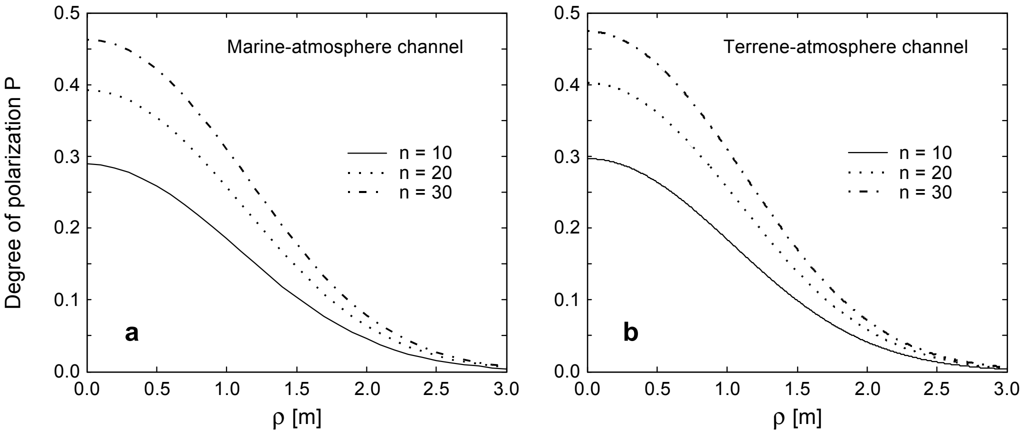

Fig. 2. The degree of polarization as a function of the transverse coordinate for different number of photons n = 10, 20, and 30 with parameters: $C_n^2 = 3 \times 10^{-13}$ m$^{-2/3}$, L_0 = 10 m, l_0 = 1 mm, ρ_{s_0} = 0.01 m, and w_0 = 0.1 m. Marine-atmosphere channel (**a**), and terrene-atmosphere channel (**b**).

marine- and terrene-atmosphere, we perform the numerical propagation described by Eqs. (16) and (17). The simulation results are shown in Figs. 1–4. We know that long -wavelength infrared radiation possesses better all-weather transmission than the shorter wavelength radiation and the infrared radiation λ = 1550 nm is a wavelength of a low-loss atmospheric window [21]. In the numerical propagation, we set wavelength λ = 1550 nm and propagation distance z = 5 km. By Figs. 1–4, we can see that under the condition of same transport parameters, the turbulence fade of marine-atmosphere channels is larger than that of terrene-atmosphere channels. This result declares that the content of small-scale turbulence eddies in marine-atmosphere is larger than that of terrene-atmosphere channels.

Figures 1 and 2 show the effects of the source transverse coherent width and the number of received photons on the degree of polarization of linearly polarized quantum beams in the marine-atmosphere and the terrene-atmosphere channels. From Figs. 1 and 2, we can find that the degree of polarization decreases with the decrease in source transverse coherent width and the number of received photons.

Figure 3 depicts the effects of the inner scale l_0 and the outer scale L_0 on the degree of polarization. The effects of the outer scale on the degree of polarization can be neglected, this result comes from the large scale eddies producing refractive effects. But, the small scale eddies produce scattering (diffractive) effects and polarization state variation, therefore, the effects of the inner scale on the degree of polarization are distinct and the degree of polarization decreases with the decrease in the inner scale. The beam spread decreases with the increase in the inner scale. From Fig. 3, it can be noted that, in the receiving plane, the detection radio range of a polarization signal increases with the decrease in the inner scale. This is caused by a large diameter of a receiving beam from the small inner scale of turbulence. As a consequence, channels with stronger turbulence strength will possess a larger detection area of the polarization signal.

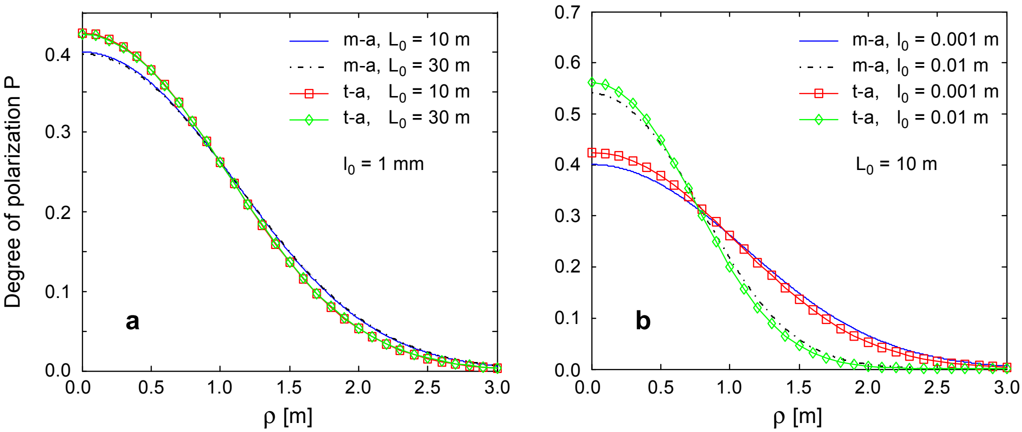

Fig. 3. The degree of polarization as a function of the transverse coordinate for different outer scale and inner scale with $C_n^2 = 3 \times 10^{-13}$ m$^{-2/3}$, $n = 21$, $\rho_{s_0} = 0.01$ m, and $w_0 = 0.1$ m; m-a – marine-atmosphere channel, t-a – terrene-atmosphere channel. Outer scale $L_0 = 10$ and 30 m, and inner scale $l_0 = 1$ mm (**a**). Inner scale $l_0 = 0.001$ and 0.01 m, and outer scale $L_0 = 10$ m (**b**).

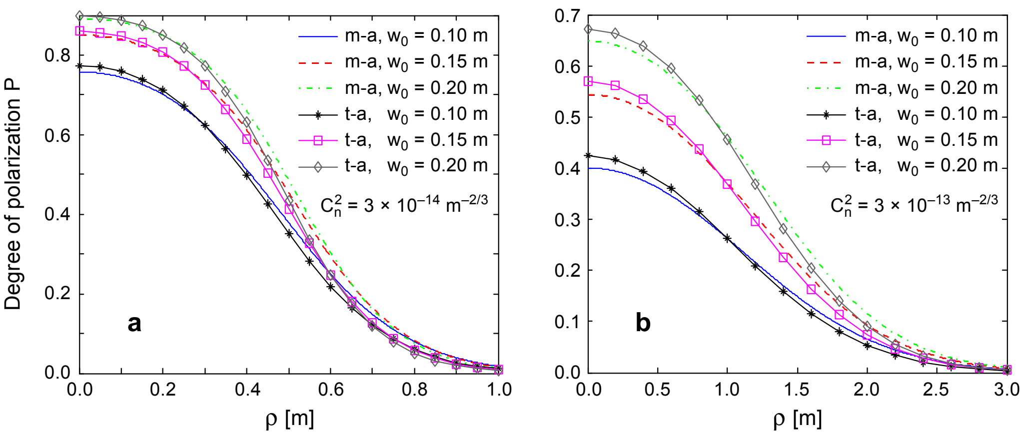

Fig. 4. The degree of polarization as a function of the transverse coordinate for different source transverse size $w_0 = 0.1$, 0.15 and 0.2 m with $l_0 = 1$ mm, $L_0 = 10$ m, $\rho_{s_0} = 0.01$ m, $n = 21$; $C_n^2 = 3 \times 10^{-14}$ m$^{-2/3}$ (**a**) and $C_n^2 = 3 \times 10^{-13}$ m$^{-2/3}$ (**b**); m-a – marine-atmosphere channel, t-a – terrene-atmosphere channel.

The effects of the source transverse size w_0 on the degree of polarization in terrene- and marine-atmosphere channels are presented in Fig. 4. The degree of polarization decreases with the decrease in the source transverse size and the increase in the refractive index structure parameter C_n^2.

5. Conclusions

In this paper, we discussed the fluctuation of the degree of polarization as a function of source transverse coherent width, transverse size, refractive index structure parameter, the outer scale and inner scale of terrene- and marine-turbulence. The polarization fluctuation model of GSM quantization field propagating through the Kolmogorov turbulence of marine-atmosphere channels is obtained based on the development of an effective photon annihilation/creation operator of linearly polarized GSM quantization field in marine-atmosphere. Our results show that under the condition of same transport parameters, the polarization fade of marine-atmosphere turbulence is larger than that of terrene-atmosphere turbulence. The degree of polarization decreases with the decrease in the source transverse coherent width and received photon number. The effects of the turbulence outer scale on the degree of polarization can be neglected, but the degree of polarization decreases with the decrease in the inner scale. The beam spread decreases with the increase in the inner scale. The degree of polarization decreases with the decrease in the source transverse size and the increase in the refractive index structure parameter. The channel with a stronger turbulence strength will have a larger detection area of a polarization signal.

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