

Research on the propagation of partially coherent cosh-Gaussian beams through an $ABCD$ optical system in non-Kolmogorov turbulence by effective tensor approach

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An efficient tensor approach is used to study the propagation of partially coherent cosh-Gaussian beams through an $ABCD$ optical system in non-Kolmogorov turbulence. Analytical expressions for the average intensity of the beam propagation are derived. The properties of the average intensity are investigated with a numerical example. One finds that the propagation of the beam with larger spatial coherence length is less affected by distance when the propagation distance is long enough, and as the Ch-parameter increases, the beam propagation is less effected by turbulent atmosphere. It is also found that the average intensity distribution of the cosh-Gaussian beams with larger spatial correlation length is more affected by the structure constant of turbulence (*i.e.*, turbulence level). By choosing a suitable Ch-parameter and spatial coherence length, the partially coherent cosh-Gaussian beams can be better transmitted in non-Kolmogorov turbulence. Our results will be useful in free-space communication.

Keywords: partially coherent cosh-Gaussian beam, $ABCD$ optical system, non-Kolmogorov turbulence.

1. Introduction

The propagation characteristics of a variety of laser beams in turbulent atmosphere have been studied extensively due to their wide applications, such as long-distance optical communications, imaging, remote sensing, *etc.* [1–4]. Kolmogorov’s power spectrum of refractive index fluctuations is widely accepted and employed in previous studies. However, experimental results reveal that turbulence in portions of the troposphere and the stratosphere deviates greatly from Kolmogorov’s model [5–7]. Therefore, it is very important to find other models more general than the Kolmogorov model

to describe non-Kolmogorov turbulence. Then a non-Kolmogorov model is presented in [8, 9], which reduces to the Kolmogorov model only for the power-law exponent value. Nevertheless, based on this model, a great deal of work has been investigated: the scintillation index, the signal-to-noise ratio, and the bit-error rate [10], the average spreading of a Gaussian–Schell model beam [11], the second-order statistics of stochastic electromagnetic beams [12]. On the other hand, in some practical systems, such as laser radar systems and remote sensing systems, *etc.*, paraxial $ABCD$ optical systems such as thin lenses are commonly encountered [13–15]. Therefore, it is very important to study the propagation of laser beams through an $ABCD$ optical system in turbulent atmosphere.

CASPERSON and TOVAR demonstrated that the Hermite–sinusoidal-Gaussian (HSG) beam is a set of solutions for the wave equation in Cartesian coordinates system [16] and derives its propagation formula. As one of the special cases of HSG beams, the hyperbolic-cosine-Gaussian beam (ChGB) has been studied in recent years due to its potential applications in remote sensing, imaging and communication [17–19]. As far as we know, no results have been reported up until now on propagation of a partially coherent ChGB through a paraxial $ABCD$ optical system in non-Kolmogorov turbulence. And more importantly, we are the first to use an efficient tensor approach (ETA) [20, 21] to simulate the propagation of partially coherent ChGB and illustrate it by a numerical example, because the ETA has obvious advantages over traditional methods. In the traditional discrete Fourier transform (DFT) related methods, since the integral window cannot be changed during the two Fourier transforms, the loss of high frequency information after the quadratic transformation may cause distortions in the reconstruction by DFT [22]. Furthermore, sampling constraints must be strictly obeyed when taking the DFT [23]. Any violation of the constraints would result in errors in reconstruction. ETA has improved both. One is to replace the integral operation with matrix multiplication, and there is no need to consider the influence of the integration window. The second is to use the sampling of points replaced by small -segment integral sampling. Under the same sample size, the calculation time of ETA is greatly shortened and the calculation result is more accurate.

In this paper, the dependences of the average intensity distribution on the spatial correlation length σ_0 , the Ch-parameter \mathcal{Q}_0 , the propagation distance z and the structure constant of turbulence C_n^2 are mainly discussed.

2. Theory

In the Cartesian coordinate system, the z -axis is taken to be the propagation axis. The cross-spectral density (CSD) of partially coherent ChGB in the source plane $z = 0$ takes the form as [24]

$$W(x_1, x_2, y_1, y_2, 0) = W(x_1, x_2, 0)W(y_1, y_2, 0) \quad (1)$$

with $W(x_1, x_2, 0)$ and $W(y_1, y_2, 0)$ given by

$$W_0(i_1, i_2, 0) = \exp\left(-\frac{i_1^2 + i_2^2}{w_0^2}\right) \cosh(\Omega_0 i_1) \cosh(\Omega_0 i_2) \exp\left(-\frac{(i_1 - i_2)^2}{2\sigma_0^2}\right) \quad (2)$$

where $j = x$ or y , w_0 denotes the waist width of the Gaussian part, Ω_0 is the parameter associated with the Ch-part, σ_0 is the spatial correlation length of the laser source in the plane $z = 0$.

Based on Huygens–Fresnel diffraction integral, the cross-spectral density transmission of partially coherent ChGB passing through an $ABCD$ optical system in turbulent atmosphere can be expressed as [20]

$$\begin{aligned} \langle I(x, y, z) \rangle &= W(x, x, y, y, z) \\ &= \left(\frac{k}{2\pi B}\right)^2 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} W_0(x_1, x_2, y_1, y_2, 0) \\ &\quad \times \exp\left\{-\frac{ik}{2B} \left[A(x_1^2 - x_2^2 + y_1^2 - y_2^2) - 2x(x_1 - x_2) - 2y(y_1 - y_2) + Dx^2 \right]\right\} \\ &\quad \times \langle \exp[\psi(x_1, y_1, x, y, z) + \psi^*(x_2, y_2, x, y, z)] \rangle dx_1 dy_1 dx_2 dy_2 \end{aligned} \quad (3)$$

where z refers to the propagation distance in the atmosphere, $k = 2\pi/\lambda$ is the wave number, $\psi(x_1, y_1, x, y)$ is the solution to the Rytov method that represents the random part of the complex phase. The angle bracket indicates the ensemble average over the medium statistics covering the log-amplitude and phase fluctuations due to the turbulent atmosphere. The asterisk means the complex conjugation. A, B, C, D denote the elements of the transfer matrix of the optical system. Moreover, there is no inherent aperture between the source and the output planes. Therefore, A, B, C, D are all real-valued.

The phase term $\langle \exp[\psi(x_1, y_1, x, y) + \psi^*(x_2, y_2, x, y)] \rangle$ can be written as [25]

$$\begin{aligned} &\langle \exp[\psi(x_1, y_1, x, y) + \psi^*(x_2, y_2, x, y)] \rangle \\ &= \exp\left\{-\frac{\pi^2 k^2 T z}{3} \left[(x_1 - x_2)^2 + (y_1 - y_2)^2 \right]\right\} \end{aligned} \quad (4)$$

where T is a turbulence parameter defined by [25]

$$T = \frac{A(\alpha)}{2(\alpha - 2)} C_n^2 \left[\kappa_m^{2-\alpha} \beta(\alpha) \exp(\kappa_0^2 - \kappa_m^2) \Gamma_1(2 - \alpha/2, \kappa_0^2 - \kappa_m^2) - 4\kappa_0^{4-\alpha} \right] \quad (5)$$

where $A(\alpha)$ and $\beta(\alpha)$ are functions of α ($3 < \alpha < 4$) and defined by:

$$A(\alpha) = \frac{\Gamma(\alpha - 1) \cos(\alpha\pi/2)}{4\pi^2}$$

$$\beta(\alpha) = 2\kappa_0^2 - 2\kappa_m^2 + \alpha\kappa_m^2$$

while

$$\kappa_0 = 2\pi/L_0$$

$$\kappa_m = \frac{1}{l_0} \left[\frac{2\pi}{3} \Gamma(5 - \alpha/2) A(\alpha) \right]^{1/(\alpha - 5)}$$

Here, l_0 and L_0 represent the internal and external scales of turbulence, respectively, $\Gamma(\cdot)$ denotes the gamma function, and $\Gamma_1(\cdot)$ refers to the incomplete gamma function, C_n^2 is the generalized refractive index structure parameter and describes the turbulence level, with a unit of $m^3^{-\alpha}$. The parameter α represents the power law exponent. Equation (5) is applicable for the description of both Kolmogorov ($\alpha = 11/3$) and non-Kolmogorov ($\alpha \neq 11/3$) turbulences.

Inserting Eqs. (1), (2) and (4) into Eq. (3), the average intensity distribution of the partially coherent ChGB through an *ABCD* optical system in turbulent atmosphere can be obtained by calculating the following integral:

$$\begin{aligned} \langle I(x, y, z) \rangle &= \frac{1}{\lambda^2 B^2} \iiint \iiint W_t(x_1, x_2, y_1, y_2) \\ &\times \exp \left\{ -\frac{ik}{2B} \left[A(x_1^2 - x_2^2 + y_1^2 - y_2^2) - 2x(x_1 - x_2) - 2y(y_1 - y_2) + Dx^2 \right] \right\} dx_1 dy_1 dx_2 dy_2 \end{aligned} \quad (6)$$

where

$$W_t(x_1, x_2, y_1, y_2) = W_0(x_1, x_2, y_1, y_2) \exp \left\{ -\frac{\pi^2 k^2 Tz}{3} \left[(x_1 - x_2)^2 + (y_1 - y_2)^2 \right] \right\} \quad (7)$$

To indicate the spatial vector points in a transverse plane, we have used $\mathbf{p} \equiv (u, v)$ and $\mathbf{r} \equiv (x, y)$ to indicate the spatial vector points in a transverse plane. To indicate discrete coordinates, we set $x_{j_1} = j_1 \Delta_1$, $y_{k_1} = k_1 \Delta_1$, $u_m = m \Delta_1$ and $v_n = n \Delta_2$, where the grid (sampling) separations in the input and output planes are Δ_1 and Δ_2 , respectively. The discrete form of Eq. (6) is as follows [21]:

$$\langle I(u_m, v_n, z) \rangle = [\mathbf{I}]_{mn} = \sum_{j_1}^{N_1} \sum_{j_2}^{N_1} \sum_{k_1}^{N_1} \sum_{k_2}^{N_1} [\mathbf{H}_y^T]_{nk_1} [\mathbf{H}_x^T]_{mj_1} [W_t]_{j_1 j_2 k_1 k_2} [\mathbf{H}_x]_{j_2 m} [\mathbf{H}_x]_{k_2 n} \quad (8)$$

where the average intensity $\mathbf{I} \equiv \{I(u_m, v_n)\}$ with $m, n = 1 \dots N_2$ is a matrix of $N_2 \times N_2$. $\mathbf{H}_x = \{H_x(x_j, u_m)\}$ and $\mathbf{H}_y = \{H_y(y_k, v_n)\}$ with $j, k = 1 \dots N_1$ represent the impulse response functions of a free propagation system in x and y directions, respectively, and

both are $N_1 \times N_2$ matrices. N_1 and N_2 denote integers, representing the numbers of sampling points in the input and output planes, respectively. T indicates the matrix transpose and complex conjugate.

Within the paraxial approximation, *i.e.*, $\Delta_1 \ll \sqrt{\lambda z}$, the response matrix through an *ABCD* system is as follows:

$$\begin{aligned} [\mathbf{H}_x]_{j_m} &\equiv H_x(x_j, u_m) \\ &= \Delta_1 \frac{\exp(i\pi B/\lambda)}{\sqrt{i\lambda B}} \operatorname{sinc}\left[\frac{(u_m - Ax_j)\Delta_1}{\lambda B}\right] \exp\left[i\frac{\pi}{\lambda B}(Ax_j^2 - 2x_j u_m + Du_m^2)\right] \end{aligned} \quad (9)$$

\mathbf{H}_y can be represented in the same way. $\mathbf{W}_t \equiv \{W_t(x_{j_1}, x_{j_2}, y_{k_1}, y_{k_2})\}$ with $j_{1,2}, k_{1,2} = 1 \dots N_1$ signifies the discrete form of the input CSD function, which is a tensor (multi-dimensional array) of $N_1 \times N_1 \times N_1 \times N_1$. The discrete form of Eq. (7) is as follows:

$$[W_t]_{j_1 j_2 k_1 k_2} \equiv [\mathbf{W}_0]_{j_1 j_2 k_1 k_2} \exp\left\{-\frac{\pi^2 k^2 Tz}{3} [(x_{j_1}^2 - x_{j_2}^2) + (y_{k_1} - y_{k_2})^2]\right\} \quad (10)$$

Because the CSD function of the optical source is *x-y* separable, Eq. (7) can be separated, *i.e.*, where

$$W_{tx}(x_1, x_2) = W_0(x_1, x_2) \exp\left\{-\frac{\pi^2 k^2 Tz}{3} (x_1 - x_2)^2\right\} \quad (11)$$

And $W_{ty}(x_1, x_2)$ can be obtained in a similar way. Equation (8) can be simplified into a matrix form

$$[I]_{mn} = [\mathbf{H}_x^T \times \mathbf{W}_{tx} \times \mathbf{H}_x]_{mm} \cdot [\mathbf{H}_y^T \times \mathbf{W}_{ty} \times \mathbf{H}_y]_{nn} \quad (12)$$

where \mathbf{W}_{tx} and \mathbf{W}_{ty} are $N_1 \times N_1$ matrices: $W_{tx} \equiv \{W_{tx}(x_{j_1}, x_{j_2})\}$, $W_{ty} \equiv \{W_{ty}(y_{k_1}, y_{k_2})\}$.

Through Eq. (12), the evolution of the average intensity of partially coherent ChGB through an *ABCD* optical system in non-Kolmogorov turbulence can be quantitatively analyzed.

3. Simulation result and analysis

In this section, we simulate the propagation of a partially coherent ChGB transmitting through an *ABCD* optical system in non-Kolmogorov turbulence. The main results of the ETA can be summarized as Eq. (12), which provides the averaged intensity for a partially coherent ChGB in turbulence. As a numerical example, a partially coherent ChGB is focused by a lens, propagates through atmospheric turbulence, and is eventually terminated at the observation plane, which is shown in Fig. 1.

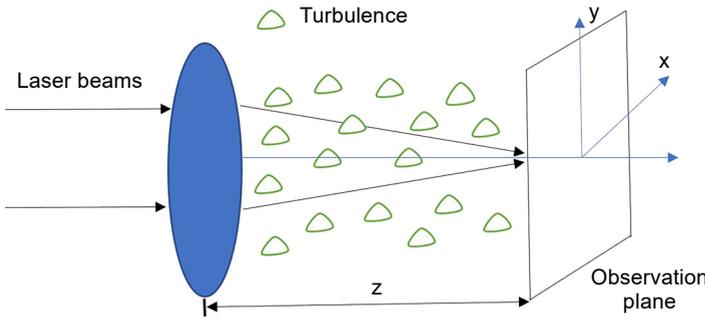


Fig. 1. Schematic diagram of the single lens system in turbulent atmosphere.

The dimension of the thin lens is assumed to be larger than the corresponding beam diameter. Therefore, the diffraction due to the lens is neglected. The distance between the lens and observation plane is z . The corresponding transfer matrix between the source plane and the observed plane in the focusing system is [26]

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} = \begin{pmatrix} 1 - z/f & z \\ -1/f & 1 \end{pmatrix} \quad (13)$$

Substituting Eq. (13) into Eq. (12), we can calculate the propagation properties of a partially coherent ChGB through an $ABCD$ optical system in non-Kolmogorov turbulence numerically. The parameters that do not change in the simulation are selected as $\alpha = 3.5$, $A(\alpha) = 0.033$, $L_0 = 10$ m, $l_0 = 0.001$ m, $\lambda = 632.8$ nm, $w_0 = 0.06$ m and $f = 50$ m in all figures.

Firstly, the effects of the spatial correlation length of the laser source σ_0 and the propagation distance z on the average intensity distribution of a partially coherent ChGB through an $ABCD$ optical system in non-Kolmogorov turbulence are investigated.

Figure 2 shows calculated results for the average intensity distribution with $\Omega_0 = 30$ m⁻¹ and $C_n^2 = 1 \times 10^{-14}$ m^{-2/3}. The curves of black, red, and green correspond to $\sigma_0 = 0.04$ m, 0.02 m, and 0.01 m, respectively. Subfigure (a), (b), and (c) correspond to $z = 1000$ m, 3000 m, and 5000 m, respectively.

One finds from Fig. 2a that the average intensity distributions of the curves of black, red, and green are very close and all have two peaks when the propagation distance is small ($z = 1000$ m). With increasing the propagation distance, the average intensity distribution gradually evolves into a Gaussian-like distribution, the width of the beam is gradually expanding and the difference between the three curves increases. This phenomenon could be explained by the fact that the average intensity distribution of the beam is almost independent of the spatial coherence length when the propagation distance is very small. Moreover, it is clear that the peak value of average intensity of

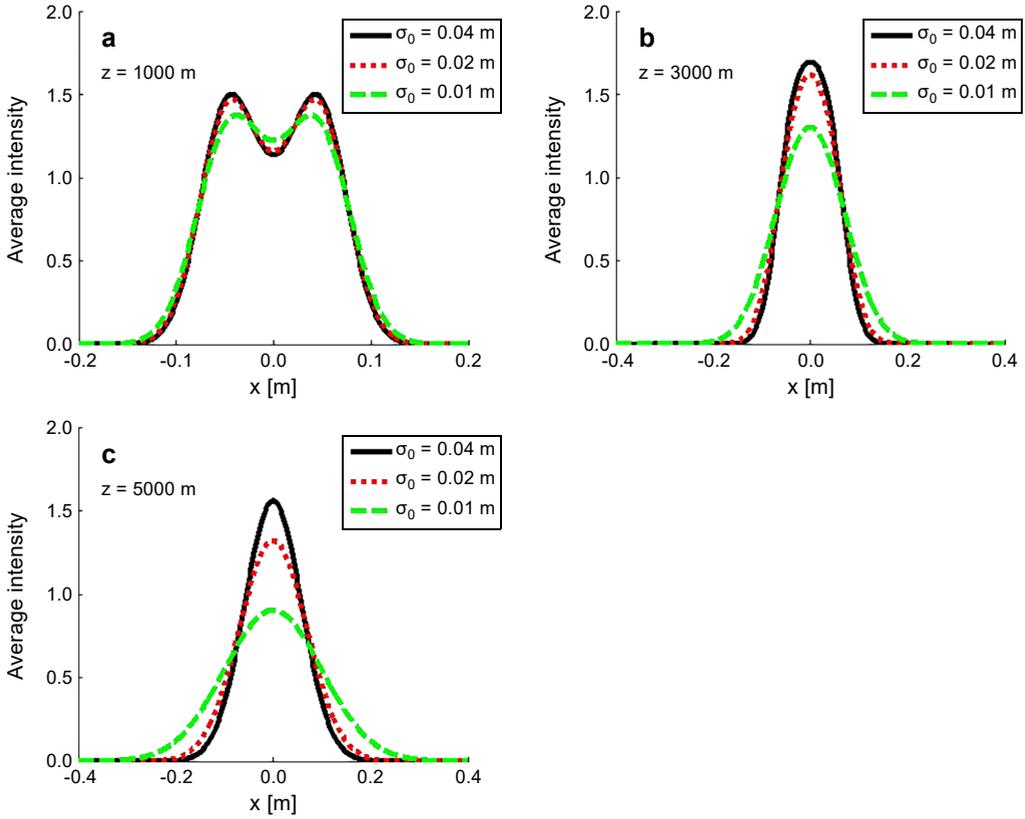


Fig. 2. Average intensity distribution in the x direction of a partially coherent ChGB with different values of spatial correlation length through an $ABCD$ optical system at different propagation distances in turbulent atmosphere at $\Omega_0 = 30 \text{ m}^{-1}$. (a) $z = 1000$ m, (b) $z = 3000$ m, and (c) $z = 5000$ m.

the curve with larger σ_0 is always higher than that of the curve with smaller σ_0 when the propagation distance is long enough. That is to say, the propagation of a partially coherent ChGB with a larger value of σ_0 is less affected by distance.

Then Fig. 3 illustrates the average intensity distributions when Ch-parameter Ω_0 changes from 30 to 10 m^{-1} , and the rest of parameters are same as those in Fig. 2. It is found that the average intensity always maintains a Gaussian-like distribution, which shows Ω_0 affects the initial intensity distribution of a partially coherent ChGB. Meanwhile, as the propagation distance increases, the peak value of the average intensity decreases gradually and the width of the beam expands gradually. Particularly, comparing Fig. 2a and Fig. 3a ($z = 1000$ m), it can be found that as the Ω_0 decreases, the peak values of the three curves are all significantly reduced. Same for $z = 3000$ m and $z = 5000$ m, which means the Ω_0 has an obvious effect on the average intensity distribution of the ChGB.

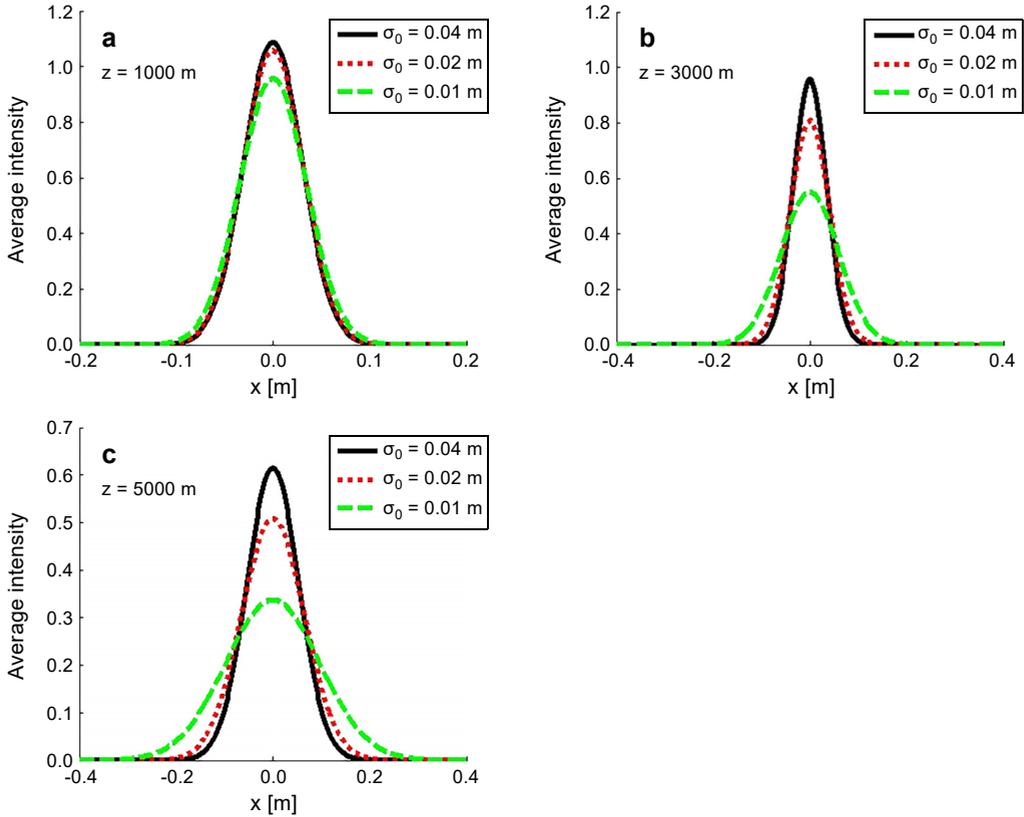


Fig. 3. Average intensity distribution in the x direction of a partially coherent ChGB with different values of spatial correlation length through an $ABCD$ optical system at different propagation distances in turbulent atmosphere at $\Omega_0 = 10 \text{ m}^{-1}$. (a) $z = 1000$ m, (b) $z = 3000$ m, and (c) $z = 5000$ m.

To further investigate the effect of Ω_0 on the average intensity distribution of the beam, Fig. 4 shows the average intensity distribution of a partially coherent ChGB of different values of Ch-parameters Ω_0 in turbulent atmosphere with $\sigma_0 = 0.04$ m, $z = 5000$ m, and $C_n^2 = 1 \times 10^{-14} \text{ m}^{-2/3}$. It is obvious that the peak value of the average intensity increases with the increase of Ω_0 , that is to say, the beam with larger Ch-parameter Ω_0 is less affected by the turbulent atmosphere.

Figure 5 illustrates the average intensity of partially coherent ChGB with different values of the structure constant of turbulence C_n^2 through an $ABCD$ optical system in non-Kolmogorov turbulence with $\Omega_0 = 10 \text{ m}^{-1}$ and $z = 3000$ m. The curves of black, and red correspond to $C_n^2 = 1 \times 10^{-14} \text{ m}^{-2/3}$, and $C_n^2 = 1 \times 10^{-16} \text{ m}^{-2/3}$, respectively. Subfigure (a), (b), and (c) correspond to $\sigma_0 = 0.04$ m, 0.02 m and 0.01 m, respectively. From Fig. 5, it can be seen that with the decreasing of turbulence structure parameter C_n^2 (*i.e.*, turbulence intensity weakening), the peak value of the average intensity decreases

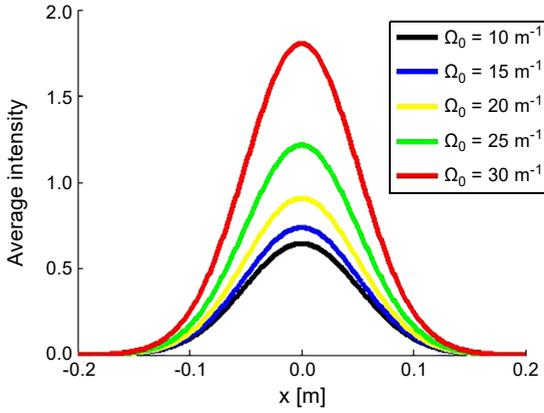


Fig. 4. Average intensity distribution in the x direction of a partially coherent ChGB with different values of Ch-parameters through an $ABCD$ optical system in turbulent atmosphere.

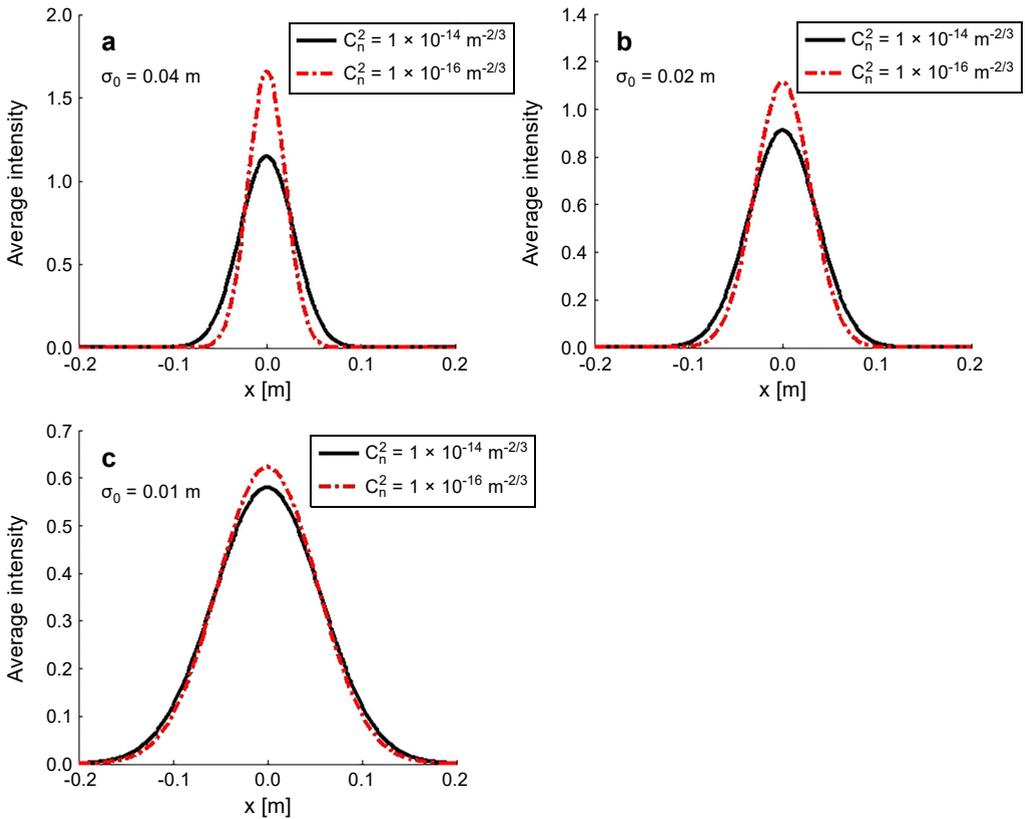


Fig. 5. Average intensity distribution in the x direction of a partially coherent ChGB with different values of spatial correlation length through an $ABCD$ optical system at different turbulence levels.

obviously. From all three figures (Figs. 5a–5c), it can be found that the peak value of ChGB with a smaller spatial coherence length σ_0 is lower than that of ChGB with a larger σ_0 . However, when $\sigma_0 = 0.01$ m, the average intensity distributions of a partially coherent ChGB with $C_n^2 = 1 \times 10^{-14} \text{ m}^{-2/3}$ and $C_n^2 = 1 \times 10^{-16} \text{ m}^{-2/3}$ are very close. This phenomenon could be explained by the fact that the turbulent structure constant C_n^2 has little effect on the average intensity distribution of the beam when the spatial coherence length σ_0 is small. Moreover, comparing Figs. 5a–5c, it is clear the difference between the two curves at different turbulence levels increases with increasing σ_0 , which means the average intensity distribution of the beam is more affected by the structure constant of turbulence (*i.e.*, turbulence level) when the spatial coherence length is larger.

4. Conclusion

In conclusion, the propagation of a partially coherent ChGB through *ABCD* optical system in non-Kolmogorov turbulence is simulated by the effective tensor approach (ETA). Analytical expressions for the cross-spectral density (CSD) and the average intensity are derived. As a numerical example, the properties of the average intensity of a partially coherent ChGB through the optical system of a thin lens in non-Kolmogorov turbulence are demonstrated. The effects of the spatial coherence length σ_0 , the Ch-parameter Ω_0 , the propagation distance z and the structure constant of turbulence C_n^2 on beam propagation are mainly discussed. We have found that the σ_0 has less influence on the average intensity distribution of the beam when the propagation distance is small. Furthermore, the propagation of a partially coherent ChGB with a larger value of σ_0 is less affected by distance when the propagation distance is long enough. It is also found that with the increase of Ω_0 , the propagation is less affected by turbulent atmosphere. And the average intensity distribution of the beam with larger σ_0 is more affected by the C_n^2 (*i.e.*, turbulence level). By selecting appropriate Ch-parameter and spatial coherence length, the partially coherent ChGB can be better transmitted in the non-Kolmogorov turbulence.

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