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Transmission stability of chirped dark vector quasi-solitons in birefringent fiber system with nonlinear gain

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In this article, we consider the coupled Ginzburg–Landau equation with variable coefficients including the nonlinear gain and obtain the exact solutions of chirped dark vector quasi-solitons via the ansatz method. Next, the propagation of chirped dark vector quasi-solitons is discussed to verify whether they can be transmitted stably in the birefringent optical fiber system. The numerical simulation shows that this can be achieved. We deeply add the small perturbation to the transmission of dark vector quasi-solitons to make the results above more general. The results further prove the correctness of our solutions.

Keywords: chirped dark vector quasi-soliton, coupled Ginzburg–Landau equation, birefringent fiber.

1. Introduction

Solitons have been known for long, the concept of which was first put forward in 1834. From the view of physics, the soliton is considered to be a special product of the nonlinear effect. Mathematically, it is a stable non-dispersive solution of some nonlinear partial differential equations with the finite energy. That is, it can keep its waveform unchanged all the time. As a stable local light wave, the soliton has paved a particular interest since it was brought into the optical field theoretically and experimentally in the 1970s and 1980s [1, 2], ascribed to its characteristics of the high peak power in optical communication systems. In 1981, HASEGAWA and KODAMA proposed to use optical solitons as the information carrier in the optical fiber communication, officially opening the era of the optical soliton communication [3]. With the characteristics of optical solitons, optical communication with the large capacity over the ultra-long distance can be easily realized.

In general, it is not comprehensive to use the nonlinear Schrödinger equation (NSE) without the cross-phase modulation (XPM) term to describe the dynamics of optical fiber solitons, considering the fact that there are two polarization modes transmitted simultaneously at different speeds in the single-mode fiber. These modes cannot be actually degenerated because of the asymmetry, the stress and the bending of optical fibers, resulting in the birefringence of fibers. Based on this, the coupled NSE including the XPM term is taken as the theoretical model instead to study the transmission of solitons in birefringent fibers. To some extent, due to the XPM between two polarization components, it is considered that solitons have the ability to resist the polarization mode dispersion (PMD) effect caused by the time delay difference. XPM exactly compensates for the delay distortion, making the group velocity between two polarization components match to the greatest extent [4, 5]. During this transmission, the solitons in two polarization directions will periodically capture each other to achieve the synchronization in the time domain, which is the phenomenon of soliton self-trapping [6–8]. Nevertheless, the NSE, combined with nonlinear gain and dispersion gain terms, is modified to the Ginzburg–Landau (G-L) equation when solitons are transmitted in the gain medium [9–11]. It plays an important role in the study of the state transformation and unstable wave theory. Therefore, a lot of attention has been directed to exploring the stable soliton solutions according to this equation in recent years. Subsequently, the transmission of solitons satisfying the solutions is simulated in birefringent fibers [12, 13]. Vector solitons are steady-state waves in birefringent fibers that hold the waveforms constant [14]. In this paper, they refer to the solitons whose polarization state and pulse envelope remain unchanged or change periodically during the transmission in birefringent fibers.

The vector soliton in birefringent fiber systems is one of the hotspots in the field of nonlinear optics. In the last few years, many researches on it have gained much more attention. The propagation of vector solitons in circularly birefringent fibers was studied by the perturbation analytically and numerically and an effect leading to the polarization stabilization of circularly polarized vector solitons was found [15]. Four types of vector solitons with different characteristics were totally observed with the usage of different lengths of polarization maintaining fibers in the highly birefringent cavity of the all-normal-dispersion ytterbium-doped fiber laser [16]. A general formalism was developed for investigating the evolution of arbitrarily polarized short pulses in a birefringent fiber. When the polarization of the dispersion wave is controlled by the linear birefringence of fibers, the polarization of vector solitons can be considerably sensitive to the nonlinear birefringence [17]. This paper deals with dark vector solitons in particular, which generally exist in the normal group velocity dispersion region. This kind of soliton has been studied theoretically and experimentally recently. The propagation of the incoherently coupled two-color vector solitons in the nonlinear self-defocusing media was investigated, which showed that the two-color vector dark and grey solitons with different wavelengths are always unstable [18]. Afterwards, incoherently coupled dark-bright vector solitons were observed experimentally for the first time in the single-mode fiber [19].

In this paper, we use the ansatz method to analyze the coupled G-L equation which includes the nonlinear gain term with variable coefficients and obtain the exact solutions of chirped dark vector quasi-solitons. We also numerically simulate the propagation of the solitons in the birefringent fiber system by means of the split-step Fourier method to testify the correctness of our solutions. For more details of the stability of solutions, we go a step further to simulate the evolution process under the circumstance of adding the small perturbation of amplitude, noise and phase position. In some ways, the results derived in this article provide a theoretical reference for the further study of dark vector solitons.

2. Theoretical solution and analysis

The coupled Ginzburg–Landau equation including the nonlinear gain term is evoked to describe the propagation of vector solitons in the gain medium, which can be written in the following form:

$$\frac{\partial p}{\partial z} + \sigma \frac{\partial p}{\partial t} + \frac{i}{2} k_1(z) \frac{\partial^2 p}{\partial t^2} - i\gamma_1(z)\left(|p|^2 + B|q|^2\right)p - g_1(z)p + \beta_1(z)|p|^2 p = 0 \tag{1a}$$

$$\frac{\partial q}{\partial z} - \sigma \frac{\partial q}{\partial t} + \frac{i}{2} k_2(z) \frac{\partial^2 q}{\partial t^2} - i\gamma_2(z)\left(|q|^2 + B|p|^2\right)q - g_2(z)q + \beta_2(z)|q|^2 q = 0 \tag{1b}$$

where p and q represent the slowly varying envelopes of solitons in the orthogonal direction in the electric field, which are related to z and t (z and t are the normalized transmission distance and the delay time, respectively), σ is the normalized birefringence coefficient to describe the group velocity mismatch between two polarization components, B is the XPM coefficient, which is generally set equal to 2/3 as large as the self -phase modulation term. As functions of the propagation distance z, $k_i(z)$, $\gamma_i(z)$, $g_i(z)$ and $\beta_i(z)$ ($i = 1, 2$) stand for the group velocity dispersion (GVD), the nonlinear coefficient, the linear gain (loss) and the nonlinear gain, respectively. In this paper, whether equals 1 or 2, $k_i(z)$, $\gamma_i(z)$, $g_i(z)$ and $\beta_i(z)$ have the same values. Therefore in the following analysis, we use $k(z)$, $\gamma(z)$, $g(z)$ and $\beta(z)$ instead.

Considering that the birefringent effect will bring great inconvenience to our calculation, we put forward the following scheme to obtain the vector soliton solutions of Eq. (1):

$$p(z, t) = u(z, t)\exp\left[i\sigma^2 z\left(-\frac{1}{2k(z)}\right) - i\sigma\left(-\frac{1}{k(z)}\right)t\right] \tag{2a}$$

$$q(z, t) = v(z, t)\exp\left[i\sigma^2 z\left(-\frac{1}{2k(z)}\right) + i\sigma\left(-\frac{1}{k(z)}\right)t\right] \tag{2b}$$

We substitute Eq. (2) into Eq. (1) and one obtains,

$$\frac{\partial u}{\partial z} + \frac{i}{2}k(z)\frac{\partial^2 u}{\partial t^2} - i\gamma(z)\left(|u|^2 + B|v|^2\right)u - g(z)u + \beta(z)|u|^2 u = 0 \tag{3a}$$

$$\frac{\partial v}{\partial z} + \frac{i}{2}k(z)\frac{\partial^2 v}{\partial t^2} - i\gamma(z)\left(|v|^2 + B|u|^2\right)v - g(z)v + \beta(z)|v|^2 v = 0 \tag{3b}$$

Adopting the ansatz method, we make the assumption that the solutions of this equation are chirped dark solitary wave solutions as follows:

$$u(z, t) = A_1(z)\tanh\left[\eta(z)(t - T(z))\right]\exp\left[i\varphi_1(z, t)\right] \tag{4a}$$

$$v(z, t) = A_2(z)\tanh\left[\eta(z)(t - T(z))\right]\exp\left[i\varphi_2(z, t)\right] \tag{4b}$$

where $A_1(z)$ and $A_2(z)$ are both amplitudes, $\eta(z)$ is the parameter related to the pulse width, $T(z)$ is the central position of the pulse. The phase can be described as follows:

$$\varphi_1(z, t) = \rho(z)\ln\left\{\operatorname{sech}\left[\eta(z)(t - T(z))\right]\right\} + a_1(z)t^2 + b_1(z)t + c_1(z) \tag{5a}$$

$$\varphi_2(z, t) = \rho(z)\ln\left\{\operatorname{sech}\left[\eta(z)(t - T(z))\right]\right\} + a_2(z)t^2 + b_2(z)t + c_2(z) \tag{5b}$$

where $a_i(z)$, $b_i(z)$ and $c_i(z)$ ($i = 1, 2$) represent the linear chirp, the frequency and the initial phase, respectively, and $\rho(z)$ is the nonlinear chirp.

We plug Eqs. (4) and (5) into Eq. (3) and separate the real part from the imaginary one. The exact solutions of chirped dark vector quasi-solitons can be determined by the following constraints:

$$\rho(z) = \rho = \text{const} \tag{6}$$

$$\eta(z) = \eta = \text{const} \tag{7}$$

$$b_1(z) = b_2(z) = b = \text{const} \tag{8}$$

$$c_1(z) = c_2(z) = 0 \tag{9}$$

$$a_1(z) = a_2(z) = \frac{1}{2}\left[b^2 k(z) + 2k(z)\eta^2\right]z \tag{10}$$

$$T(z) = -k(z)bz \tag{11}$$

$$g(z) = \frac{3k(z)\eta^2\rho\beta(z) - \beta'(z)}{2\beta(z)} \tag{12}$$

$$\gamma(z) = \frac{2-\rho^2}{5\rho}\beta(z) \tag{13}$$

$$A_1(z) = A_2(z) = \sqrt{\frac{3k(z)\eta^2\rho}{2\beta(z)}} \tag{14}$$

Considering the heterogeneity of fibers, it is necessary to adopt the variable coefficient form to set the fiber parameters, which can be assumed in the following form:

$$\beta(z) = \beta_0\left[1 + C_1\sin(\delta_1 z)\right]\exp(\delta_2 z) \tag{15}$$

where β_0 is the ideal birefringent fiber parameter, C_1 represents the amplitude of the fiber parameter fluctuation, which is a tiny amount, δ_1 represents the period of the change of fiber parameters, δ_2 is a real constant. And then one obtains,

$$g(z) = \frac{3}{2}\left[k(z)\eta^2\rho\right] - \frac{\beta_0 C_1\cos(\delta_1 z)\exp(\delta_2 z) + \delta_2\beta(z)}{2\beta(z)} \tag{16}$$

3. Numerical analysis

In the process above, we just analyzed the theoretical expressions of various parameters and we need to combine numerical simulations by means of the split-step Fourier scheme to make the results more comprehensive. Dark vector solitons satisfying the solutions are selected as incident pulses in two polarization directions of birefringent fibers. For simplicity, we need to determine initial values: $c(0) = 0$ and $T(0) = 0$ so that the initial phase and the initial position of incident pulses are both set to 0. In this paper, we assume that $k(z) = k$ is a constant. Other parameter values are as follows: $k = 0.5$, $\beta_0 = 0.1$, $\eta = 0.2$, $\rho = 1.3$, $C_1 = 0.01$, $\delta_1 = 1$, and $\delta_2 = 0$.

To verify the stability of our solutions, the propagation of dark vector solitons corresponding to the solutions is numerically simulated over the long distance during four different situations. Under the theoretical model we study, solitons are transmitted over 250 dispersion lengths by selecting the appropriate parameters. When adding no perturbation, the evolution of solitons is shown in Fig. 1. As we can see, whether along the fast axis or the slow axis, there is a slight fluctuation on both sides of solitons at first, but it tends to stabilize gradually with the increase of the transmission distance. Moreover, it is worth noting that the propagation of solitons is accompanied with the amplitude periodic oscillation.

Based on the analysis above, we can deduce the stability of solitons preliminarily. However, it is too one-sided. So in order to clarify the stability more comprehensively in birefringent fibers, we consider the transmission of vector dark solitons under the small perturbation. Firstly, with other parameters kept unchanged, 5% amplitude perturbation is added to solitons, as is shown in Fig. 2. As is visible, solitons can keep

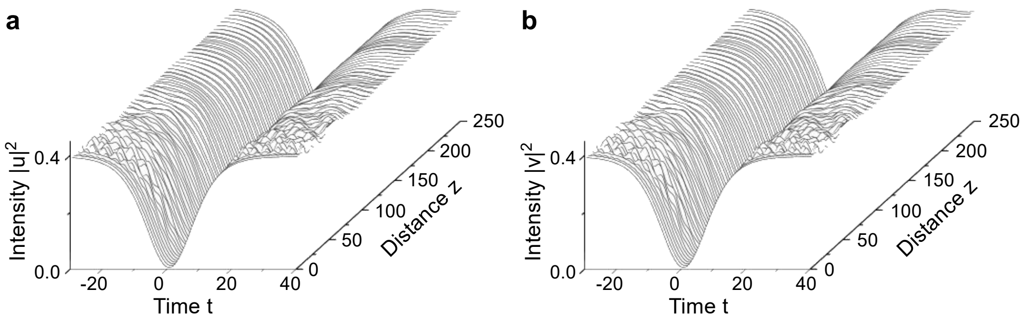

Fig. 1. The evolution of dark vector solitons without the perturbation.

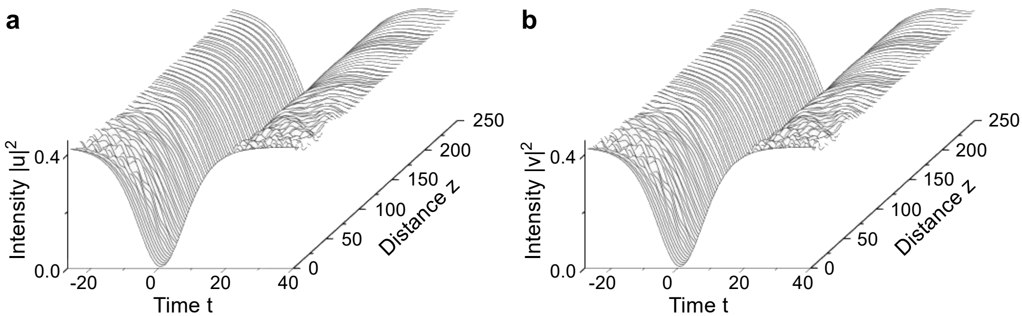

Fig. 2. The evolution of dark vector solitons with the 5% amplitude perturbation.

stable transmission in two polarization directions, almost without the influence of the amplitude perturbation.

Subsequently, keeping other parameters retaining the same, we add 3% noise perturbation to the transmission of solitons and the results are shown in Fig. 3. It should be pointed out that whether along the slow axis or the fast axis, although the noise is superimposed at the beginning, both fluctuation and noise are weakened or even disappear with the increase of the transmission distance, further demonstrating the stability of solitons under the circumstance of this perturbation.

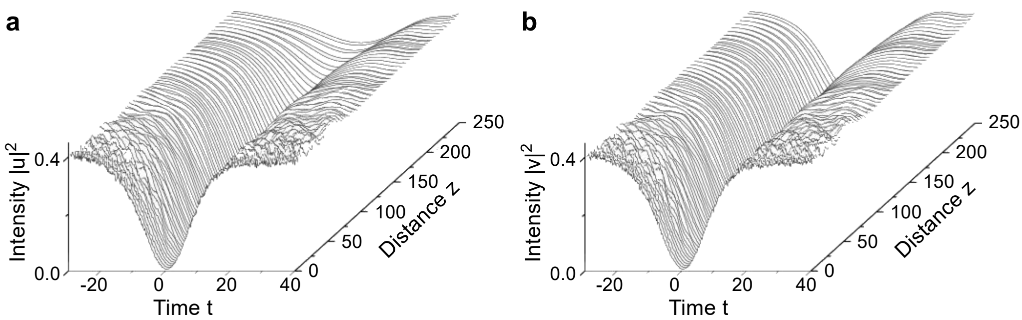

Fig. 3. The evolution of dark vector solitons with the 3% noise perturbation.

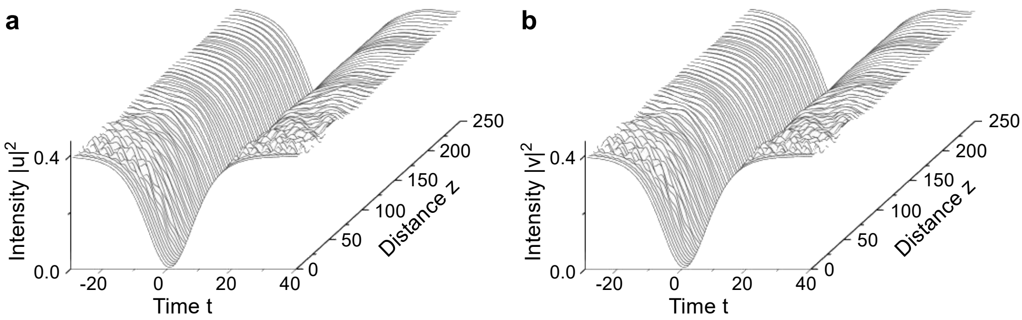

Fig. 4. The evolution of dark vector solitons with the 5% phase perturbation.

Finally, retaining invariant from other parameters, 5% phase perturbation is available to add to solitons. The evolution in this case is displayed in Fig. 4, which is almost identical to the case without any perturbation. In other words, the stable transmission of solitons in this case can also be well achieved.

4. Conclusions

In this paper, we obtained exact solutions of chirped dark vector quasi-solitons, based on the coupled Ginzburg–Landau equation. In addition, without loss of generality, we provided the deeper insight into the stability of solitons. After carrying out the numerical simulation, dark vector solitons satisfying the solutions can be transmitted stably. As is evident from above, as long as we choose the appropriate parameters, the stable transmission of solitons in two polarization directions can be achieved, which is consistent with our expected results. This indicates the correctness of our calculation. As for the stability of dark vector solitons under the small perturbation, we came to a conclusion that no matter the amplitude, noise or phase perturbation are added to dark vector solitons, the stable propagation of solitons is almost not affected.

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