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The far-zone behaviors of light waves on scattering from particulate medium with various distribution

QIHANG DAI, ZHONGYUAN XIE, XIAONING PAN, XIAOLING JI, TAO WANG*

Department of Physics, Sichuan Normal University, Chengdu 610068, China

*Corresponding author: towerwang@126.com

The far-zone spectral density of light waves scattered on a particulate medium was discussed, and the influence of characteristics of the medium on the far-zone scattered spectral density was investigated. It is shown that the normalized spectral density of the scattered field is closely related with the structural characteristics of the particles collection, including the relative size of particles and the distribution information of particles in the collection. These results may provide potential application in the reconstruction of the structure information of particulate medium.

Keywords: weak scattering, particulate collection, spectral density.

1. Introduction

Since Wolf and his collaborators found that the far field spectrum of light waves may change as it is scattered by a random medium [1], the topic of light waves scattering has attracted increasing attention. During the past few years, the behaviors of far-zone scattered field of various media were discussed due to its potential applications in areas such as remote sensing, detection, medical diagnosis and other fields [2–22]. For example, the phenomenon of spectral shift and spectral switches during scattering was discovered by studying the spectral behavior of light waves on scattering [9–12]; the reciprocity relation of light waves during scattering has been derived, which presents the specific relationship between the properties of scattered field and the characteristics of scattering medium [13–16]; the equivalence theorem of light waves on scattering from different media has been presented, and the conditions that different scattering media may produce far field with identical spectral density or that with identical spectral degree of coherence have been obtained [17–19]. These discussions greatly enrich the research area of light waves scattering.

In practice, the scattering medium which we frequently encounter may be particulate medium, such as tissue, and cells in the blood [20–22]. The optical properties of this kind of medium are described by the superposition of the scattering potential of each particle in the collection [23]. Recently, researchers have paid more attention to the particulate medium and a lot of papers have been published on behaviors of its scattered field (see, for examples, Refs. [24, 25]). However, to the best of our knowledge, a general relation between the properties of the scattered field and the characteristics of the particulate medium have hardly been discussed, which is important in areas such as inverse problem, *i.e.*, the determination of the structural information of media from the measurements of the properties of the scattered field [26]. These will be discussed in this manuscript. Especiaally, the far field spectral density of light waves scattered by a particulate medium will be discussed and the distribution of the spectral density produced by particulate medium with different sizes or with different distribution characteristics will be discussed. These results may be applied to reconstruct the structure information of unknown particulate medium, which is of great significance in areas such as medical diagnostics.

2. Theory

We begin our analysis by assuming that a spatially coherent monochromatic plane light wave propagating in a direction specified by a unit vector $\mathbf{s}_0$, is incident on a scattering medium (see as Fig. 1). The cross spectral density (CSD) function of the incident field at a pair of points specified by a couple of position vectors $\mathbf{r}'_1$ and $\mathbf{r}'_2$ can be characterized by the expression of [23]

$$W^{(i)}(\mathbf{r}'_1, \mathbf{r}'_2, \mathbf{s}_0, \omega) = \langle U^{(i)*}(\mathbf{r}'_1, \mathbf{s}_0, \omega) U^{(i)}(\mathbf{r}'_2, \mathbf{s}_0, \omega)\rangle \quad (1)$$

where the asterisk denotes complex conjugate and the angular brackets denote the ensemble average, and $U^{(i)}(\mathbf{r}', \omega)$ represents a collection of frequency-dependent, re-

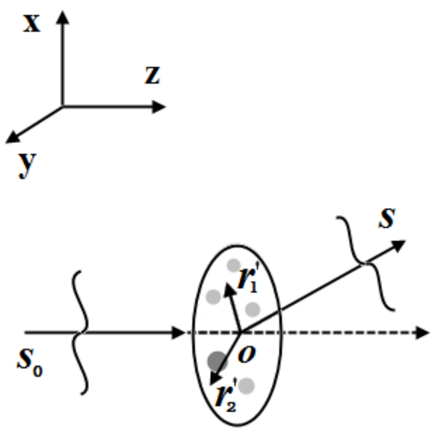

Fig. 1. Illustration of scattering process from a collection of particles.

alizable state statistical ensembles with ω being the frequency of incident scalar monochromatic light wave, which can be expressed as

$$U^{(i)}(\mathbf{r}', \mathbf{s}_0, \omega) = a(\omega)\exp(ik\mathbf{s}_0 \cdot \mathbf{r}') \tag{2}$$

where $a(\omega)$ is a random function, and $k = \omega/c$ where c is the speed of light in vacuum. Upon substituting from Eq. (2) into Eq. (1), the CSD function of the incident field can be rewritten as

$$W^{(i)}(\mathbf{r}_1', \mathbf{r}_2', \mathbf{s}_0, \omega) = S^{(i)}(\omega)\exp\left[ik\mathbf{s}_0 \cdot (\mathbf{r}_2' - \mathbf{r}_1')\right] \tag{3}$$

where

$$S^{(i)}(\omega) = \langle a^*(\omega)a(\omega)\rangle \tag{4}$$

represents the spectrum of the incident field.

Assume that weak scattering occurs after the light wave is incident on the scattering medium, so that the scattering progress can be analyzed within the accuracy of the first-order Born approximation [27]. In this case, the CSD function of the far-zone scattered field at two positions specified by a pair of position vectors $r\mathbf{s}_1$ and $r\mathbf{s}_2$ can be represented as

$$W^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) = \frac{S^{(i)}(\omega)}{r^2}\tilde{C}_\mathrm{F}(\mathbf{K}_1, \mathbf{K}_2, \omega) \tag{5}$$

where

$$\tilde{C}_\mathrm{F}(\mathbf{K}_1, \mathbf{K}_2, \omega) = \iint_D C_\mathrm{F}(\mathbf{r}_1', \mathbf{r}_2', \omega)\exp\left[i(\mathbf{K}_1 \cdot \mathbf{r}_1' - \mathbf{K}_2 \cdot \mathbf{r}_2')\right]\mathrm{d}r_1'\,\mathrm{d}r_2' \tag{6}$$

is the six dimensional Fourier transform of the correlation function of the scattering potential of the medium, where $\mathbf{K}_1 = k(\mathbf{s}_1 - \mathbf{s}_0)$, $\mathbf{K}_2 = k(\mathbf{s}_2 - \mathbf{s}_0)$, and $C_\mathrm{F}(\mathbf{r}_1', \mathbf{r}_2', \omega)$ is the correlation function of the scattering potentials, which can be expressed as [23]

$$C_\mathrm{F}(\mathbf{r}_1', \mathbf{r}_2', \omega) = \langle F^*(\mathbf{r}_1', \omega)F(\mathbf{r}_2', \omega)\rangle \tag{7}$$

where $F(\mathbf{r}', \omega)$ is the scattering potential of medium, and the asterisk and the angular brackets denote complex conjugate and ensemble average, respectively.

We assume that the scattering medium is composed of a collection of particles where the properties of each particle are deterministic, and the locations of each particle of the collection are represented by position vectors $\mathbf{r}_1', \mathbf{r}_2', ..., \mathbf{r}_n'$. In this case, the scattering potential of the whole particle collection can be defined as [23]

$$F(\mathbf{r}', \omega) = \sum_n^N f_n(\mathbf{r}' - \mathbf{r}_n', \omega) \tag{8}$$

where f_n is the scattering potential of the n-th particle and N is the total number of particles. For convenience of discussion, we assume that the scattering potential of each particle in the collection satisfies the Gaussian distribution, *i.e.*, [23]

$$f_n(r, \omega) = A \exp\left(-\frac{r^2}{2\sigma_n^2}\right) \tag{9}$$

where A is a constant and σ_n represents the effective width of the scattering potential of the n-th particle. By substituting from Eq. (9) into Eq. (8), the scattering potential of particles in the collection can be expressed as

$$F(\mathbf{r}', \omega) = A \sum_{n}^{N} \exp\left[-\frac{(\mathbf{r}' - \mathbf{r}'_n)^2}{2\sigma_n^2}\right] \tag{10}$$

Upon substituting from Eq. (10) into Eq. (7), then into Eq. (6), and taking the six dimensional Fourier transform, one can obtain

$$\begin{aligned}\tilde{C}_{\mathrm{F}}(\mathbf{K}_1, \mathbf{K}_2, \omega) = {} & A(2\pi)^2 \sum_{n=1}^{N} \sigma_n^3 \exp\left(-\frac{1}{2}\sigma_n^2 \mathbf{K}_1^2\right) \exp(i\mathbf{K}_1 \cdot \mathbf{r}'_n) \\ & \times \sum_{n=1}^{N} \sigma_n^3 \exp\left(-\frac{1}{2}\sigma_n^2 \mathbf{K}_2^2\right) \exp(-i\mathbf{K}_2 \cdot \mathbf{r}'_n)\end{aligned} \tag{11}$$

When the position vector $r\mathbf{s}_1$ and $r\mathbf{s}_2$ coincide, the scattered spectral density of the far field can be obtained from the CSD function, with a form of

$$\begin{aligned}S^{(\mathrm{s})}(r\mathbf{s}, \mathbf{s}_0, \omega) \equiv {} & W^{(\mathrm{s})}(r\mathbf{s}, r\mathbf{s}, \mathbf{s}_0, \omega) \\ = {} & \frac{S^{(\mathrm{i})}(\omega)}{r^2} \tilde{C}_{\mathrm{F}}\Big[k(\mathbf{s} - \mathbf{s}_0), k(\mathbf{s} - \mathbf{s}_0), \omega\Big]\end{aligned} \tag{12}$$

Upon substituting from Eq. (11) into Eq. (12), the far-zone spectral density of the light wave scattered on a collection of particles can be expressed as the following form

$$\begin{aligned}S^{(\mathrm{s})}(r\mathbf{s}, \mathbf{s}_0, \omega) = {} & \frac{A(2\pi)^2 S^{(\mathrm{i})}(\omega)}{r^2} \sum_{n=1}^{N} \sigma_n^3 \exp\left(-\frac{1}{2}\mathbf{K}^2\sigma_n^2 + i\mathbf{K} \cdot \mathbf{r}_n\right) \\ & \times \sum_{n=1}^{N} \sigma_n^3 \exp\left(-\frac{1}{2}\mathbf{K}^2\sigma_n^2 - i\mathbf{K} \cdot \mathbf{r}_n\right)\end{aligned} \tag{13}$$

where $\mathbf{K} = k(\mathbf{s} - \mathbf{s}_0)$.

3. Numerical simulations

In this section, we will discuss the behaviors of far field spectral density of light waves scattered on particles collection with different distributions, including one-dimensional distribution and two-dimensional distribution. In what follows, the normalized spectral density of far field will be plotted, and the yellow line in all graphs represents the normalized spectral density of the scattered field at the dotted line.

3.1. Particles with one dimensional distribution

In this part, we will consider the influence of the characteristics of particles collection on the normalized spectral density of the scattered field in the case of particles with one-dimensional distribution.

First of all, we will consider the influence of the size of particles on the spectral density of the scattered field. As shown in Figs. 2**a**–2**c**, we gradually decrease the size of one particle by keeping the distance between the two particles as a constant, and the corresponding spectral density of the far field is also presented in Figs. 2**d**–2**f**, respectively. In all graphs of spectral density, α is the angle between the scattered direction $\mathbf{s}$ and the incident direction $\mathbf{s}_0$, while β is the angle between the x-axis and the projection of the scattered direction in the xoz plane. As shown in Fig. 2**d**, when the size of the two particles is the same (Fig. 2**a**), there is a minimum between the two principal maxima of the scattered spectral density at the dotted line, and the minimum value is zero. However, the minimum value of the normalized spectral density will become larger when the size of two particles is different as shown in Fig. 2**e**. Moreover, when the difference between the size of two particles is large enough (Fig. 2**c**), the dis-

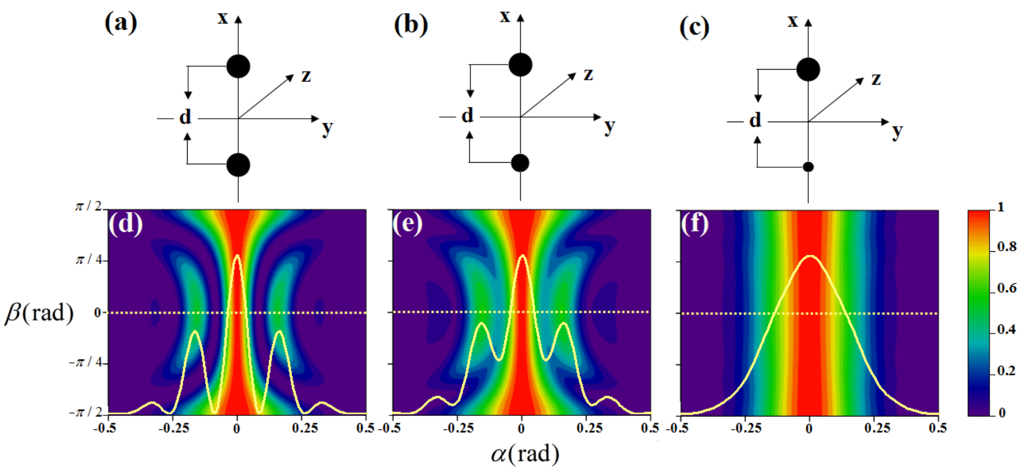

Fig. 2. Illustration of particle distribution (**a**–**c**) and its normalized spectral density of scattered field (**d**–**f**). The parameters for calculations are: $\lambda = 0.6283$ μm, $d = 6\lambda$, $k = 2\pi/\lambda$, $\sigma_{11}/\sigma_{12} = 5/5$, $\sigma_{21}/\sigma_{22} = 5/3$, and $\sigma_{31}/\sigma_{32} = 5/1$.

tribution of the scattered spectral density will tend to a Gaussian profile (Fig. 2**f**). This phenomenon is caused by the superposition of the scattered field of each particle. When the size of these two particles is the same, the effect of the superposition on the scattered field is obvious. However, when the size of the two particles is evidently different, the intensity of the scattered field is mainly determined by one of the particles.

In Fig. 3, we consider the influence of the distribution of particles on the spectral density of the scattered field. As shown in Figs. 3**a**–3**c**, we assume that one of the particles in the collection drifts farther and farther away, and its corresponding spectral

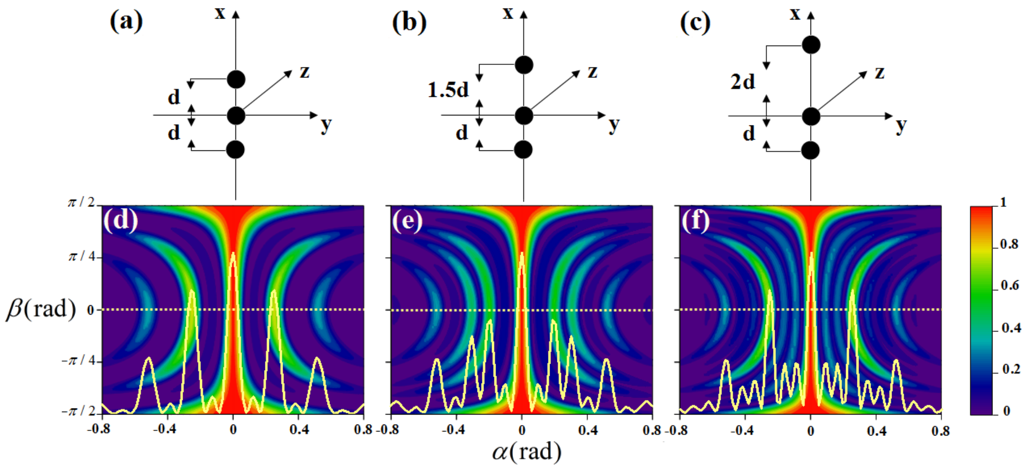

Fig. 3. Illustration of particle distribution (**a**–**c**) and its normalized spectral density of scattered field (**d**–**f**). The parameters for calculations are: $k\sigma = 2$, $d = 4\lambda$, $d_{11}/d_{12} = 1/1$, $d_{21}/d_{22} = 1.5/1$, and $d_{31}/d_{32} = 2/1$.

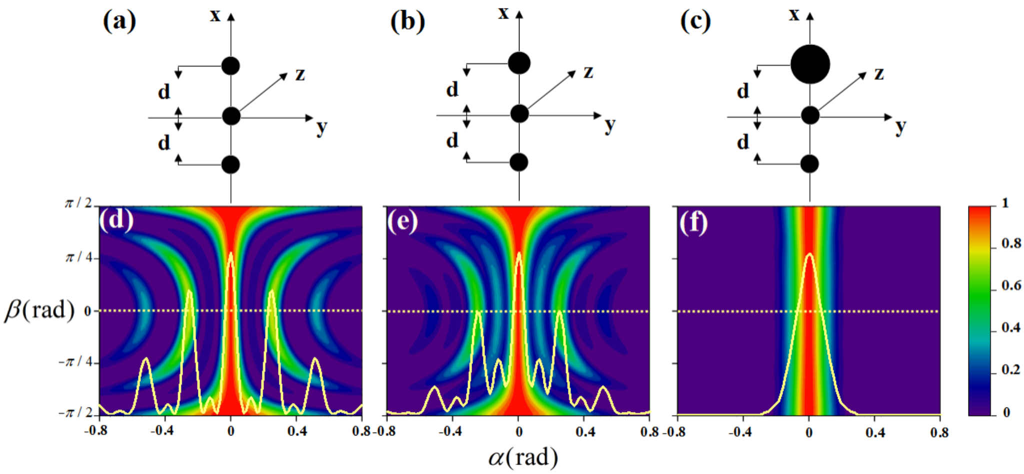

Fig. 4. Illustration of particle distribution of scattering medium (**a**–**c**) and normalized spectral density distribution of scattered field (**d**–**f**). The calculated parameters are shown below: $d = 4\lambda$, $k\sigma_{11}/k\sigma_{12}/k\sigma_{13} = 2/2/2$, $k\sigma_{21}/k\sigma_{22}/k\sigma_{23} = 3/2/2$, and $k\sigma_{31}/k\sigma_{32}/k\sigma_{33} = 8/2/2$.

density distributions are presented in Figs. 3**d**–3**f**, respectively. It is shown from Fig. 3**d** that there are two minima with zero values between the two principal maxima of the scattered spectral density at the dotted line when the distances between the particles are the same (Fig. 3**a**). However, as shown in Fig. 3**b** and Fig. 3**c**, when one of the particles in the collection drifts farther and farther, the number of maxima in the scattered spectral density increases gradually, and in this case the minimum values are no longer equal to zero (Fig. 3**e** and Fig. 3**f**).

In Fig. 4, we consider the influence of the size of one particle in the collection on the spectral density of the scattered field. As shown in Figs. 4**a**–4**c**, we assume that there are three different particles collections, whose corresponding normalized spectral densities are presented in Figs. 4**d**–4**f**, respectively. In Fig. 4**d**, one can find that there are two minima between the two main maxima, and the values are zero. However, with the increase of the size of one particle in the particles collection (Fig. 4**b**), the minimum values of far field scattered spectral density are no longer equal to zero, as shown in Fig. 4**e**. What is more, when the size of the particle is large enough (Fig. 4**c**), the scattered spectral density will show a Gaussian profile, as shown in Fig. 4**f**.

3.2. Particles with two dimensional distribution

In this part, we will discuss the influence of different distribution of two dimensional particles collection on the far field scattered spectral density.

In Fig. 5, we consider the influence of orientation of particulate collection on the distribution of the far-zone scattered field. As shown in Figs. 5**a**–5**c**, we assume that the particles are located in x-axis, y-axis and a random direction, respectively, and the corresponding spectral densities are presented in Figs. 5**d**–5**f**. By Comparing Figs. 5**d** –5**f**, one can find that the normalized spectral density of the far field will vary with the

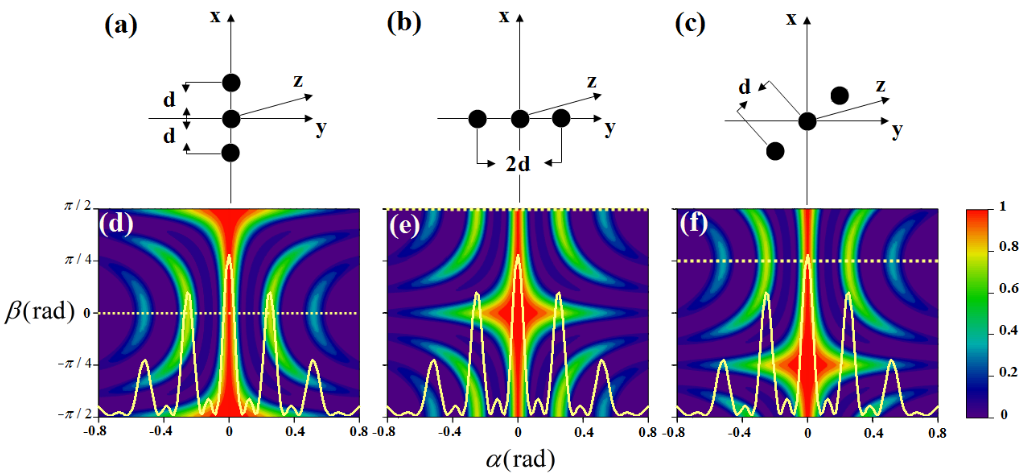

Fig. 5. Illustration of particle distribution of scattering medium (**a**–**c**) and normalized spectral density distribution of scattering field (**d**–**f**). The calculated parameters are shown below: $k\sigma = 2$, and $d = 4\lambda$.

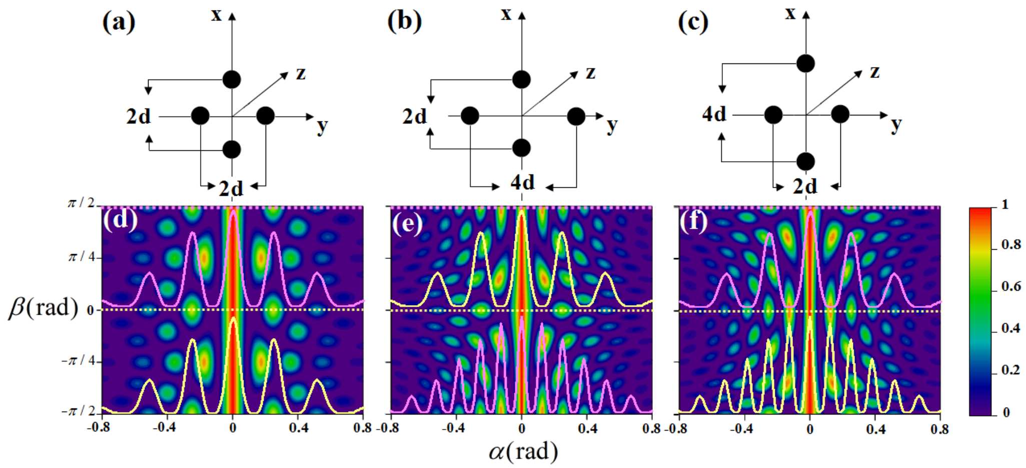

Fig. 6. Illustration of particle distribution of scattering medium (**a**–**c**) and normalized spectral density distribution of scattering field (**d**–**f**). The calculated parameters are shown below: $k\sigma = 2$, and $d = 4\lambda$.

orientation of particles, while the normalized spectral density of the scattered field at the dotted line in all graphs are identical, *i.e.* the distribution of spectral density is related to the orientation of particles.

In Fig. 6, we consider the influence of the asymmetric distribution of particles on the spectral density of the scattered field. As shown in Figs. 6**a**–6**c**, three different particulate media are presented, and the corresponding spectral density distribution is presented in Figs. 6**d**–6**f**, respectively. As shown in Fig. 6**a**, when the distances between the two particles along the *x*-axis and that along the *y*-axis are the same (*i.e.* the symmetric distribution), the distributions of the spectral density in the *xoz* plane and the *yoz* plane are identical (as shown in Fig. 6**d**). However, as shown in Fig. 6**b** and Fig. 6**c**, if the distance between the particles in the two axes is not equal (*i.e.*, the asymmetric distribution), the spectral densities of scattered field will be different in *xoz* plane and *yoz* plane. In particular, when the distance along the *x*-axis is smaller than that along the *y*-axis (see Fig. 6**b**), the number of maximum value of scattered spectral density distribution in *yoz* plane is larger than that in the *xoz* plane, as shown in Fig. 6**e**. An opposite phenomenon can be found by letting the distance along the *y*-axis smaller than that along the *x*-axis, as shown in Fig. 6**c** and Fig. 6**f**.

4. Conclusions

In summary, we have discussed the behaviors of far field spectral density scattered by particles collection with different characteristics. The influences of particles size and particles distribution on the normalized spectral density of the far field are discussed. These results show that the normalized spectral density of scattered field, including the location, the number and the amplitude of extreme points, is affected by the structural characteristics of the media. This phenomenon may provide a possible method to reconstruct the structural information of an unknown medium.

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