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Gravitational waves in a quantum electrodynamic perspective

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Resent scientific progress has suggested that gravitational waves can, under certain conditions, be reflected in a similar manner to electromagnetic waves. As a consequence, it is necessary for scientists to begin incorporating the theories of electromagnetic wave optics unto gravitational waves in order to better understand their physical behavior. This manuscript attempts to extrapolate the behavior of a gravitational wave trapped in a Fabry–Pérot cavity by utilizing theories used to model an electromagnetic wave trapped under similar conditions. Using classical optical theories, it is determined that a pair of microscopic black holes with a mass of $\sim 7.38 \times 10^{14}$ kg separated by a distance of 1 micron would generate a gravitation wave of approximately 100 GHz. Moreover, the shortcomings of classical theories of gravity are discussed as they are applied to microscopic scales.

Keywords: optical resonator, cavity, quantum, gravity, electrodynamics.

1. Introduction

Perhaps one of the most intriguing questions in modern science is the connection, or lack thereof, between quantum mechanics and general relativity. Both theories appear to be valid, with quantum mechanics being able to describe the behavior of molecules and atoms in detail [1,2]. At the same time, it is well known that relativistic effects must also be accounted for when a strong gravitational field is encountered. Perhaps one of the most intriguing scientific endeavors is finding connections between the two theories. The photon is the particle responsible for being the force carrier of electromagnetic radiation, and is arguably the most important bosonic particle in quantum mechanics [1,3,4]. For example, the photon's interaction with matter is responsible for spectroscopic phenomena such as photosynthesis, which is the key to life as we currently understand it. Interestingly, a photonic counterpart is hypothesized to exist in general relativity that is the force carrier of gravity, that is, the graviton [5]. While the particle-wave duality of photons is unquestioned in the scientific community, with the double slit experiment being replicated countless times, such is not the case for the graviton [5,6]. Nevertheless, the theoretical backing for its existence is rather strong [5,7,8].

Most recently, scientists have experimentally observed gravitational waves, proving the theoretical existence of such phenomena with experimental evidence [9]. The device used to detect the waves was a highly specialized Fabry–Pérot type interferometer [9]. Note that such devices are routinely used in quantum electrodynamic experiments on the nano scale to study the interactions between light and matter [3, 10]. When a molecule is placed in a Fabry–Pérot nano scale cavity, it can interact with the trapped photon. If the cavity is built such that the photon is of the same energy as the molecular electronic transition of the molecule, and the lifetime of the photon is longer than the relaxation rate of the excited state, strong light-matter coupling may occur forming cavity polaritons. Note that gravitational waves need not be produced from astronomical objects. Physicists have hypothesized that energetic particles, such as those found inside particle accelerators, can be converted into microscopic black holes [11, 12]. Indeed, it has been hypothesized that regular matter may serve as a source of high frequency gravitational waves due to changes in the quadrupole moment of a particle [7]. However, because the change in the quadrupole moment is negligible, any such contribution would be as well unless a large, dense mass is considered, at which point it is more sensible to consider microscopic black holes as a viable candidate. Interestingly, the frequency of a gravitational wave is proportional to the mass of the objects involved as well as the orbit size, therefore, the gravitational waves produced by a micro black hole would be on a more reasonable scale, requiring a smaller cavity than the massive LIGO laboratory [13]. For example, particle colliders may produce such structures as technology advances [12, 14]. Moreover, recent work has suggested that gravitational waves may be deflected by certain material, in the similar manner to electromagnetic waves [15]. Therefore, one must postulate: what would occur if a microscopic black hole pair, which is capable of producing gravitational waves, would be placed inside a cavity surrounded by such a reflective material?

2. Theory

2.1. Basics of optical cavities

First, the well-established theory of strong light matter coupling will be discussed. The bulk of this theory was established independently by John Hopfield and Vladimir Agranovich, with Thomas Ebbesen and David Lidzey performing a significant amount of experimental contribution [3, 10, 16, 17]. In simple terms, a molecule placed in a specialized quantum cavity can interact with a photon to form a hybrid light-matter state under the right conditions. The hybridized states are referred to as polaritons. The required conditions for successful strong light-matter coupling include: the energy of the cavity photon being the same as the excitation of the molecule, the decay rate of the photon not exceeding the relaxation rate of the molecule, and the oscillator strength of the molecular component being sufficient to allow for a strong transition. Environmental factors also play an important component, for example early polariton structures had to be kept at cryogenic temperatures, otherwise the Boltzman energy at room temperature was sufficient to overcome the separation between the energy levels of the polaritons [18,19]. The polariton structures are true hybrid states, with each state having to be referred to with the inclusion of the photonic (boson) and the molecular (fermion) part, as described in Eq. (1) [3,19]. In the equation, the polariton energy level is described in terms of the excited state with 0 photons $|e\rangle_e|0\rangle_c$, and the ground state with 1 photon $|g\rangle_e|1\rangle_c$.

$$|\mathrm{UP}\rangle/|\mathrm{LP}\rangle = \frac{1}{\sqrt{2}}\left[|e\rangle_e|0\rangle_c \pm |g\rangle_e|1\rangle_c\right] \tag{1}$$

While it is important to remember the quantum nature of the polariton structure, it is noted that it is possible to use classical wave optic theories to describe the interference behavior of light with great accuracy. In such a treatment, the free spectral range (FSR) of a cavity is calculated using the well-established formula listed in Eq. (2), in which the FSR is inversely proportional to the cavity length and the index of refraction (nL) [19,20]:

$$\mathrm{FSR} = \frac{c}{2nL} \tag{2}$$

Note that on a molecular scale the L term requires the cavity to be on the nano scale as the wavelength of most optical transitions in a molecule typically lie in the visible range of the electromagnetic spectrum [1,20]. The finesse of the cavity must also be accounted for, which is related to the reflectance of the mirrors used in the cavity construction, as seen in Eq. (3) [19,20].

$$F = \frac{2\pi}{\ln\left(\frac{1}{R_1 R_2}\right)} \tag{3}$$

It should also be noted that no mirror system is perfect, and the decay of the photon must be logged. The absorptive losses of the mirror material, as well as the photon leakage from the cavity due to finite reflectance will allow light to escape the cavity. This factor is accounted for by including an attenuation coefficient term α, as seen in Eq. (4) [20].

$$\alpha = \frac{1}{2nd}\ln\left(\frac{1}{R_1 R_2}\right) \tag{4}$$

With the help of Eqs. (1-4) the final intensity of the photon in an empty Fabry–Pérot cavity can now be described in Eqs. (5-7):

$$r = R_1 R_2 \exp(-2\alpha n d) \tag{5}$$

$$I_{\max} = \frac{I_o}{(1-|r|)^2} \tag{6}$$

$$I_{\text{final}} = \frac{I_{\max}}{1 + \left(\frac{2F}{\pi}\right)^2 \sin^2\left(\frac{\pi \nu_{\text{incidance}}}{\nu_{\text{FSR}}}\right)} \tag{7}$$

Equation (7) describes the intensity of a photon in a Fabry–Pérot cavity, while Eqs. (2) and (3) describe the photon's full width half maximum (FWHM) in such a structure. The FWHM is directly lined to the FSR and finesse: FWHM = FSR/F. Note that when a molecule is placed in such a cavity structure, the photon will interact with the molecule. The wave transfer matrix can be used to determine how light at each layer of the cavity will behave [20]:

$$M = \begin{bmatrix} t_{12}t_{21} - r_{12}r_{21} & \dfrac{r_{21}}{t_{12}} \\ \dfrac{r_{12}}{t_{21}} & \dfrac{1}{t_{12}} \end{bmatrix} \tag{8}$$

where r and t are the Fresnel coefficients describing the reflectance and transmittance between each layer of the structure. The matrix must be solved for each layer to determine the final transmittance. While solving for the layer that includes the absorbing molecule, the absorbance loss is incorporated along with the photon attenuation coefficient as another source that is responsible for decreasing the signal intensity.

2.2. Applying QED principles to gravitational waves

Similarly to how the wave-matrix method describes the behavior of cavity confined photons, this treatment is now extended to cavity confined gravitons. Note that the photon was treated as an entirely a wave-like particle to describe its cavity behavior. Likewise, the graviton will be treated entirely as being wave-like. Moreover, since the behavior of the light inside the cavity can be well described using classical physics, this manuscript will describe the behavior of the graviton in a cavity in classical terms as well. The strong coupling description of light-matter interaction also requires certain specific conditions. One such assumption is that the photon energy and the molecular excitation must be of similar energy. Thus, this limitation will be imposed on the description of gravitational waves in a cavity structure. That is, the frequency of the gravitational waves must be resonant with that of the spin of a pair of microscopic black holes.

The frequency of a gravitation wave is directly related to the orbit of the binary system from which the gravitational waves originated, as seen in Eq. (9) [13]. Using the well-established Kepler's laws and Newtonian mechanics, Eq. (13) relates the orbital frequency to the masses of the objects involved [21]. To accomplish this, a simple derivation is provided. First, Newton's second law is invoked in Eq. (10). In Eq. (11) the values for acceleration due to gravity and the mass terms are incorporated and related to orbital frequency ω. Note that the factors L_a and L represent the distance from the barycenter to each mass, and in case of a perfectly circular orbit $L = L_a$. Finally, Eq. (13) is rearranged such that the mass and gravitational constant terms are all placed on the same side and the need to take the square root of this term arises so as to relieve ω of its exponent. Note that in order to produce a gravitation wave, the two objects must rotate 180 degrees about their orbits. Therefore, the frequency of a gravitational wave can be estimated by extrapolating it from the period of rotation of two bodies, which is listed for convenience as Eq. (14).

$$\omega_g = 2\omega_{\text{orbit}} \tag{9}$$

$$F = ma \tag{10}$$

$$F = \left[\left(\frac{G}{L^2}\right)(m_1 m_2)\right] = m_1 \frac{(\omega L_a)^2}{L_a} \tag{11}$$

$$L_a = \frac{m_2 L}{m_1 + m_2} \tag{12}$$

$$\omega_{\text{orbit}} = \frac{1}{2\pi}\sqrt{\frac{G(m_1 + m_2)}{L^3}} \tag{13}$$

$$\frac{T}{2} = \pi\sqrt{\frac{L^3}{G(m_1 + m_2)}} \tag{14}$$

Estimates on the size of microscopic black holes generally limit their size based on the fact that below a certain mass the Schwarzschild radius of such a structure would become smaller than the Planck length [14,22]. Therefore, a lower limit of $\sim 10^{12}$ kg is given to the size of a microscopic black hole, as below such a mass current theories predict that their radius quickly approaches the $\sim 10^{-35}$ meter limit. Using Eqs. (9) and (13) while setting L at a constant 10^{-6} meters, the estimated frequency of the gravitational wave generated from a pair of black holes with a mass of 7.384×10^{14} kg would be 10×10^{10} Hz. For simplicity, it is assumed that both m_1 and m_2 have the same mass. Using the relationship in Eq. (2), in which the index is ignored due to the extremely negligible interaction gravitational waves with matter, it is estimated that a cavity of 1.495×10^{-3} meters would be resonant with such a wave. The design of the cavity can be seen in Fig. 1.

The distance used for the separation of the two microscopic black holes (L in Eq. (13)) was not random, rather it is set to generate gravitational waves in the gigahertz range, which is the necessary frequency required to produce reflection by gravitation waves according to Minter *et al.* While gravitational waves and electromagnetic waves posses some similarities in that they are both “wave-like” and are expected to travel

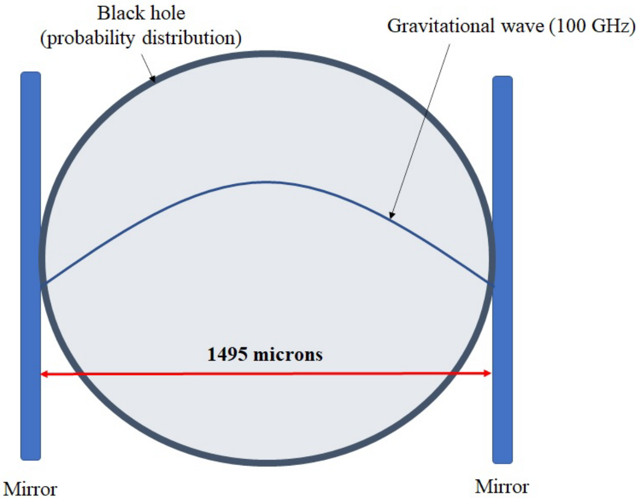

Fig. 1. Overview of a black hole cavity. The black hole is distributed in a Fabry–Pérot cavity, with the resulting gravitational waves being reflected by the superconducting mirrors. Note that while the distance between the pair of microscopic black holes to generate the 100 GHz gravitation wave was set to 1 micron, the figure displays the pair of black holes as a probability distribution as it is not known where inside the cavity they are located.

at the same velocity in vacuum, $\sim 2.99\times10^8$ m/s, electromagnetic waves readily interact with matter, while gravitational waves almost entirely do not [15,23]. Therefore, the control of a gravitational wave in a Fabry–Pérot type cavity through reflective and deflective mechanisms is of particular difficulty. However, recent studies have shown that gravitational waves at the microwave frequency may be reflected using super conducting Cooper pairs [15]. The authors of the study point out that a gravitational wave of any size must carry energy, however, this energy is typically radiated as heat by molecules as low energy gravitational waves pass through matter. The authors argue that at higher energies, such as the MHz–GHz range, the energy of the gravitational waves would displace the Cooper pairs inside the super conductor, as the ions snapp back into place, their combined mass would produce a gravitational wave in the opposite direction, not unlike the Lorentz model of dielectrics [24]. According to MINTER *et al.* only the reflection of higher frequency gravitation waves is possible, as the reflection is directly related to the London penetration depth of the material, therefore, reflection of low frequency waves would require impossibly large superconducting materials [15].

Another important component required to sustain strong coupling is that the mirror reflectivity must be sufficient enough so as to allow the graviton to exist in the cavity long enough for the coupling to occur, as defined in Eq. (4). This is analogous to the photonic example in which the lifetime of the photon must be longer than the decay rate of the molecular excitation so as to allow the two components to successfully hybridize. Interestingly, MINTER *et al.* estimate gravitational wave reflection near unity at temperatures near 0 kelvin. Note that the superconducting model of gravitational mirrors is somewhat similar to the early models of strong light-matter coupling which also required cryogenic temperatures [10,16].

$$r = \frac{2GM}{c^2} \tag{15}$$

$$T = \frac{\frac{h}{2\pi}c^3}{k_B 8\pi GM} \tag{16}$$

Using Eqs. (15) and (16) to estimate the Schwarzschild radius and the Hawking temperature of a black hole required to produce 10×10^{10} Hz (100 GHz) microwave radiation results in 1.84×10^{-21} meters and a temperature of 1.63×10^{9} kelvin [25,26]. Note that in Eq. (15) M is the mass of the individual microscopic black hole. These equations suggest that such a structure would be approximately 6 orders of magnitude smaller than the radius of an atomic nucleus, while emitting a temperature that is similar to that of the core of the Sun [27]. These values being un-physical is not surprising as the equations are designed to estimate the properties of massive bodies, and a theory of quantum gravity is necessary to accurately describe the behavior of gravity at quantum scales. This is not unlike the “ultraviolet catastrophe” which scientists faced while describing the irradiance of a black body prior to the advent of a sufficient quantum explanation for electromagnetic radiation [2]. Indeed, the similarity of an infinitely increasing radiation as the size of the black hole decreases in the Hawking approximation is eerily similar to the Rayleigh–Jeans law in which radiation of a black body increases infinitely as the wavelength decreased. Therefore, a Planck’s-like correction to the Hawking model is required in order to adequately explain gravitational behavior at quantum scales. Moreover, at the quantum level particles are typically treated in terms of their probability distribution. Treating a microscopic black hole as a “point -like” source is largely non-sensical in quantum mechanics. Indeed, from a quantum perspective instead of the 1.84×10^{-21} meter radius of the black hole as calculated in Eq. (15), it would be equally as valid to state the black hole is 1.495×10^{-3} meters in diameter as that is the size of the cavity, and thus the object must exist at some location inside the cavity along its orbit. However, the exact location is undeterminable, that is, it is treated as entirely a wave-like system. This is essentially the application of the Heisenberg principle [28]. Indeed, it would be physically impossible to know the exact location of a microscopic black hole as any attempt to measure it would alter its location, as with any particle on a quantum scale.

$$V = \frac{1}{2}kx^2 \tag{17}$$

$$k = m\omega^2 \tag{18}$$

$$E\psi_x = \frac{-\hbar^2}{2m}\frac{\mathrm{d}^2\psi_x}{\mathrm{d}x^2} + \frac{1}{2}m\omega^2 x^2 \psi_x \tag{19}$$

$$E = \frac{h\omega}{2\pi}\left(n + \frac{1}{2}\right) \tag{20}$$

Finally, as a gravitational wave is emitted by a pair of black holes inside a cavity, their orbits are expected to decay and lose energy, eventually evaporating [29]. However, in a cavity the emitted wave would be reflected back and could constructively interfere to stimulate the black hole pair back to their original energy level, as seen in Fig. 2. This process could in theory continue until the energy of the gravitational wave is dissipated due to the finite nature of the reflection of the mirrors, as well as the small, but non-zero, absorption of gravitational waves by the surrounding matter. This is a similar process to what would be expected in a cavity undergoing light-matter coupling as described in Section 2.1 [3,30]. Moreover, such a view suggests that there should exist a "oscillator strength" like parameter in relation to the absorption of gravitational waves. The derivation of a quantum oscillator model is shown in Eqs. (17)−(20) [31]. The potential energy surface is estimated in Eq. (17), with the spring constant defined in Eq. (18). Rewriting the energy in the form of the Schrödinger equation in Eq. (19) it is possible to solve for the energy *E*, with the resulting equation shown in Eq. (20). Applying the quantum oscillator model to the cavity structure in Eq. (20) indicates that such a structure would absorb energies in quanta of 1.054×10^{-23} joules.

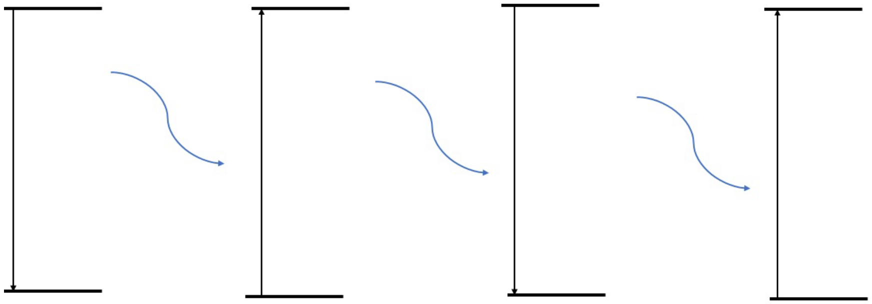

Fig. 2. The black hole pair emits a gravitational wave, which is reflected within the cavity and stimulates the black hole pair back into its original state. The process of emitting a gravitational wave and stimulation to the excited state is repeatable.

3. Conclusion

This manuscript attempts to describe the behavior of gravitational waves in a micro cavity environment using classical optical theories. While no theory can be complete without a true definition of quantum gravity, some notable takeaways can be made from this work. First, using the theory developed by MINTER *et al.*, it can be concluded that gravitational waves in the microwave frequency could be trapped by reasonably sized

Fabry–Pérot cavities which would be on the micron scale. The technology to develop cavities down to the nanoscale is well documented [3,10,32]. The temperature limitation presented by the superconducting mirrors could also be overcome as cavities have been constructed to operate in near 0 kelvin temperatures [18,33]. Second, the resonant nature of a cavity structure must force constructive interference unto the gravitational waves. Unlike in the electromagnetic analogy, the interference between two gravitational waves, particularly at the quantum level, has not been experimentally observed. As per quantum mechanics, it is likely the energy of the microscopic black holes is quantized, in a similar manner to that of a diatomic. As a consequence, the microscopic binary black hole would absorb gravitational waves which match the energy equaling the energy gap of the harmonic oscillator as defined by $(h/2\pi)\omega$. It is hopeful that this work can be further applied to help optical scientists design instruments to detect gravitational wave phenomena, as well as help further refine theories of gravitational behavior on the microscopic scale. Lastly, microscopic black holes are a candidate for the source of dark matter. Therefore, designing a device to detect high energy gravitational waves could aid in the detection of one of nature's most illusive forces.

Disclosures

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