

Light-induced magnetization field distributions of azimuthally polarized Laguerre–Gaussian beams modulated by binary optical elements in a 4π focusing system

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According to the vector diffraction theory and the inverse Faraday effect, this paper explores the light-induced magnetization field distributions of azimuthally polarized Laguerre–Gaussian beams in a 4π focusing system modulated by adjusting convergence angles for each zone of the binary optical elements. The findings demonstrate that a dumbbell shaped magnetic chain can be obtained by changing the value of the truncation parameter β in a one-belt optical element. By changing the width of the central zone in the two-belt optical element, the position of the magnetic chain can be flexibly changed, and by changing the width of the middle annular zone in the three-belt optical element, an ultra-long magnetic chain can be obtained. Moreover, the five-belt optical element will have an impact on the distance between magnetization spots and changing the five-belt optical element can change the length and number of magnetic chains. These unique magnetization field distributions may find potential applications in atom control, all-optical magnetic recording, magnetic resonance microscopy and magneto-optical data storage.

Keywords: vector diffraction theory, the reverse Faraday effect, magnetizing field, binary optical element.

1. Introduction

Laguerre–Gaussian beams, also known as multi ring beams, have attracted the attention of many researchers due to their unique characteristics [1-6]. GHOTRA *et al.* studied the self-focusing characteristics of Laguerre–Gaussian laser beams in collision free Gaussian plasmas, and the results showed that low order Laguerre–Gaussian laser beams have better focusing characteristics [7]. ALKELLY *et al.* found that azimuthally polarized Laguerre–Gaussian vortex beams exhibit different focal shapes when interacting with different optical components [8]. PAN *et al.* found through their research

that the modulation of Gaussian beams through helical phase plates is suitable for generating Laguerre–Gaussian beams with low topological charges [9]. YUWEN *et al.* found that fractional order vortex beams can damage the annular structure of Laguerre–Gaussian beams [10]. YANG *et al.* studied the electric field distribution of azimuthally polarized Laguerre–Gaussian beams under different conditions in a 4π focusing system [11].

Since JIANG *et al.* discovered that combining azimuthally polarized beams with helical phase can generate sub wavelength pure longitudinal magnetization fields at the focal plane [12], more and more researchers have invested in related research [13–15]. NIE *et al.* investigated the photo induced magnetization characteristics of a 4π compact focusing system illuminated by two backpropagating radially polarized hollow Gaussian vortex beams. The results showed that spherical and subwavelength longitudinal magnetization can be achieved by adjusting the beam order and truncation parameters [16]. GONG *et al.* used an azimuthally polarized beam to form an ultra-long magnetic chain under the condition of a 4π compact focusing system, and calculated the magnetic force of the azimuthally polarized beam on magnetic particles [17]. LUO *et al.* generated high-purity magnetic needles with arbitrary orientation control and orthogonal orientation magnetic needle arrays by reverse calculating the far-field radiation field of electric dipole arrays [18]. However, there is limited research on the magnetization distributions of azimuthally polarized Laguerre–Gaussian beams in a 4π focusing system.

In this paper, we selected azimuthally Laguerre–Gaussian polarized beams. We have achieved progress by modulating tightly focused azimuthally polarized Laguerre–Gaussian beams, and have generated magnetization needle chains with a long focal depth and magnetization needles with a high aspect ratio. This was accomplished through the employment of multi-belt binary optical elements, variation of element designs, and modulation of Laguerre–Gaussian beam parameters. These synergistic effects allow us to achieve a gradual increase in both the number of magnetization spots and the length of the magnetic chain. Moreover, the size and depth of each magnetization spot, as well as the distance between different spots, can be precisely controlled. This enables the flexible adjustment of the magnetization chain position, thereby achieving the goal of capturing multiple particles

2. Theory

Figure 1 shows a schematic diagram of a typical 4π focusing system. Two counter-propagating azimuthally polarized Laguerre–Gaussian (LG) beams pass through a binary optical element and then focused by two high numerical apertures, and finally converge on a magneto-optical (MO) film with IFE.

Five concentric zones make up the binary optical element: center zone, first annular zone, second annular zone, third annular zone and outer annular zone. For the convenience of calculation, we define a , b , c , d , and 1 as the ratio of convergence angles in

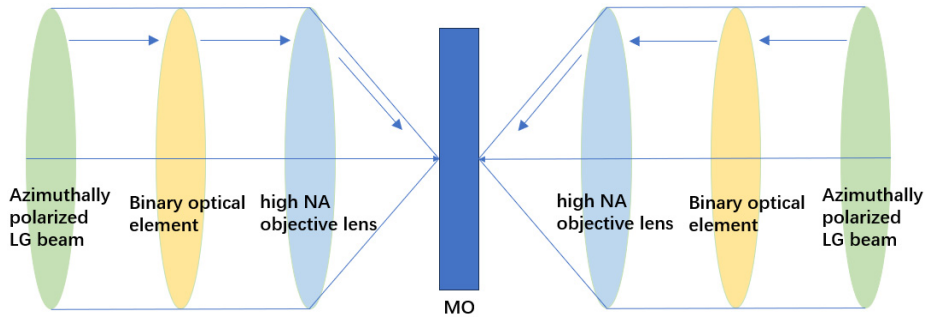


Fig. 1. System structure diagram.

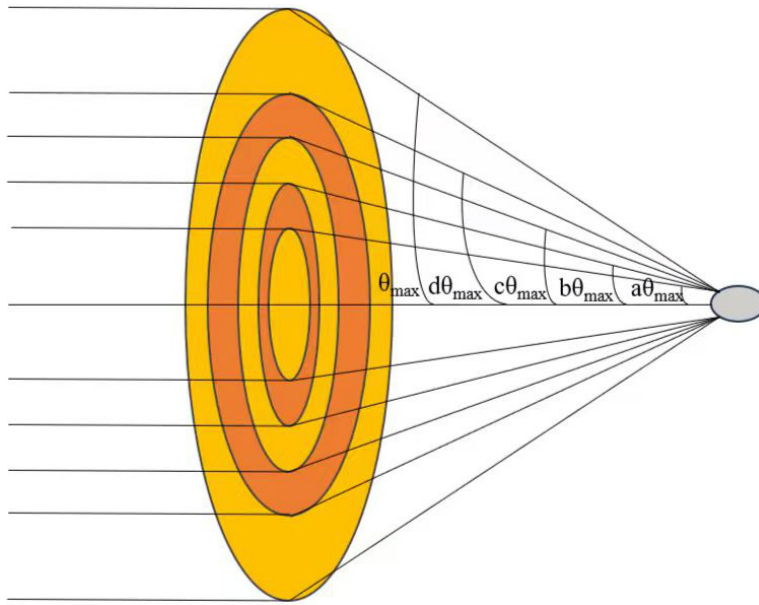


Fig. 2. Binary optical element illustration.

different regions to normalize the maximum convergence angle $\theta_{\max}$. The structure of the binary optical element is illustrated in Fig. 2. The variables a , b , c , and d can be modified to form optical elements with different numbers of belts.

In the focusing optical system, the incident beam is an azimuthally polarized Laguerre–Gaussian beam and the relative amplitude of it can be written in the form:

$$E(\theta) = \frac{\beta \sin \theta}{\sin \theta_{\max}} \exp\left(-\frac{\beta \sin^2 \theta}{\sin^2 \theta_{\max}}\right) L_1^1 \left[2 \left(\frac{\beta \sin \theta}{\sin \theta_{\max}} \right)^2 \right] \quad (1)$$

where L_1^1 is the Laguerre polynomial with radial and angular mode numbers equal to 1, θ is the convergence angle, and $\theta \in [0, \arcsin(\text{NA})]$. NA is the numerical aperture of this focusing system and the maximum convergence angle is $\theta_{\max}$. β is the truncation parameter, which denotes the ratio of the pupil radius to the beam waist. It is important to note that β should be greater than 1 to ensure proper beam truncation. According to the vectorial diffraction theory [19-20], the electric field distribution of azimuthally polarized Laguerre–Gaussian beam modulated by binary optical element can be represented as:

$$\mathbf{E}(r, \varphi, z) = \begin{bmatrix} E_r \\ E_\varphi \\ E_z \end{bmatrix} = A \begin{bmatrix} -I_1 \\ iI_2 \\ 0 \end{bmatrix} \quad (2)$$

with I_i ($i = 1$ and 2) are three integrals of

$$I_1(r, \varphi, z) = \int_0^{\theta_{\max}} P(\theta) E(\theta) \sqrt{\cos \theta} \sin \theta \left[J_2(kr \sin \theta) + J_0(kr \sin \theta) \right] \exp(ikz \cos \theta) d\theta \quad (3)$$

$$I_2(r, \varphi, z) = \int_0^{\theta_{\max}} P(\theta) E(\theta) \sqrt{\cos \theta} \sin \theta \left[J_2(kr \sin \theta) - J_0(kr \sin \theta) \right] \exp(ikz \cos \theta) d\theta \quad (4)$$

where r , φ and z are the cylindrical coordinates, $k = 2\pi/\lambda$ is called wavenumber and J_0 is the 0th-order Bessel function of the first kind and J_2 is the 2nd-order Bessel function of the first kind. λ is the wavelength of the incident beam and A is the amplitude constant. The $P(\theta)$ denotes the transmittance function of the binary optical element.

The electric field near the focus of the 4π focusing system can be described as:

$$\mathbf{E}(r, \varphi, z) = \mathbf{E}_1(r, \varphi, z) + \mathbf{E}_2(-r, \varphi, -z) \quad (5)$$

where $\mathbf{E}_1$ is the total electric field focused by the left objective lens and $\mathbf{E}_2$ is the total electric focused by the right objective lens. The negative signs of r and z in $\mathbf{E}_2$ indicate that the instantaneous polarization direction and propagation direction of the two incident azimuthally polarized Laguerre–Gaussian beams are opposite, respectively. Based on the inverse Faraday effect [21-22], the magnetization field induced by azimuthally polarized Laguerre–Gaussian beams in the vicinity of the focal point is defined as

$$\mathbf{M}(r, \varphi, z) = i\gamma \mathbf{E} \times \mathbf{E}^* \quad (6)$$

with γ is a magneto-optical constant, $\mathbf{E}$ is the electric field, and $\mathbf{E}^*$ is its conjugate. By substituting Eq. (1) and Eqs. (3)–(5) into Eq. (6), one gets

$$\begin{aligned}
\mathbf{M}(r, \varphi, z) &= \begin{bmatrix} M_r \\ M_\varphi \\ M_z \end{bmatrix} \\
&= \begin{bmatrix} 0 \\ 0 \\ -\text{Re} \left\{ \left[I_1(r, \varphi, z) + I_2(-r, \varphi, -z) \right] \left[I_2(r, \varphi, z) + I_2(-r, \varphi, -z) \right]^* \right\} \end{bmatrix} \quad (7)
\end{aligned}$$

According to Eq. (7), the magnitude of total magnetization is calculated as $\mathbf{M} = M_z$ and it can be concluded that the magnetization field generated by azimuthally polarized beams in a 4π focusing system only has a longitudinal component, which is a pure longitudinal magnetization field.

3. Numerical results

The distributions of magnetization on the focal plane under various values of a , b , c , and d are simulated in accordance with the formulas above. In this article, the parameters are selected as follows: the numerical aperture of the focusing optical system $\text{NA} = 0.95$ and the wavelength $\lambda = 633 \text{ nm}$.

Firstly, we investigate the effect of truncation parameter β on magnetization distributions. Figure 3 illustrates the magnetization distributions when $a = b = c = 0$, $d = 1$ and different β . The binary optical element at this time is a one-belt optical element. The transmission function $P(\theta)$ at this point can be expressed as:

$$P(\theta) = 1, \quad \theta \in [0, \theta_{\max}] \quad (8)$$

When $\beta = 1$, the magnetization spot with the highest magnetization intensity is located at the focal point, and the two weak magnetization spots are symmetrically distributed, as shown in Fig. 3(a). When $\beta = 2.5$, the magnetization intensity of the central magnetization spot decreases, while the magnetization intensity of the weak magnetization spots on both sides of the central magnetization spot increases and exceeds the magnetization intensity of the central magnetization spot, and more symmetric weak magnetization spots appear, as shown in Fig. 3(b). When $\beta = 4.5$, the magnetization intensity of the magnetization spots near the central magnetization spot becomes smaller, and more magnetization spots appear on both sides, with the magnetization intensity of these magnetization spots gradually increasing, as shown in Fig. 3(c). Overall, a dumbbell shaped magnetic chain has been formed. When $\beta = 5$, the length of the magnetic chain continues to increase, as shown in Fig. 3(d). It can be seen that

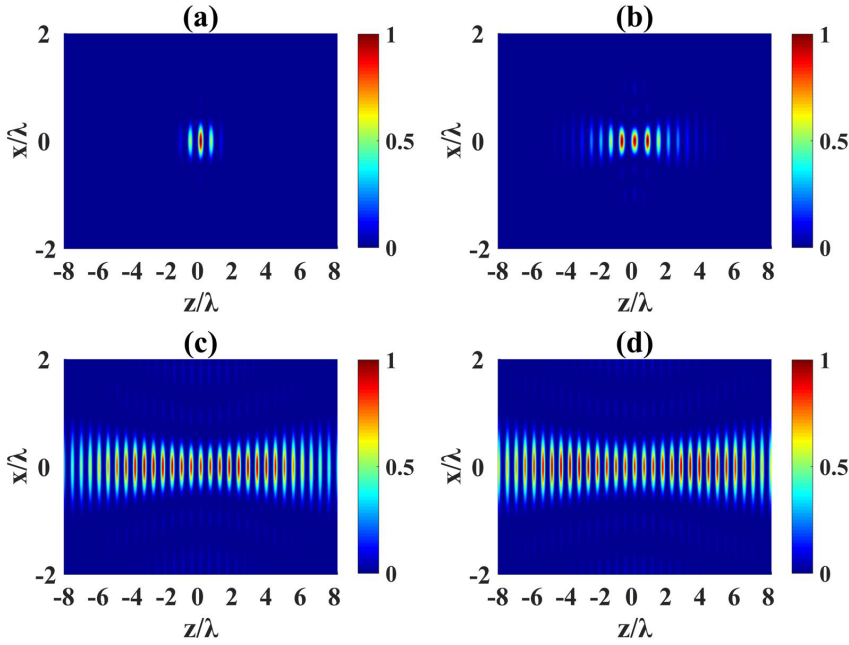


Fig. 3. Normalized magnetization distributions when $a = b = c = 0$, $d = 1$, and (a) $\beta = 1$, (b) $\beta = 2.5$, (c) $\beta = 4.5$, and (d) $\beta = 5$.

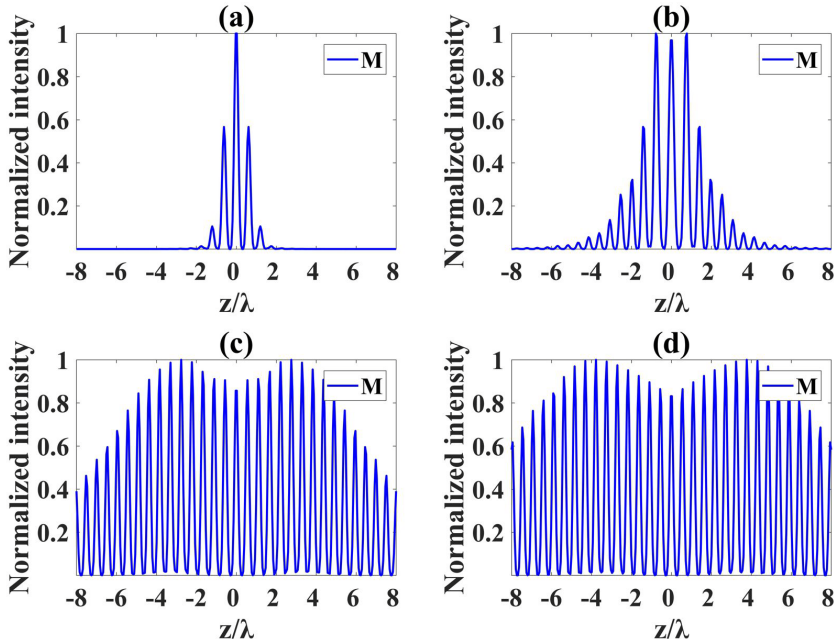


Fig. 4. Normalized magnetization distributions curves along the z -axis at $x = 0$ when $a = b = c = 0$, $d = 1$, and (a) $\beta = 1$, (b) $\beta = 2.5$, (c) $\beta = 4.5$, and (d) $\beta = 5$.

the value of β will affect the magnetization intensity of each magnetization spot. As β gradually increases, the number of magnetization spots will gradually increase, and the length of the magnetic chain will also gradually become longer.

Figure 4 shows the normalized magnetization distribution curves along the z -axis at $x = 0$ when $a = b = c = 0$, $d = 1$ and different β , which intuitively illustrates the estimated size and depth of each magnetization spot, as well as the distance between different magnetization spots.

Then, we change the binary optical element to a two-belt optical element. Figure 5 illustrates the magnetization distributions when $a = b = c = 0$, $\beta = 4.5$ and different d . The transmission function $P(\theta)$ at this point can be expressed as:

$$P(\theta) = \begin{cases} 1, & \theta \in [0, d\theta_{\max}] \\ 0, & \theta \in [d\theta_{\max}, \theta_{\max}] \end{cases}, \quad d \in (0, 1) \quad (9)$$

When $d = 0.4$, the magnetization intensity of the magnetic spot near the geometric focal point of the optical axis is extremely small, forming two symmetrical magnetic chains along the optical axis, as shown in Fig. 5(a). When d increases from 0.5 to 0.7, the magnetization intensity of the magnetic spot near the geometric focal point of the optical axis gradually increases, and at the same time, the two symmetrical magnetic chains in Fig. 5(a) tend to form one magnetic chain, as shown in Fig. 5(b)–(d). When d is small, the magnetization distribution forms two magnetic chains. As d increases, the

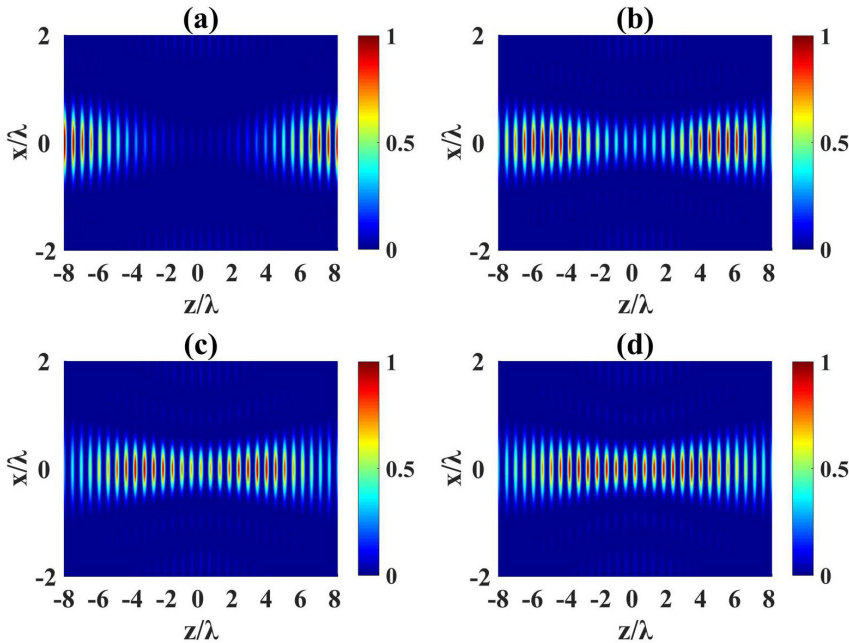


Fig. 5. Normalized magnetization distributions when $a = b = c = 0$, $\beta = 4.5$, and (a) $d = 0.4$, (b) $d = 0.5$, (c) $d = 0.6$, and (d) $d = 0.7$.

two magnetic chains gradually approach and tend to form one magnetic chain. From this, it can be seen that the position of the magnetization chain can be flexibly adjusted to achieve the goal of capturing multiple particles.

Figure 6 shows the normalized magnetization distribution curves along the z -axis at $x = 0$ when $a = b = c = 0$, $\beta = 4.5$ and different d , which intuitively illustrates the estimated size and depth of each magnetization spot, as well as the distance between different magnetization spots.

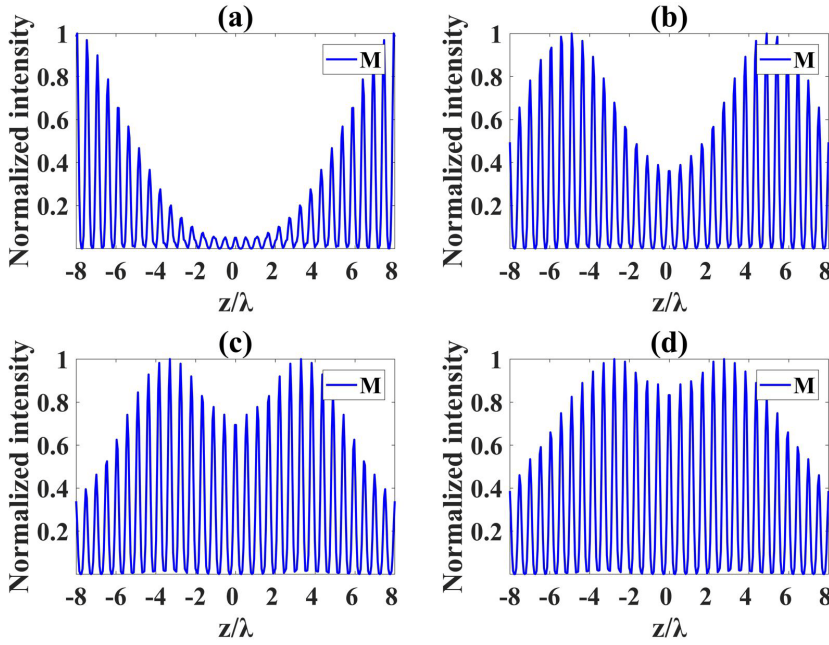


Fig. 6. Normalized magnetization distributions curves along the z -axis at $x = 0$ when $a = b = c = 0$, $\beta = 4.5$, and (a) $d = 0.4$, (b) $d = 0.5$, (c) $d = 0.6$, and (d) $d = 0.7$.

Next, the binary optical element is set as a three-belt optical element. Figure 7 illustrates the magnetization distributions when $a = b = c = 0.785$, $\beta = 4.5$, and different d . The transmission function $P(\theta)$ at this point can be expressed as:

$$P(\theta) = \begin{cases} 0, & \theta \in [0, a\theta_{\max}] \\ 1, & \theta \in [a\theta_{\max}, d\theta_{\max}], \\ 0, & \theta \in [d\theta_{\max}, 1] \end{cases}, \quad a, d \in (0, 1) \quad (10)$$

When $d = 0.79$, a super long magnetic chain is formed, which contains 21 magnetization spots, as shown in Fig. 7(a). When $d = 0.85$, the magnetization intensity of magnetic spots far from the focus decreases, as shown in Fig. 7(b). When d increases from 0.9 to 0.985, while the magnetization intensity of the magnetization spot further away from the focus continues to decrease, the length of the magnetic chain also gradually

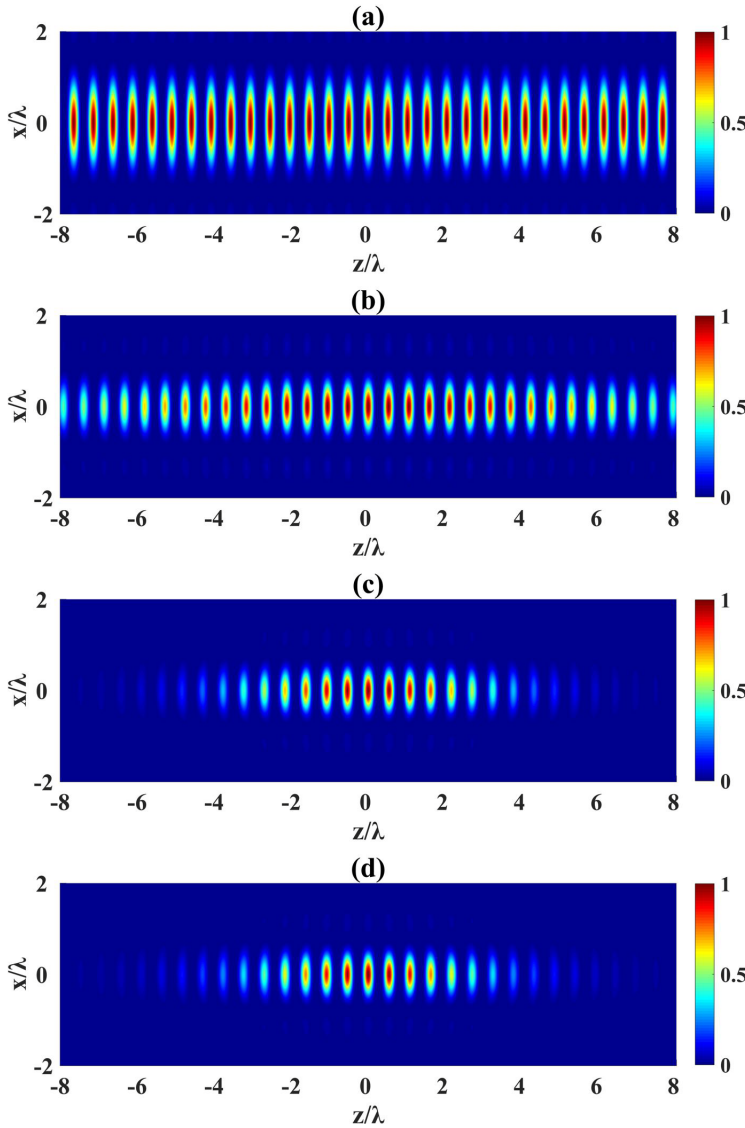


Fig. 7. Normalized magnetization distributions when $a = b = c = 0.785$, $\beta = 4.5$, and (a) $d = 0.79$, (b) $d = 0.85$, (c) $d = 0.9$, and (d) $d = 0.985$.

decreases, as shown in Figs. 7(c) and (d). In summary, it can be seen that as the width of the middle annular zone gradually increases, the length of the magnetic chain gradually shortens, and the magnetization intensity of each magnetization point also gradually decreases.

Figure 8 shows the normalized magnetization distribution curves along the z -axis at $x = 0$ when $a = b = c = 0.785$, $\beta = 4.5$, and different d , which intuitively illustrates the estimated size and depth of each magnetization spot, as well as the distance between

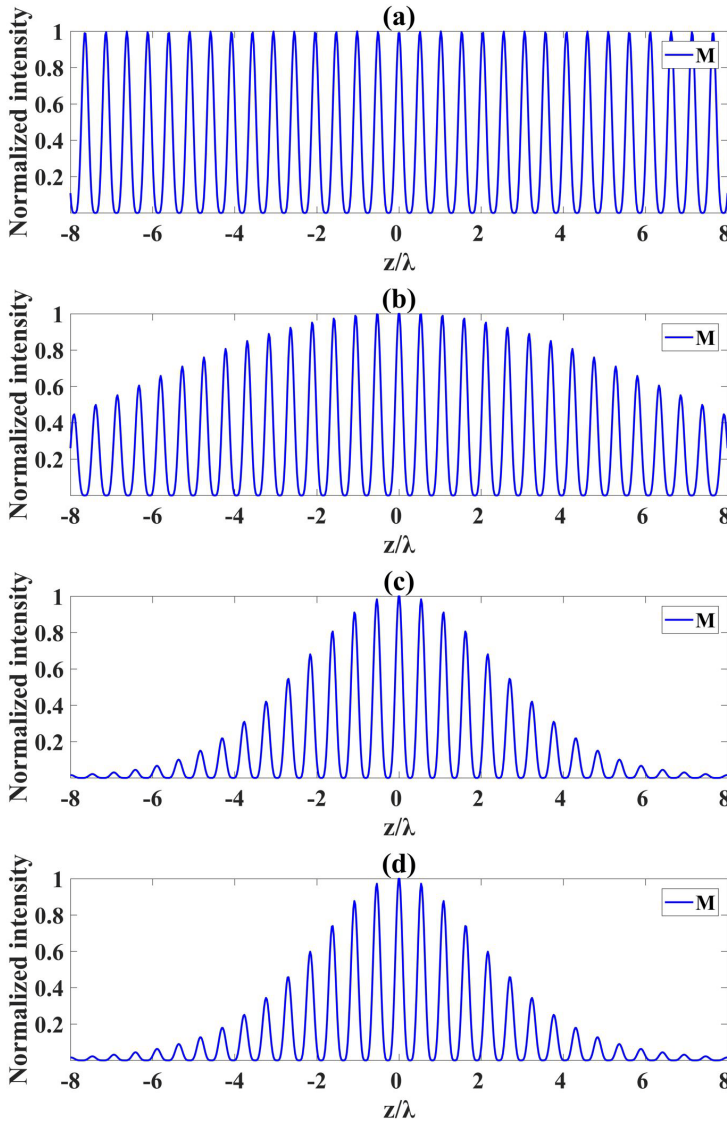


Fig. 8. Normalized magnetization distributions curves along the z -axis at $x = 0$ when $a = b = c = 0.785$, $\beta = 4.5$, and (a) $d = 0.79$, (b) $d = 0.85$, (c) $d = 0.9$, and (d) $d = 0.985$.

different magnetization spots. Specifically, Fig. 7 forms a 16λ long magnetization chain consisting of 21 identical magnetization spots, with each spot having a transverse width of 0.77λ , a longitudinal width of 0.78λ , and a distance of 0.75λ between adjacent magnetization spots.

In order to further investigate the influence of the width of different zones on magnetization distributions, we further change the binary optical element to a five-belt optical element. Figure 9 illustrates the magnetization distributions when $a = 0.1$,

$d = 0.9$, $\beta = 4.5$ and different b and c . The transmission function $P(\theta)$ at this point can be expressed as

$$P(\theta) = \begin{cases} 0, & \theta \in [0, a\theta_{\max}] \\ 1, & \theta \in [a\theta_{\max}, b\theta_{\max}] \\ 0, & \theta \in [b\theta_{\max}, c\theta_{\max}] \\ 1, & \theta \in [c\theta_{\max}, d\theta_{\max}] \\ 0, & \theta \in [d\theta_{\max}, 1] \end{cases}, \quad a, d, c, d \in (0, 1) \quad (11)$$

When $b = 0.15$ and $c = 0.85$, the magnetization spot reaches its maximum value at the focal point, while other weak magnetization spots are symmetrically distributed, as shown in Fig. 9(a). When $b = 0.2$ and $c = 0.8$, the magnetization intensity of the weak magnetization spots on both sides gradually decreases, and the overall length of the magnetic chain also shows a trend of shortening, as shown in Fig. 9(b). When $b = 0.3$ and $c = 0.7$, the magnetization intensity of the magnetic spot at the focal point decreases, and two magnetization chains gradually form on both sides of the focal point, as shown in Fig. 9(c). When $b = 0.4$ and $c = 0.6$, the magnetization spot at the focal point completely disappears, while the lengths of the two magnetization chains gradually shorten, as shown in Fig. 9(d). It can be seen that when the width of the zones with $P(\theta) = 1$ is small, the magnetization distribution still follows a magnetization chain,

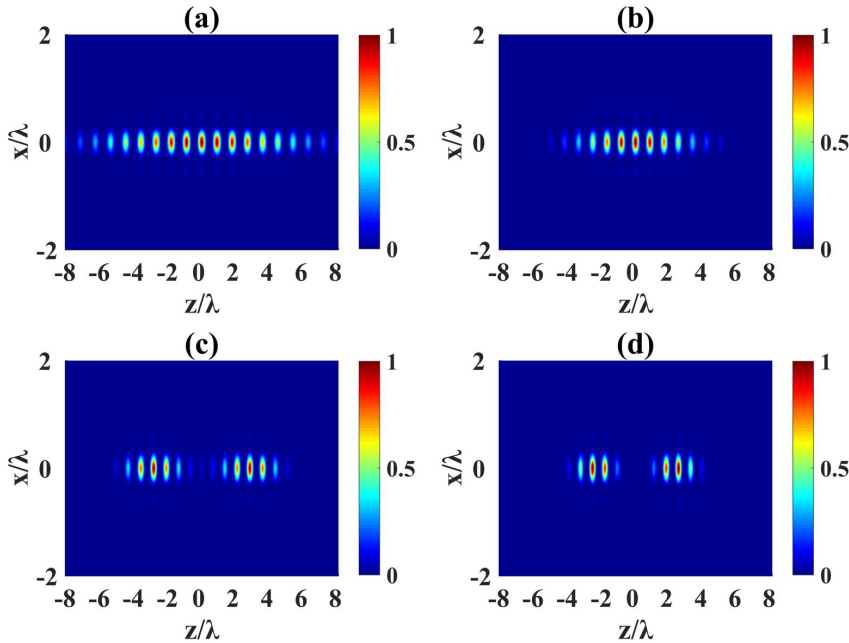


Fig. 9. Normalized magnetization distributions when $a = 0.1$, $d = 0.9$, $\beta = 4.5$, and (a) $b = 0.15$, $c = 0.85$, (b) $b = 0.2$, $c = 0.8$, (c) $b = 0.3$, $c = 0.7$, and (d) $b = 0.4$, $c = 0.6$.

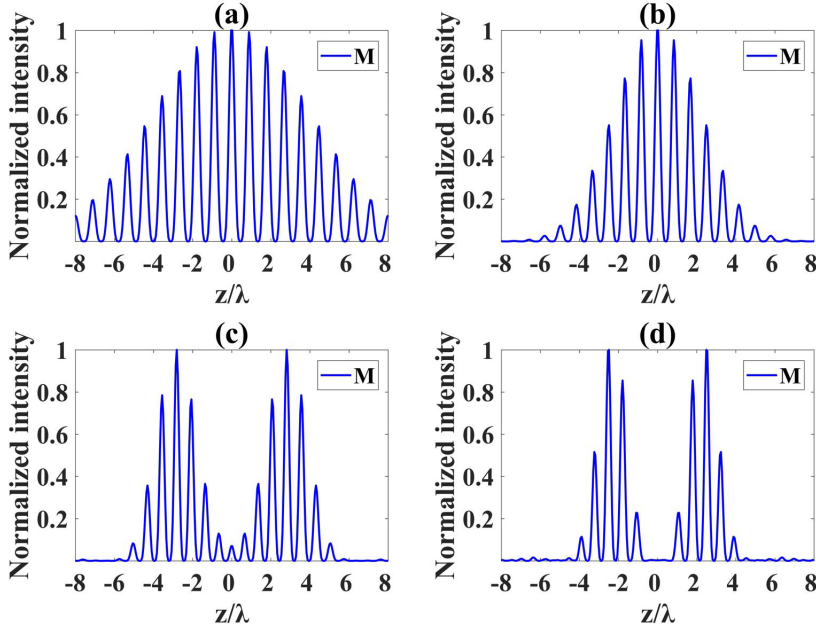


Fig. 10. Normalized magnetization distributions curves along the z -axis at $x = 0$ when $a = 0.1$, $d = 0.9$, $\beta = 4.5$, and (a) $b = 0.15$, $c = 0.85$, (b) $b = 0.2$, $c = 0.8$, (c) $b = 0.3$, $c = 0.7$, and (d) $b = 0.4$, $c = 0.6$.

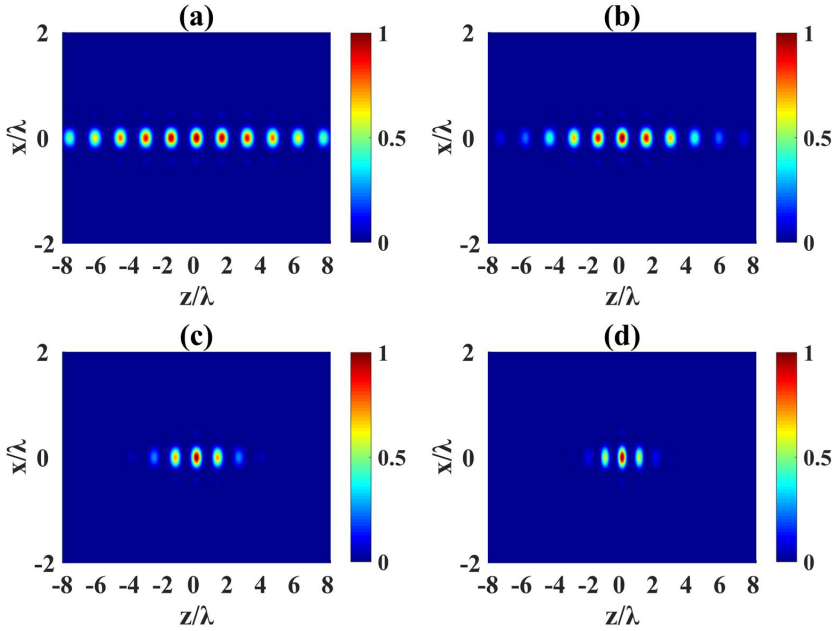


Fig. 11. Normalized magnetization distributions when $\beta = 4.5$ and (a) $a = 0.02$, $b = 0.49$, $c = 0.51$, and $d = 0.98$, (b) $a = 0.04$, $b = 0.45$, $c = 0.55$, and $d = 0.96$, (c) $a = 0.1$, $b = 0.45$, $c = 0.55$, and $d = 0.9$, (d) $a = 0.2$, $b = 0.4$, $c = 0.6$ and $d = 0.8$.

and as the width gradually increases, the magnetization distribution tends to evolve into two magnetization chains.

Figure 10 shows the normalized magnetization distribution curves along the z -axis at $x = 0$ when $a = 0.1$, $d = 0.9$, $\beta = 4.5$, and different b and c , which intuitively illustrates the estimated size and depth of each magnetization spot, as well as the distance between different magnetization spots.

Figure 11 illustrates the magnetization distributions when $\beta = 4.5$ and different a , b , c , and d . The transmission function $P(\theta)$ at this point can be expressed as

$$P(\theta) = \begin{cases} 1, & \theta \in [0, a\theta_{\max}] \\ 0, & \theta \in [a\theta_{\max}, b\theta_{\max}] \\ 1, & \theta \in [b\theta_{\max}, c\theta_{\max}] \\ 0, & \theta \in [c\theta_{\max}, d\theta_{\max}] \\ 1, & \theta \in [d\theta_{\max}, 1] \end{cases}, \quad a, d, c, d \in (0, 1) \quad (12)$$

When $a = 0.02$, $b = 0.49$, $c = 0.51$, and $d = 0.98$, the magnetization distribution still forms a magnetic chain, as shown in Fig. 11(a). As the width of the three zones with $P(\theta) = 1$ gradually increases, the magnetic chain shows a trend of gradually shortening, as shown in Fig. 11(b)–(d).

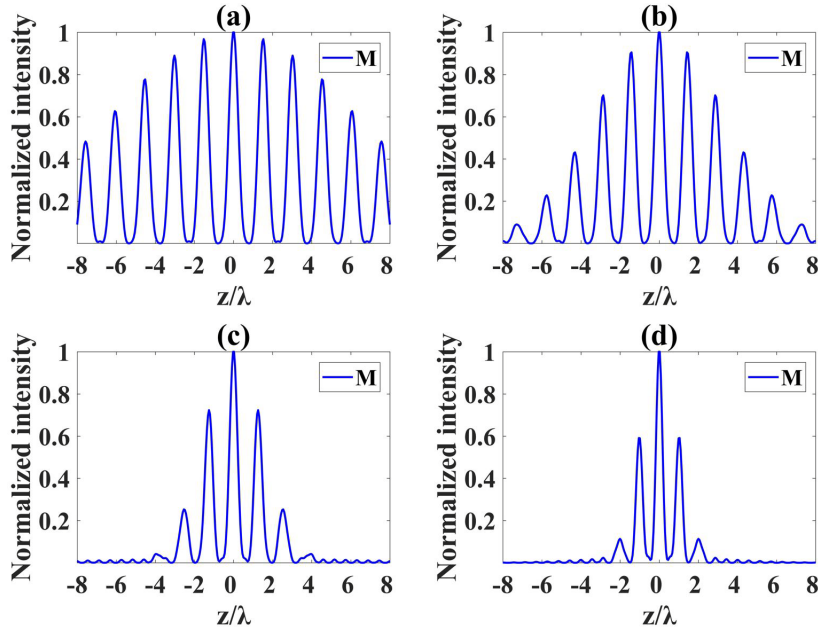


Fig. 12. Normalized magnetization distributions curves along the z -axis at $x = 0$ when $\beta = 4.5$, and (a) $a = 0.02$, $b = 0.49$, $c = 0.51$, and $d = 0.98$, (b) $a = 0.04$, $b = 0.45$, $c = 0.55$, and $d = 0.96$, (c) $a = 0.1$, $b = 0.45$, $c = 0.55$, and $d = 0.9$, (d) $a = 0.2$, $b = 0.4$, $c = 0.6$, and $d = 0.8$.

Figure 12 shows the normalized magnetization distribution curves along the z -axis at $x = 0$ when $\beta = 4.5$ and different a , b , c and d , which intuitively illustrates the estimated size and depth of each magnetization spot, as well as the distance between different magnetization spots.

The results of Figs. 9 and 11 indicate that when the structure of the five-belt optical element is changed, the distance between adjacent magnetization spots changes, and the length and number of magnetization chains are affected by the width of the zones with $P(\theta) = 1$.

4. Conclusions

This paper investigates the distributions of magnetization on the focal plane of azimuthally polarized Laguerre–Gaussian beam modulated by binary optical elements in a 4π high NA focusing system. The findings indicate that when $a = b = c = 0$, and $d = 1$, the value of β will affect the magnetization intensity of each magnetization spot. As β gradually increases, the number of magnetization spots will gradually increase, and the length of the magnetic chain will also gradually become longer. When $a = b = c = 0$, $\beta = 4.5$, and $d = 0.4$, the magnetization distribution forms two magnetic chains. As d increases, the two magnetic chains gradually approach and tend to form one magnetic chain. When $a = b = c = 0.785$, $\beta = 4.5$, and the width of the middle annular zone gradually increases, the length of the magnetic chain gradually shortens, and the magnetization intensity of each magnetization point also gradually decreases. When $\beta = 4.5$ and the binary optical element is a five-belt optical element, the distance between adjacent magnetization points increases, and the length and number of magnetization chains are affected by the width of the zones with $P(\theta) = 1$. Overall, these unique magnetization field distributions may find potential applications in atom control, all-optical magnetic recording, magnetic resonance microscopy and magneto-optical data storage.

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Data availability

Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

Declarations

The authors declare no conflicts of interest.

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