

Resistance of Bessel beams to strong turbulence over short transmission distances

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LiDAR emits and receives laser beams to perceive three-dimensional spatial information, providing support for environmental perception. Scintillation, Distortion, and drift occur on the laser beams when they propagate through strong atmospheric turbulences even for short distances. Methods to eliminate or mitigate the impacts of strong turbulences on laser beams can effectively enhance the LiDAR accuracy, which are of great significance for automotive autonomous driving application requiring high precision in short-range measurements. This paper investigates the anti-turbulence performance of the Bessel beams as they transmit in strong atmospheric turbulences over short distances. Compared to Gaussian beams, Bessel beams demonstrate greater resistance to strong turbulences, which is due to their self-healing property, more concentrated intensity distribution, and smaller beam divergence angle. The scintillation and distortion of the Bessel beam is an order of magnitude and 20% lower than those of the Gaussian beam, respectively, as they propagated through strong turbulences of the same strength. The beam drift caused by strong turbulence is characterized by the maximum beam drift range, and its radius for the Gaussian beam is twice the radius for the Bessel beam. This research contributes to enhancing the measurement accuracy of LiDAR systems in autonomous vehicles under conditions of strong turbulence.

Keyword: atmospheric turbulence, short-range transmission, Bessel beams, scintillation index, laser beam drift, beam distortion.

1. Introduction

It is well known that atmospheric turbulence causes distortions of light beams and severely degrades the performance of long-distance laser communication systems. It also has a significant impact on short-range laser application systems. LiDAR measures spatial distances in real time by emitting laser beams and receiving reflected signals, and thereby constructs a 3D model of the surrounding environment [1-2]. LiDAR provides high-precision, highly reliable 3D environmental perception capabilities, making

it an essential component of autonomous driving systems [3]. Researches have focused on enhancing the data processing capabilities of autonomous driving systems and increasing the resolution of LiDAR to improve the systems' performances. In real-world autonomous driving scenarios, intense solar radiation, heat generated by tire friction, vehicle engine emissions and wind might cause strong convections in the air near road surface, leading to the formation of strong atmospheric turbulences [4]. Atmospheric turbulence significantly distorts the wave front of a light beam propagated through it, leading to the scintillation, drift, jitter, and intensity spreading of the beam [5-7]. Atmospheric turbulence-induced laser beam distortion introduces interferences to LiDAR and degrades its measurement accuracy, placing autonomous vehicles at significant risk [8]. Mitigating the impact of atmospheric turbulence on laser beams is critical to enhance the autonomous driving systems' environmental perception capabilities and enable them to operate effectively under complex weather and geographical condition [9-10]. Therefore the study of real-time approaches that are capable of suppressing turbulence's effects over short distances, is of great significance for autonomous driving systems.

The topic of maintaining laser beam propagation stability by suppressing the effects of atmospheric turbulence has attracted widespread interest and been extensively studied [11-15]. Adaptive optics technology is an effective approach to counteract the interference of environmental turbulence by correcting wave-front distortions [16]. This technology employs sophisticated optical components to correct distorted beams, providing an active compensation scheme for atmospheric turbulence effects [17]. However, adaptive optics systems are characterized by complex structures and high costs. Moreover, adaptive optics technology cannot meet the requirement of high real-time performance for autonomous driving systems, which leads to its absence in automotive LiDAR. Another approach to suppress atmospheric turbulence is signal processing technologies, which suppress the interference of atmospheric turbulence on signals through methods such as channel coding and channel equalization, but exhibit limited effectiveness for signals of low signal-to-noise ratio caused by strong turbulences [14-15, 18]. Over the past decade, the field of beam shaping has attracted significant research attention. Beams with self-healing characteristic exhibit excellent performance of turbulence resistance during transmission. For example, when propagating through atmospheric turbulence, the non-diffracting Bessel beam which travels in a straight line, offers several advantages over traditional Gaussian beams and can be used in optical communication [11, 19]. Airy beams propagate along curved or self-bending trajectories. They also show characteristics of non-diffraction and self-healing, which make them resistant to distortion caused by turbulence [20]. A recently proposed non-diffracting Bessel-like needle-shaped beam exhibits excellent turbulence suppression characteristic during long-distance transmission, achieved by counteracting its transverse wave vector through the superposition of an Airy-type beam component [21]. All of the above beams exhibit a certain degree of turbulence suppression capability. However, owing to the nonlinear phase distribution characteristic inherent in Airy beams, when these beams propagate in free space, phenomena like beam bend-

ing will arise. This renders them unsuitable for practical free-space applications [22]. The trajectory of a needle-shaped laser beam over short transmission distances is non-linear, meaning it does not follow a straight path. Consequently, it does not meet LiDAR requirement and is more suitable for long-range applications [21]. For automotive LiDAR systems requiring suppression of strong turbulence over short distances (from a few meters to tens of meters), Bessel beams are more desirable. In this paper, the characteristics of Bessel beams propagating through strong atmospheric turbulence over short distances are investigated, in order to identify a solution for automotive LiDAR systems that can effectively suppress the interference in reflected signals caused by strong turbulence, thereby enhancing the measurement accuracy of LiDAR systems. Parameters such as beam drift, power fluctuations, and beam spreading caused by turbulence were quantitatively analyzed and subsequently compared with those of conventional Gaussian beams.

2. The impact of strong turbulence on LiDAR

The impact of atmospheric turbulence on optical signal transmission exhibits a cumulative effect. That is, the longer the transmission distance, the greater the beam distortion, and the more severe the interference to the optical signal. In environments with strong atmospheric turbulence, the impact of turbulence on optical signals over short distances (from several meters to tens of meters) is also considerable. Intense solar heat radiation (especially during summer), wind, exhaust fumes from vehicles, and heat released by engine operation and tire friction cause strong localized thermal convections between vehicles, leading to the generation of intense turbulence within confined spaces. For the theoretical analyses of such intense turbulence over short distances, we employed the method of Zernike polynomial expansion to generate turbulence phase screens to simulate it. Zernike polynomials are a complete set of orthogonal basis functions defined on a unit circle, which can be used to simulate and fit distorted wave-front surfaces. Firstly, we generate a turbulence phase screen using Zernike polynomial expansion. By superimposing different polynomial modes $Z_n^m(\rho, \theta)$ with random coefficients $\alpha_{n,m}$, a two-dimensional phase distribution $\varphi(x, y)$ is formed to describe the wave-front distortion caused by atmospheric turbulence. To simulate strong atmospheric turbulence in short-range channels, we assume the refractive-index structure constant of atmospheric turbulence to be $C_n^2 = 1 \times 10^{-13} \text{ m}^{-2/3}$ and the propagation distance L to be 50 m. Subsequently, this phase screen is applied to the optical system's aperture function, yielding an exit-pupil plane electric field distribution that incorporates turbulence effects. Finally, a Fourier transform is performed on this electric field distribution to obtain the point spread function (PSF) in the far field. Figures 1(f) and (g) represent the phase screen generated using Zernike polynomials and the corresponding far-field point spread function (PSF), respectively. Finally, by convolving a clear image with the point spread function (PSF) generated from the turbulent phase screen, we obtain the image distorted by turbulence, as shown in image (b) in Fig. 1. Such kind of distorted image can often be observed in real-world scenarios, and this

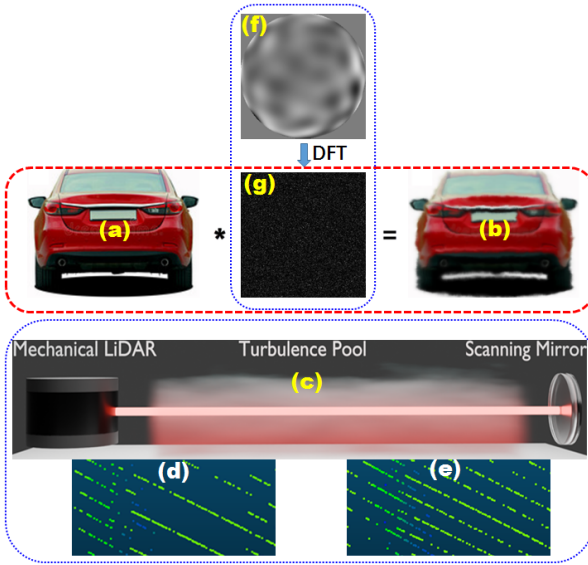


Fig. 1. The impact of strong turbulence on LiDAR target detection in short-range channels. (a) Image of the rear end of a car. (b) Image distorted by turbulence. (c) Schematic diagram of LiDAR (Velodyne-VLP-16) detecting a mirror through a turbulence cell. (d) Point cloud image of the mirror obtained from LiDAR scanning without turbulence in turbulence cell. (e) Point cloud image of the mirror obtained from LiDAR scanning without turbulence in turbulence cell. (f) Phase screen for simulating atmospheric turbulence generated by Zernike polynomials with $C_n^2 = 1 \times 10^{-13} \text{ m}^{-2/3}$ and $L = 50 \text{ m}$. (g) Point spread function (PSF) corresponding to the phase screen in (f).

phenomenon is precisely caused by strong atmospheric turbulence in short-range channels.

Experiment was carried out to study the impact of strong turbulence on the detection accuracy of LiDAR, whose setup is shown in Fig. 1(c). A LiDAR (Velodyne-VLP-16) was employed to scan a fully reflective mirror and a turbulence cell was placed between the LiDAR and the mirror. An electric heating plate is installed at the bottom of the turbulence cell, capable of heating from room temperature to 150°C . A cold plate filled with the ice-water mixture is installed at the top of the turbulence cell, maintaining a temperature of 0°C . The distance between the hot and cold plates is 15 centimeters. The significant temperature difference causes intense convection in the air between the hot and cold plates, resulting in a strong turbulence environment in the turbulence cell. In the absence of thermal and cold sources within the turbulence cell, short-range turbulence effects can be neglected, and a clear point cloud image can be successfully acquired using LiDAR, as illustrated in Fig. 1(d). Then, the hot plate was then heated to 150°C , and a mixture of ice and water was applied to the cold plate. As the temperature difference between the hot and cold plates was 150°C , the LiDAR was activated to scan the target mirror again and resultant point cloud image obtained under strong turbulence condition is shown in Fig. 1(e). Comparing Figs. 1(d)

and (e), we can clearly observe that strong turbulence introduced severe interference to LiDAR scanning. The results indicate that strong turbulence along the laser transmission path can cause severe distortion of the laser beam, leading to errors in the spatial data obtained from LiDAR measurements and degradation of LiDAR performance. To mitigate the interference introduced by strong turbulence over short distances on vehicle-mounted LiDAR systems, one solution is to replace the traditional Gaussian beam with anti-turbulence laser beams.

3. Propagation characteristics of Bessel beams over short distances in strong turbulence

Figure 1 demonstrates that intense localized turbulence poses significant challenge for short-range laser applications such as automotive LiDAR systems, as it interferes with reflected laser signals and reduces LiDAR measurement accuracy. Turbulence-induced drift, scintillation, and spot distortion of the laser beams are the primary causes of degraded LiDAR measurement accuracy. If a laser beam remains unaffected by turbulence, in other words, if it does not experience drift, scintillation, or beam distortion after propagating through turbulent, the degradation of LiDAR measurement accuracy can be avoided. Both Airy beams and needle-shaped optical beam have the characteristic of anti-turbulence, but Bessel beams are preferable for automotive LiDAR application that requires short-distance transmission of laser beams in strong turbulence. If Bessel beams demonstrate strong resistance to turbulence, simply modulating the Gaussian beams currently used in automotive LiDAR systems into Bessel beams for output could effectively enhance LiDAR measurement accuracy. Experiments were carried out to study the propagation of Bessel beams in strong turbulence within short propagation distance and the propagation of Gaussian beams under the same conditions was also studied for comparison.

The experimental setup is similar to that shown in Fig. 1(c). The turbulence cell composed of a hot plate and a cold plate was employed to produce strong turbulence of short range. A Gaussian beam emitted from a laser passed through the turbulence cell and was reflected back by the total reflecting mirror. After traversing the turbulence cell once more, it was incident onto the sensor of an optical power meter. The acquisition frequency of the optical power meter was set to 60 Hz. After completing the instantaneous power acquisition of the Gaussian beam, this beam was modulated into a Bessel beam and subjected to the same experiment under identical conditions. The instantaneous power of the reflected Bessel beam was also recorded. Atmospheric turbulence causes laser beams passing through it to scintillate, so the scintillation index is often used as a benchmark for turbulence intensity. It can also be employed to quantitatively assess the degree of interference that turbulence imposes on optical signals. For evaluating a laser beam's resistance to turbulence in our experiments, the smaller the scintillation index calculated from the reflected laser signal, the stronger the laser beam's resistance to turbulence. Based on the instantaneous power data collected by

the optical power meter, the scintillation index of the reflected laser signal can be calculated using the following equation [23]:

$$\beta_I^2(D) = \frac{\langle I^2(t) \rangle - \langle I(t) \rangle^2}{\langle I(t) \rangle^2} \quad (1)$$

In this equation, the symbol $\langle \cdot \rangle$ denotes the statistical averaging operation, D represents the diameter of the detector's receiving aperture, and $I(t)$ represents the power received by the detector at a certain moment.

By taking the ensemble average of 60 instantaneous laser power values in one second and then applying it to Eq. (1), one scintillation index value per second can be achieved. The sensor of the optical power meter employed in our experiments has an effective diameter of 8 mm, and the wavelength of the laser beam is 632 nm. In each experiment, the cold plate temperature was maintained at 0 °C, while the hot plate was heated up from room temperature to 150 °C. By altering the temperature difference between the hot and cold plates, the strength of turbulence in the turbulence cell was also changed. During the experiment, the temperature of the hot plate and cold plate was continuously monitored using temperature sensors, and the temperature difference was recorded synchronously with the optical power measurements. The acquisition of the scintillation index and the temperature difference was time-aligned, ensuring a one-to-one correspondence between turbulence strength and laser intensity fluctuation at each moment. Experiments were conducted with the Gaussian beam and the Bessel beam respectively and the scintillation indices were calculated. The scintillation index of the Gaussian beam and the temperature difference between the hot and cold plates as functions of time are plotted in Fig. 2(a) and those for the Bessel beam are plotted in Fig. 2(b). It can be seen that the scintillation indices of both Gaussian and Bessel beams vary with the change of temperature difference between the hot and cold plates. This indicates that the temperature difference between the hot and cold plates produces turbulence in the turbulent cell, and this turbulence does cause the laser beams' scin-

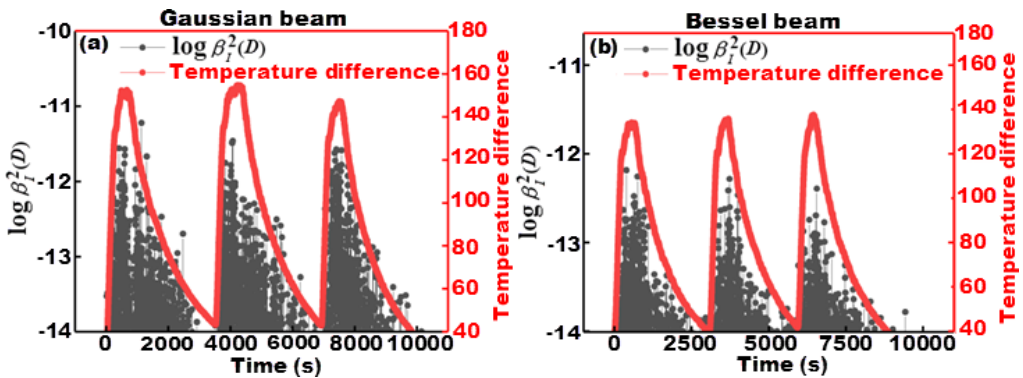


Fig. 2. Scintillation indices of laser beams and temperature difference between the hot and cold plates as functions of time. (a) Gaussian beam, (b) Bessel beam.

tillation. The greater the temperature difference, the stronger the turbulence, and the more pronounced the laser beam's scintillation becomes. However, comparing Fig. 2(a) and (b) reveals that the same temperature difference or turbulence intensity affects Gaussian beam and Bessel beam to different degrees. The scintillation index of the Gaussian beam is about an order of magnitude higher than that of the Bessel beam. For instance, when the temperature difference between the hot and cold plates is 120 °C, the $\log \beta_I^2(D)$ of the Gaussian beam is approximately -11.5 ($\beta_I^2(D) = 1.01 \times 10^{-5}$), whereas it is approximately -12.5 ($\beta_I^2(D) = 3.73 \times 10^{-6}$) for the Bessel beam. Therefore, from the perspective of laser power fluctuations caused by atmospheric turbulence, the Bessel beam exhibits stronger turbulence resistance compared to the traditional Gaussian beam.

The scintillation index of a laser beam propagating through atmospheric turbulence indicates the strength of the turbulence. But more commonly, the strength of turbulence is characterized by the atmospheric refractive index structure constant C_n^2 , which can be calculated from the scintillation index via Eq. (2) [24]. L and k in this equation represent the length of the laser propagation path and the wavenumber of the laser, respectively. For example, when the temperature difference between the hot plate and cold plates is 120 °C, $\beta_I^2(D)$ of the Gaussian beam was calculated as 1.01×10^{-5} using Eq. (1). It was then applied to Eq. (2) with the length of the turbulence cell ($L = 1.5$ m), and the atmospheric refractive index structure constant of the turbulence produced in the turbulence cell by the temperature difference of 120 °C can be determined to be $4.68 \times 10^{-12} \text{ m}^{-2/3}$. In the subsequent discussion of the impact of atmospheric turbulence on beam drift and distortion, the atmospheric refractive index structure constant – a metric that more directly reflects turbulence intensity – is employed.

$$C_n^2 = \frac{\beta_I^2(D)}{0.496k^{7/6}L^{11/6}\left[1 + 0.214(kD^2/4L)^{7/6}\right]^{-1}\left[0.44\exp(-L/1079.23) + 0.5\right]} \quad (2)$$

To investigate the beam drift and distortion caused by turbulence in laser beams after short-distance transmission, experiments were conducted using the setup shown in Fig. 3. The Gaussian beam emitted by a laser passes through a beam expander to widen its divergence before incident onto a beam splitter. The beam splitter reflects the laser beam onto a spatial light modulator (SLM FSLM-2K39-P02), which transforms the incident Gaussian beam into a Bessel beam before reflecting it back to the beam splitter. The beam splitter then directs the Bessel beam into the turbulence cell. To investigate beam drift and distortion of the Gaussian beam, the beam splitter and spatial light modulator are removed, allowing the expanded Gaussian beam to enter the turbulence cell directly. Two total reflection mirrors are positioned on opposite sides of the turbulence cell. The beam is reflected twice and passes through the turbulence cell three times, with a total transmission distance of 1.5 meters in the turbulence. The beam ultimately strikes a CCD camera (GT1920), where its instantaneous beam

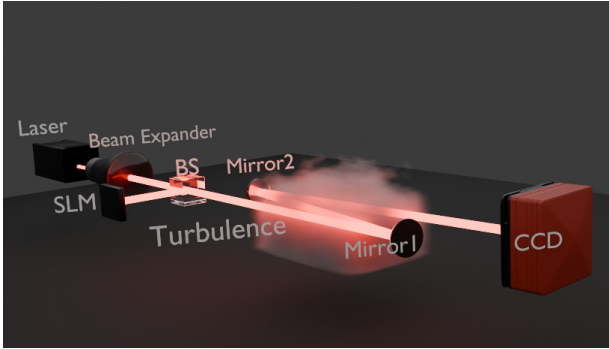


Fig. 3. Schematic diagram of laser beam propagation through short-range with strong turbulence.

spots are recorded. By employing the CCD camera instead of the optical power meter, we can quantitatively investigate the drift and distortion of the laser beams caused by turbulence.

The beam drift and distortion of both Gaussian beam and Bessel beam were first investigated qualitatively. The temperature difference between the hot and cold plates in the turbulence cell was set to 120 °C, and the instantaneous beam spot images of

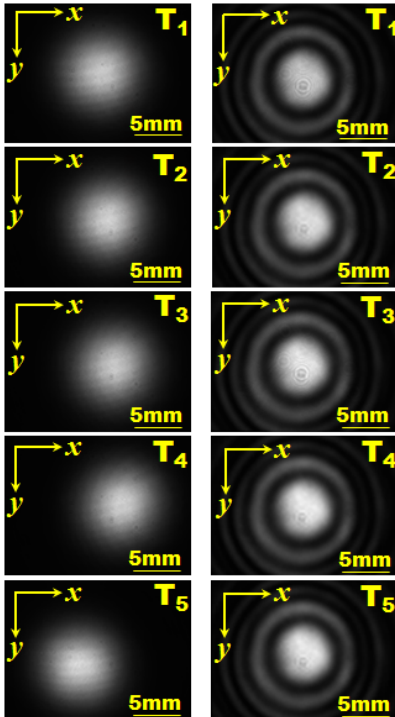


Fig. 4. Intensity distributions of Gaussian beams and Bessel beams at different moments after propagating through a short-range channel with strong turbulence.

Gaussian beam and Bessel beam were captured by the CCD camera. Based on the preceding experimental results, at a temperature difference of 120 °C, the turbulence strength in the turbulence cell is $C_n^2 = 4.68 \times 10^{-12} \text{ m}^{-2/3}$. Figure 4 shows images captured by the CCD camera of the Gaussian beam and the Bessel beam at different moments. From the intensity distribution of the images on the left, the Gaussian beam was distorted in shape, and from the positions of the spots in the images, it drifted randomly over time from a central point under the influence of strong turbulence. It can be predicted that if Gaussian beams are employed in LiDAR systems, the beam distortion and drift induced by turbulence will result in significant errors in measurement. Unlike the Gaussian beam, the Bessel beam spots in the images on the right show no significant beam distortion or drift. Therefore, compared to the Gaussian beam, the Bessel beam exhibit superior resistance to strong turbulence over short transmission distances, making it highly suitable for automotive LiDAR applications.

We further quantitatively characterize the beam distortion using the structural similarity index (SSIM) and normalized cross-correlation (NCC) as metrics. The beam spot image obtained without turbulence in the turbulence cell serves as the reference. The image of the laser beam interfered by strong turbulence in short-distance transmission are compared with the reference to obtain the SSIM and NCC. Higher SSIM and NCC values indicate greater similarity between the beam and the reference beam, signifying lower beam distortion. The SSIM and NCC for both Gaussian beam and Bessel beam were calculated and are listed in Table 1. It can be seen that the SSIM and NCC values for the Bessel beam at all five time points are significantly higher than those for the Gaussian beam. The maximum SSIM value for the Bessel beam is 0.9568, whereas it is 0.74 for the Gaussian beam subjected to turbulence of the same strength. In summary, the Bessel beam outperforms the Gaussian beam by approximately 20% in resisting beam distortion caused by strong turbulence.

T a b l e 1. SSIM and NCC for Gaussian beam and Bessel beam passing through strong turbulence over short distance.

Parameter	Beam	$C_n^2 = 4.68 \times 10^{-12} \text{ m}^{-2/3}$				
		T_1	T_2	T_3	T_4	T_5
SSIM	Gaussian	0.8688	0.8183	0.8072	0.7494	0.7986
	Bessel	0.9314	0.9423	0.9247	0.9568	0.9478
NCC	Gaussian	0.8880	0.8759	0.86695	0.8513	0.8943
	Bessel	0.9929	0.9911	0.9800	0.9762	0.9665

It can be observed from Fig. 4 that strong turbulence not only deforms the laser beam passing through it but also causes the beam drift randomly over time. To intuitively visualize the beam drift caused by turbulence, we employed image stacking technique to process images of the Gaussian beam and the Bessel beam passing through turbulences of varying intensities. The temperature difference between the hot plate and the cold plate was altered to produce turbulences of strengths of $C_n^2 = 2.24 \times 10^{-15} \text{ m}^{-2/3}$,

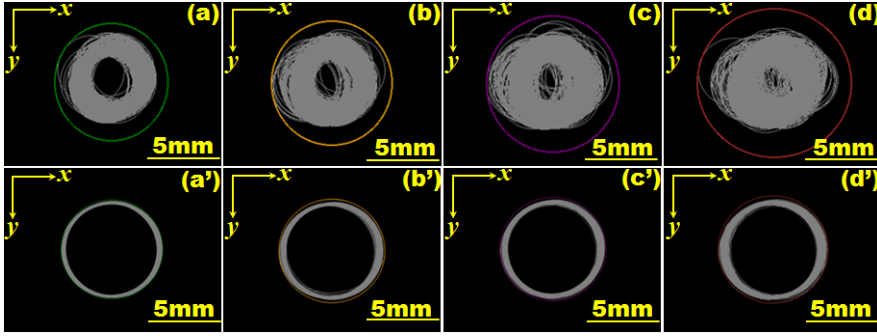


Fig. 5. Images of stacked effective beam spot circles of beams affected by turbulences of different intensities. (a)–(d) images for Gaussian beam, and (a')–(d') images for Bessel beam. (a) and (a') $C_n^2 = 2.24 \times 10^{-15} \text{ m}^{-2/3}$, (b) and (b') $C_n^2 = 2.17 \times 10^{-14} \text{ m}^{-2/3}$, (c) and (c') $C_n^2 = 1.12 \times 10^{-13} \text{ m}^{-2/3}$, (d) and (d') $C_n^2 = 4.68 \times 10^{-12} \text{ m}^{-2/3}$.

$C_n^2 = 2.17 \times 10^{-14} \text{ m}^{-2/3}$, $C_n^2 = 1.12 \times 10^{-13} \text{ m}^{-2/3}$, and $C_n^2 = 4.68 \times 10^{-12} \text{ m}^{-2/3}$, respectively. For either beam and each turbulence strength, 1,000 images were continuously captured. For each image in the group, a circle indicating the effective beam spot is drawn with the brightest pixel at the beam spot center as its center and a radius equal to the distance between the center and the pixel point whose grayscale drops to 20% of the center's grayscale, and the beam spot pattern is then removed from the image. The 1,000 images with circle patterns are stacked to form an image resembling a long-exposure photograph. Finally, another circle encircling all drifted circles and indicating the maximum drift range is drawn on the image in a different color. This circle has its center at the center of the beam shot in the image captured without turbulence in the turbulence cell, and its radius is the maximum distance from the center to the farthest point on the drifted circles. Figure 5 shows images of stacked beam circles of the Gaussian beam and the Bessel beam affected by turbulences of different intensities. Due to random beam drift caused by turbulence, the pattern of stacked beam circles turns from a single-pixel circle into an annular pattern. From the stacked images of Fig. 5(a) to (d) for the Gaussian beam, the maximum drift range increases with the intensity of turbulence, which certifies that the greater the intensity of turbulence, the larger the amplitude of beam drift it causes. Especially in the case of strong turbulence with the intensity of $C_n^2 = 4.68 \times 10^{-12} \text{ m}^{-2/3}$, the pattern of stacked circles appears as a tangled mess and the radius of maximum drift range circle is almost twice the radius of the effective beam spot circle. The patterns of stacked circles in Fig. 5(a') to (d') for the Bessel beam appear as neat annular rings, and the maximum drift range also increases with the increase in turbulence intensity. However, this increase is negligible in magnitude compared to that of the Gaussian beam. It can be concluded that the Bessel beam outperforms the Gaussian beam in resisting beam drift caused by turbulence.

4. Conclusion

In applications such as automotive LiDAR systems, laser beams may traverse intense turbulence over short transmission distances, introducing significant interference and errors into the systems. Finding a laser beam with anti-turbulence characteristic is crucial for these applications. This paper provides a qualitative and quantitative analyses of the scintillation, distortion, and drift of the Gaussian and Bessel beams after they pass through strong turbulences of short range. Compared to Gaussian beams, Bessel beams are characterized by self-healing property, a more concentrated intensity distribution, and a smaller beam divergence angle, which enable Bessel beams to better resist the interference caused by turbulence. This is confirmed by our experimental results. The scintillation index of the Bessel beam is about an order of magnitude lower than that of the Gaussian beam in strong turbulence environment. From calculated SSIM and NCC, the beam distortion of the Bessel beam are 20% lower than those of the Gaussian beam as they pass through turbulences of identical strength. The maximum range of beam drift caused by strong turbulence was analyzed, with the radius for the Gaussian beam being twice that of the Bessel beam. The Bessel beam's resistance to strong turbulence over short transmission distances makes it highly suitable for replacing traditional Gaussian beams in applications such as automotive LiDAR, thereby reducing measurement errors and enhancing system performance.

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